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Original Article

The High Energy Scattering Amplitude One-loop Gravitation in the Effective Field Theory

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Abstract: In this work, the Newton potential, together with the low-key corrective energy, an attractive loop of two large non-relativistic masses has been found. The asymptotic behavior of the scattering amplitude for two scalar particles at high energies with fixed momentum transfers was studied in the one-loop gravitation effective field theory.

Keywords: Eikonal scattering theory, one loop effective gravitation field theory.

1. Introduction

The high energy scattering amplitude for all types of interactions, including gravitational interaction, is one of the central problems of elementary particle physics. The gravitational scattering occurs at energies $\sqrt{s} = 2E = M_{PL} -$ and is described by "effective field theory" [1-5], where s is the squared energy of the center of mass, M_{PL} is the Planck mass, G is the universal gravitational constant, which is characterized by the effective coupling constant $\alpha_G = Gs / \mathrm{h} \geq 1$.

The standard method of quantum field theory is based on perturbation theory. This method is suitable when the energy of individual particles is not very high, and the effective coupling constant is not very large. When the energy is increased, the effective coupling constant also increases so that the corrections calculated by perturbation theory play a crucial role.

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Hitherto, the different approaches tackling this problem have only received the leading term of the scattering amplitude. Searching for non-leading terms has failed [6-8] even though they contain many classical and quantum effects of unknown nature [7-8]. The determination of these corrections to gravitational scattering is currently an open problem. The path-integral method with a modified perturbation theory and Logunov-Tavkhelidze quasipotential are used to give the analytic expression of the first correction [9-10].

This work arms to make a more detailed investigation of the approach, which is based on modified perturbation theory, to find the correction terms to the leading eikonal amplitude by solving the Logunov–Tavkhelidze quasi-potential equation [11-14].

The paper is organized as follows: In the second section, we briefly introduce the scattering matrix and the non-relativistic potential. Calculating the one-loop leading eikonal scattering amplitude and its first correction to leading amplitude in the effective theory of quantum gravity are constructed in the third section. In the last section, we draw our conclusion.

2. Scattering Matrix and the Non-relativistic Potential

The scattering amplitude for two scalar massive particles as a function of the momentum transfer $q^2 = (p - p')^2$ in the mixed gravity-scalar theory can be expanded as [1]:

$$M \sim \left[A + Bq^2 + ... + \alpha\kappa^4 \frac{1}{q^2} + \beta_1\kappa^4 \ln(-q^2) + \beta_2\kappa^4 \frac{m}{\sqrt{-q^2}} + ... \right] \tag{1}$$

where the coefficients $A; B; ...$ and α, β_1, β_2 depend on the particle masses m_1, m_2. The terms with $A; B; ...$ in Eq. (1) are analytical in q^2 and correspond to local interactions and the other terms with α, β_1, β_2 correspond to the non-local, long-ranged interactions, described by the nonanalytic potential.

The space parts of the non-analytical terms are performed Fourier transformation:

$$\int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\vec{r}} \frac{1}{|\vec{q}|^2} = \frac{1}{4\pi r}; \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\vec{r}} \frac{1}{|\vec{q}|} = \frac{1}{2\pi^2 r^2}; \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\vec{r}} \ln\left(\frac{|\vec{q}|^2}{\mu^2} \right) = -\frac{1}{2\pi r^3} \tag{2}$$

So clearly these terms will contribute to the longrange corrections. It should be noted that such nonanalytic pieces of the scattering amplitude are essential to the unitarity of the S matrix.

In the quantization of General Relativity, the definition of potential is certainly not obvious. The choice of potential, which includes all one-loop diagrams [3-5] seems to be the simple, gauge invariant definition of the potential. We will calculate the non-relativistic potential using the full amplitude. Here we simply relate the expectation value for the S matrix to the Fourier transformation of the potential $\tilde{V}(\vec{q})$:

$$< p_1', p_2' | S | p_1, p_2 > = -i\tilde{V}(\vec{q})(2\pi)\delta(E - E') \tag{3}$$

Where p, p' is the incoming, outgoing four-momentum, respectively, and $(E - E')$ is the energy difference between the incoming and outgoing states. Comparing this to the definition of the invariant matrix element M we get from diagrams:

$$< p_1', p_2' \left| S \right| p_1, p_2 > = (2\pi)^4 \delta^{(4)}\left(p_1 + p_2 - p_1' - p_2'\right)(iM) \tag{4}$$

We have divided the above equation with $(2m_1 2m_2)$ to obtain the non-relativistic limit $\left(q = \left(0, \vec{q}\right)\right)$.

$$\tilde{V}(\vec{q}) = -\frac{1}{2m_1}\frac{1}{2m_2}M \tag{5}$$

So that:

$$V(\vec{r}) = -\frac{1}{2m_1}\frac{1}{2m_2}\int \frac{dq^3}{(2\pi)^3} e^{i\vec{q}\vec{r}} M(\vec{q}) \tag{6}$$

This will be our definition of the non-relativistic potentialgenerated by the considered non-analytic parts, where M is the non-analytical part of the amplitude of the scattering process to a given loop order.

