



Original Article

Theoretical Study of the Nernst Effect in Compositional Superlattices in the Presence of Strong Electromagnetic Wave

Cao Thi Vi Ba^{1,*}, Nguyen Thu Huong²,
Tran Khuong Duy¹, Nguyen Thi Thanh Nhan¹

¹VNU University of Science, 334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam

²Air Defense Air Force Academy, Kim Son, Son Tay, Hanoi, Vietnam

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Abstract: The Nernst effect has been theoretically studied in compositional superlattice in the presence of a strong electromagnetic wave. Using the quantum kinetic equation for electrons with two cases electrons-acoustic phonons scattering and electrons-optical phonons scattering, we obtained the analytic expression of the Nernst coefficient and kinetic tensors as a function of the magnetic field, temperature, frequency, and amplitude of the electromagnetic wave and parameters of the compositional superlattice. The dependence of the Nernst coefficient on the magnetic field and temperature is achieved by numerical calculations for AlGa/AlGaAs material. The result indicates that in the case of electron-acoustic phonon scattering, the Shubnikov-de Hass oscillation appears. Whereas, in the case of electrons-optical phonons scattering, the peak of magneto-photon-phonon resonance appears. In both cases, when temperature increases, the Nernst coefficient decreases rapidly, and for electrons-optical phonons scattering, the resonance peak has a movement.

Keyword: Nernst effect, compositional superlattice, quantum kinetic equation, electron-phonon scattering, Shubnikov-de Hass oscillation.

1. Introduction

Nowadays, many modern electronic devices are made of semiconductors, so examining semiconductors is a fundamental problem. When examining the semiconductor, the researcher realized that the low dimensional semiconductor material has many special properties which is very different

* Corresponding author.

E-mail address: caothiviba@hus.edu.vn

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from bulk semiconductor material [1-26]. One special phenomenon is the Nernst effect. The Nernst effect was examined first by Albert Von Ettingshausen and his PhD student Walther Nernst when investigating Hall effect [1, 2]. The Nernst effect is a thermoelectric phenomenon observed when a sample allowing electrical conduction is subjected to a magnetic field and a temperature gradient normal (perpendicular) to each other; an electric field will be induced normal to both [1, 2]. In low-dimensional material, the difference of electron wave function and energy spectrum compared to bulk semiconductors cause the different properties [3-24]. There are some researches about Nernst effect in some low-dimensional material, f.i. the Nernst effect in cylindrical quantum wire [14]. Another low-dimensional material is compositional superlattice, which is not only interesting for basic research because of its unique structure but also provides an important application for future technology [25]. The superlattice is a multi-well structure. We can create multiple-quantum-well structures by changing the semiconductor layers order in the crystal formation process. The compositional superlattice is the material made of two different semiconductor materials with difference energy band gap differences. The Nernst effect in compositional superlattice is not examined fully so we performed this research.

2. Calculation of the Quantum Nernst Coefficient and the Kinetic Tensor in A Two-dimensional Compositional Superlattice under the Influence of an EMW

2.1. Wave Function and Energy Spectrum of the Electron in A Two-dimensional Compositional Superlattice

We are examining a two-dimensional compositional superlattice subjected to a magnetic field. $\vec{B} = (0,0,B)$ and a static electric field $\vec{E}_1 = (E_1, 0,0)$. So, the wave function and the energy spectrum of the electron are, respectively, given by [26]:

$$|\xi\rangle = |N, n, \vec{k}_y, \vec{k}_z\rangle = \frac{1}{\sqrt{L_y}} \exp(ik_y y) \phi_N(x = x_0) \otimes |n, \vec{k}_z\rangle. \quad (1)$$

$$\varepsilon_\zeta(\vec{k}_y) = \varepsilon_{N,n,k_z}(\vec{k}_y) = \left(N + \frac{1}{2}\right) \hbar \omega_H + \varepsilon_{n,\vec{k}_z} - \hbar v_d k_y + \frac{1}{2} m_e v_d^2 \quad (2)$$