We evaluate all diagrams which contribute to the one loop scattering amplitude. Finally, we find the leading corrections to the nonrelativistic gravitational potential.

$$V_{Newton}(r) = -G\frac{m_1 m_2}{r}\left[1 + 3\frac{G(m_1 + m_2)}{c^2 r} + \frac{41}{10\pi}\frac{G\hbar}{c^3 r^2}\right], \tag{7}$$

Which includes the lowest-order relativistic correction, and the lowest-order quantum correction (also relativistic).

3. The Leading Eikonal Behavior and the First Correction of the Scattering Amplitude in the One-loop Effective Gravitation Field Theory

The low energy effective theory of quantized gravity is currently our most successful attempt at unifying general relativity and quantum mechanics [1-5]. In this theory, gravity is similar to other fundamental interactions, the results are true for energies below the Planck energy scale, and quantum correction effects can be calculated at current energy.

Therefore, we attempt to extend the above approach to calculating the high energy scattering amplitude of two “nucleons” for the graviton exchange based on the Newtonian potential with low-energy leading one gravitational loop corrections of two large non-relativistic masses [1].

From item 2 above, we have found the leading corrections to the nonrelativistic gravitational potential (7). It is important to note that the classical post - Newtonian term in expression (7) corresponds to the lowest-order scattering potential and agrees with Eq. (2.5) of Iwasaki [2]. The correct result for the quantum corrections was first published [1].

The Newtonian potential with low-energy leading one-loop gravitational corrections Eq. (7) can be rewritten as:

$$V_{Newton}(r,s) = C_1\frac{\kappa^2 s}{r} + C_2\frac{\kappa^4 s}{r^2} + C_3\frac{\kappa^4 s}{r^3} \tag{8}$$

Where $C_1 = \frac{1}{4.(32\pi)}; C_2 = \frac{3(m_1 + m_2)}{4c^2.(32\pi)^2}; C_3 = \frac{41\hbar}{40\pi c^3.(32\pi)^2}$.

Asymptotic behavior of the scattering amplitude at high energy $s \to \infty$ and fixed t-momentum transfer [14]:

$$\begin{aligned}
T(s,t)\Big|_{\substack{s\to\infty;\\ t-fixed}} &= -\frac{is}{2(2\pi)^3}\int d^2 r_\perp dz e^{i\vec{\Delta}_\perp \vec{r}_\perp}\left\{\exp\left[\frac{2ig}{s}\int_{-\infty}^{\infty} dzV\left(\sqrt{\vec{r}_\perp^{\,2}+z^2};s\right)\right]-1\right\} \\
&-\frac{6g^2}{(2\pi)^3 s\sqrt{s}}\int d^2 r_\perp e^{i\vec{\Delta}_\perp \vec{r}_\perp}\exp\left[\frac{2ig}{s}\int_{-\infty}^{\infty} dz'V\left(\sqrt{\vec{r}_\perp^{\,2}+z'^2};s\right)\right]\times\int_{-\infty}^{\infty} dzV^2\left(\sqrt{\vec{r}_\perp^{\,2}+z^2};s\right) \\
&-\frac{ig}{(2\pi)^3\sqrt{s}}\int d^2 r_\perp e^{i\vec{\Delta}_\perp \vec{x}_\perp}\times\int_{-\infty}^{\infty} dz\left\{\begin{array}{l}\exp\left[\frac{2ig}{s}\int_{z}^{\infty} dz'V\left(\sqrt{\vec{r}_\perp^{\,2}+z'^2};s\right)\right]- \\ -\exp\left[\frac{2ig}{s}\int_{-\infty}^{\infty} dz'V\left(\sqrt{\vec{r}_\perp^{\,2}+z'^2};s\right)\right]\end{array}\right\} \\
&\times\left\{\int_{-\infty}^{\infty} dz''V\left(\sqrt{\vec{r}_\perp^{\,2}+z''^2};s\right)-\frac{2ig}{s}\left[\int_{z}^{\infty} dz'\vec{\nabla}_\perp V\left(\sqrt{\vec{r}_\perp^{\,2}+z'^2};s\right)\right]^2\right\} \\
&-\frac{ig}{(2\pi)^3\sqrt{s}}\vec{\Delta}_\perp^2\times\int d^2 r_\perp e^{i\vec{\Delta}_\perp \vec{r}_\perp}\int_{-\infty}^{\infty} zdzV\left(\sqrt{\vec{r}_\perp^{\,2}+z^2};s\right)\exp\left[\frac{2ig}{s}\int_{z}^{\infty} dz'V\left(\sqrt{\vec{r}_\perp^{\,2}+z'^2};s\right)\right]+\ldots \\
&= T^{(0)}_{Scalar}(s,t)\Big|_{\substack{s\to\infty;\\ t-fixed}}+T^{(1)}_{Scalar}(s,t)\Big|_{\substack{s\to\infty;\\ t-fixed}}+T^{(2)}_{Scalar}(s,t)\Big|_{\substack{s\to\infty;\\ t-fixed}}+T^{(3)}_{Scalar}(s,t)\Big|_{\substack{s\to\infty;\\ t-fixed}}\ldots
\end{aligned} \tag{9}$$

In Eq. (9) the first term describes the leading eikonal behavior of the scattering amplitude, while the remaining terms determine the corrections of relative magnitude. Due to the smoothness of the potential V at high energy $s\to\infty$ the change of the particle momentum $\vec{\Delta}_\perp$, is relatively small. Therefore, the terms proportional to $\vec{\nabla}_\perp V$ and $\vec{\nabla}^2_\perp V$ in Eq. (9) can be neglected, now we have the leading eikonal scattering amplitude and the first correction term.

$$T^{(0)}_{Scalar}(s;t)\Big|_{\substack{s\to\infty\\ t-fixed}} = -\frac{is}{2(2\pi)^3}\int d^2 r_\perp e^{i\vec{\Delta}_\perp \vec{r}_\perp}\left\{\exp\left[\frac{2ig}{s}\int_{-\infty}^{\infty} dzV\left(\sqrt{\vec{r}_\perp^{\,2}+z^2};s\right)\right]-1\right\} \tag{10}$$

$$T^{(1)}_{Scalar}(s;t)\Big|_{\substack{s\to\infty\\ t-fixed}} = -\frac{6g^2}{(2\pi)^3 s\sqrt{s}}\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp \vec{r}_\perp}\exp\left[\frac{2ig}{s}\int_{-\infty}^{\infty} dzV\left(\sqrt{\vec{r}_\perp^{\,2}+z^2};s\right)\right]\int_{-\infty}^{\infty} dzV^2\left(\sqrt{\vec{r}_\perp^{\,2}+z^2};s\right) \tag{11}$$

When substituting Newtonian potential (8) into Eq. (10) mention the interaction peaks of gravitons between two nucleons, by the way changing V_{Newton} by sV_{Newton} [9], graviton still has mass μ, we have:

$$\begin{aligned}
T^{(0)}_{graviton}(s,t) &\approx \frac{\kappa}{(2\pi)^3}\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp \vec{r}_\perp}\left[\begin{array}{l} C_1\kappa^2 s\int_{-\infty}^{\infty}\frac{1}{\sqrt{r_\perp^2+z^2}}dz+ \\ +C_2\kappa^4 s\int_{-\infty}^{\infty}\frac{1}{r_\perp^2+z^2}dz+C_3\kappa^4 s\int_{-\infty}^{\infty}\frac{1}{\left(r_\perp^2+z^2\right)^{3/2}}dz\end{array}\right] \\
&+\frac{i\kappa^2}{(2\pi)^3 s}\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp \vec{r}_\perp}\left[C_1\kappa^2 s\int_{-\infty}^{\infty}\frac{1}{\sqrt{r_\perp^2+z^2}}dz\right]^2-\frac{2\kappa^3}{3(2\pi)^3 s^2}\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp \vec{r}_\perp}\left[C_1\kappa^2 s\int_{-\infty}^{\infty}\frac{1}{\sqrt{r_\perp^2+z^2}}dz\right]^3
\end{aligned} \tag{12}$$

Perform some necessary calculations, we receive the leading term of the scattering amplitude:

$$T^{(0)}_{graviton}(s,t)=\frac{\kappa^2 s}{(4\pi)^4}\left(\frac{1}{\mu^2-t}-\frac{\kappa^4}{2.(32\pi)^2}F_1(t)+\frac{2\kappa^7}{3.(16\pi)^5}F_2(t)\right)$$
$$-\frac{6(m_1+m_2)}{(32\pi)^3c^2}\frac{\kappa^5 s}{\sqrt{\mu^2-t}}+\frac{41\kappa^5 s\hbar}{80.(4\pi)^5c^3}F_2(t) \tag{13}$$

where

$$F_1(t)=\frac{1}{t\sqrt{1-\frac{4\mu^2}{t}}}\ln\left|\frac{1-\sqrt{1-4\mu^2/t}}{1+\sqrt{1-4\mu^2/t}}\right| \tag{14}$$

$$F_2(t)=\int_0^t dy\frac{1}{\left(ty+\mu^2\right)\left(y-1\right)}\ln\left|\frac{\mu^2}{t\left(ty+\mu^2-t\right)}\right| \tag{15}$$

By the same way, Eq. (11) is now available:

$$T^{(1)}_{graviton}\left(s,t\right)\Big|_{s\to\infty;t-fixed}\approx-\frac{6\kappa^2}{(2\pi)^3 s\sqrt{s}}.\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp\vec{r}_\perp}$$
$$\times\left\{\int_{-\infty}^{\infty}dz\left[\left(C_1\frac{\kappa^2 s}{\sqrt{r_\perp^2+z^2}}\right)^2+2C_1C_2\frac{\kappa^6 s^2}{\sqrt{\left(r_\perp^2+z^2\right)^3}}+2C_2C_3\frac{\kappa^6 s^2}{\left(r_\perp^2+z^2\right)^2}\right]\right\}$$
$$-\frac{12i\kappa^9\sqrt{s}C_1^3}{(2\pi)^3}\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp\vec{r}_\perp}\left[\int_{-\infty}^{\infty}\frac{1}{\sqrt{r_\perp^2+z^2}}dz\right]^3 \tag{16}$$

Perform some necessary calculations for the first term of Eq. (16), we have

$$J_1=\frac{3i\kappa^6\sqrt{s}}{(8\pi)^6}F_1(t)+\frac{9(m_1+m_2)}{4(8\pi)^3c^2}\frac{\kappa^6\sqrt{s}}{\sqrt{\mu^2-t}}+\frac{123\kappa^6\hbar\sqrt{s}}{10(4\pi)^5c^3}F_2(t) \tag{17}$$

and the second term of Eq. (16) gives the result:

$$J_2=\frac{6i\kappa^{10}\sqrt{s}}{(8\pi)^9}F_2(t) \tag{18}$$

The final result for the first correction term of the scattering amplitude has the form:

$$T^{(1)}_{graviton}\left(s,t\right)=J_1+J_2=$$
$$=\frac{3i\kappa^6\sqrt{s}}{(8\pi)^6}\left[F_1(t)+\frac{2\kappa^4}{(8\pi)^3}F_2(t)\right]+\frac{9(m_1+m_2)}{4(8\pi)^3c^2}\frac{\kappa^6\sqrt{s}}{\sqrt{\mu^2-t}}+\frac{123\kappa^6\hbar\sqrt{s}}{10(4\pi)^5c^3}F_2\left(t\right) \tag{19}$$

From the Eqs. (11) and (19) above, we see that the leading eikonal term and the first correction term of scattering amplitude have the same structure, including three small terms: (i) The first term is the scattering amplitude by exchanging gravitons, which in its non-relativistic limit will be Newton potential; (ii) The second term is the relativistic correction for the scattering amplitude [the term containing $\left(m_1+m_2\right)$]. This term corresponds to the non-analytic contribution because of exchanging gravitons. The relativistic correction term is explained as the "zitterbewegung" fluctuation when the distance between two interacting particles is shifted by one Compton wavelength [1-2]; and (iii) The last term (the term proportional to h) is quantum correction, obtained from the contribution of the one loop diagram in the high energy scattering process. The quantum correction term found in the linear gravitational field corresponds to the local interaction that is related to the analytical properties of the scattering amplitude. The Newtonian potential and its quantum corrections are related to the non-locality of the quasi-potential; non-analytic terms are also related to the non-locality of the Newtonian potential.

4. Conclusion

In the framework of the effective field theory, we obtained the expression for the scattering amplitude in Newtonian potential, taking into account the contribution of relativistic and quantum corrections from the one-loop diagram. The difference with the above case includes: Relativistic correction terms are calculated from non-analytical contributions and explained as a result of "zitterbewegung" fluctuations when the distance between particles is shifted one Compton wavelength. Quantum correction terms related to Planck's constant was also found.

The contributions to the high energy scattering amplitude are divided into analytic contributions related to the locality and non-analytic contributions associated with the non-locality. This division is associated with two ways of describing particles in quantum mechanics and relativistic quantum mechanics in that the particle has mass m. If the particle has mass, it is not possible to localization of the particle in a volume with linear dimensions less than the Compton wavelength of the corresponding particle.

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References

[1] B. Bohr, N. E. J. Donoghue, J. F. Holstein, Quantum Gravitational Corrections to the Nonrelativistic Scattering Potential of Two Masses, Physical Review D, Vol. 67, No. 8, 2003, pp. 1-12, https//doi.org/10.1103/physrevd.67.084033.

[2] Y. P. Iwasaki, Quantum Theory of Gravitation vs. Classical Theory Fourth - Order Potential, Progress of Theoretical Physics, Vol. 46, No. 5, 1971, pp. 1587-1609, https://doi.org/10.1143/PTP.46.1587.

[3] Hamber, H. W. Liu, on the Quantum Corrections to the Newtonian Potential, Physics Letters B, Vol. 357, No. 1-2, 1995, pp. 51-56, https//doi.org/10.1016/0370-2693(95)00790-r.

[4] Gribuk, T. S. Kazakov, K. A. Pronin, Gauge Dependence of the Effective Gravitational Field, Theoretical and Mathematical Physics, Vol. 141, No. 3, 2004, pp. 1654-1669, https//doi.org/10.1023/b: tamp.0000049760.331.

[5] Kazakov, K. A. Pronin, Gauge Independent Effective Gauge Fields, Nuclear Physics B, Vol. 573, No. 1-2, 2000, pp. 536-552, https//doi.org/10.1016/s0550-3213(99)00654-9.

[6] T. Hooft, on the Factorization of Universal Poles in a Theory of Gravitating Point Particles, Nuclear Physics B, Vol. 304, 1988, pp. 867-876, https//doi.org/10.1016/0550-3213(88)90659-1.

[7] D. Amati, M. Ciafaloni, G. Veneziano, Superstring Collisions at Planckian Energies, Physics Letters B, Vol. 197, No. 1-2, 1987, pp. 81-88, https//doi.org/10.1016/0370-2693(87)90346-7.

[8] D. Amati, M. Ciafaloni, G. Veneziano, Towards an S-matrix Description of Gravitational Collapse, Journal of High Energy Physics, Vol. 2008, No. 2, 2008, pp. 049-049, https//doi.org/10.1088/1126-6708/2008/02/049.

[9] N. S. Han, Straight-line Path Approximation for High Energy Elastic and Inelastic Scattering in Quantum Gravity, the European Physical Journal C, Vol. 16, No. 3, 2000, pp. 547-553, https//doi.org/10.1007/s100520000328.

[10] N. S. Han, N. N. Xuan, Planck Scattering Beyond the Eikonal Approximation in the Functional Approach, The European Physical Journal C - Particles and Fields, Vol. 24, No. 4, 2002, pp. 643-651, https//doi.org/10.1007/s10052-002-0962-6.

[11] A. A. Logunov, A. N. Tavkhelidze, Quasi-optical Approach in Quantum Field Theory, Il Nuovo Cimento, Vol. 29, No. 2, 1963, pp. 380-399, https//doi.org/10.1007/bf02750359.

[12] N. V. Hieu, R. N. Faustov, Quasi-optical Potential in Quantum Field Theory, Nuclear Physics, Vol. 53, 1964, pp. 337-344, https//doi.org/10.1016/0029-5582(64)90612-1.

[13] N. S. Han, L. T. H. Yen, N. N. Xuan, Nguyen, High Energy Scattering in the Quasi-Potential Approach. International Journal of Modern Physics A, Vol. 27, No. 1, 2012, pp. 1250004, https//doi.org/10.1142/s0217751x12500042.

[14] N. S. Han, N. N. Xuan, V. T. Thang, Applying the Modified Pertubation Theory to High Energy Scattering in the Quasipotential Approach, Journal of Physical Science and Application, Vol. 7, No. 4, 2017, pp. 47-58, https//doi.org/10.17265/2159-5348/2017.04.006.