Where N is the Landau level index, n denotes level quantization and N, n=0,1,2,3,..., and $k_y(k_z)$ and L_y are the wave vector and normalization length in the y(z) direction. $\phi_N(x)$ represents harmonic oscillator wave function centered $x_0 = -\frac{\hbar}{m_e \omega_H} (k_y - \frac{m_e v_d}{\hbar})$, where ω_H is the cyclotron frequency,

$v_d = \frac{E_1}{B}$, m_e is the electron's effective mass, $\varepsilon_{n,k_z} = \varepsilon_n - t_n \cos(k_z f)$, $f = f_I + f_{II}$ is the superlattice period, $\varepsilon_n = \frac{\hbar^2 \pi^2 (n+1)^2}{2m_e f_I^2}$, and t_n is the half-width of the nth mini-band given by:

$$t_n = -4(-1)^n \frac{f_I}{f_I + f_{II}} \varepsilon_n \frac{\exp(-2\sqrt{\frac{2m_e(f - f_I)^2 W}{\hbar^2}})}{\sqrt{\frac{2m_e(f - f_I)^2 W}{\hbar^2}}} \quad (3)$$

2.2. Quantum Kinetic Equation for Electron

Electron-phonon Hamiltonian under the influence of an intense EMW with the electric field vector $\vec{E} = (0, E_0 \sin \Omega t, 0)$ in a two-dimensional compositional superlattice is [26]

$$\begin{aligned}
 H = & \sum_{N,n,\vec{k}_y,\vec{k}_z} \varepsilon_{N,n,\vec{k}_y,\vec{k}_z}(\vec{k}_y - \frac{e}{\hbar c} \vec{D}(t)) a_{N,n,\vec{k}_y,\vec{k}_z}^+ a_{N,n,\vec{k}_y,\vec{k}_z} + \sum_{\vec{q}} \hbar \omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} \\
 & + \sum_{N,N',n,n',\vec{k}_y,\vec{k}_z,\vec{q}} K_{N,n,N',n'}(\vec{q}) a_{N',n',\vec{k}_y+\vec{q}_y,\vec{k}_y+\vec{q}_z}^+ a_{N,n,\vec{k}_y,\vec{k}_z} (b_{-\vec{q}}^+ + b_{\vec{q}})
 \end{aligned} \tag{4}$$

where $\hbar \omega_{\vec{q}}$ is the energy of a phonon with the wave vector $\vec{q} = (\vec{q}_\perp, q_z)$; $\vec{D}(t)$ is the vector potential of the laser field given by $-\frac{1}{c} \frac{\partial \vec{D}(t)}{\partial t} = \vec{F} \sin(\Omega t)$, with $\vec{F} = e \vec{E}_1 - \frac{\varepsilon - \varepsilon_F}{T} \Delta T$, $a_{N,n,\vec{k}_y,\vec{k}_z}^+$ and $a_{N,n,\vec{k}_y,\vec{k}_z}$ ($b_{-\vec{q}}^+$ and $b_{\vec{q}}$) being the creation and the annihilation operators of a electron (a phonon), respectively;

$$|K_{N,n,N',n'}| = |V_{\vec{q}}| |I_{n,n'}(k_z, k'_z)|^2 |J_{N,N'}(b)|^2 \tag{5}$$

With $V_{\vec{q}}$ being the electron-phonon interaction constant and

$$\begin{aligned}
 & I_{n,n'}(k_z, k'_z, q_z) \\
 = & 2^{-1} \sin \left\{ \frac{[q_z \pm (k_{n'} \pm k_n)] f_I}{2} \right\} \left\{ \frac{[q_z \pm (k_{n'} \pm k_n)] f_I}{2} \right\}^{-1} \exp \left\{ \frac{[q_z \pm (k_{n'} \pm k_n)] f_I}{2} \right\}
 \end{aligned} \tag{6}$$

Here $k_n = \left(\frac{2m_e \varepsilon_{n,k_z}}{\hbar^2} \right)^{\frac{1}{2}}$, $|J_{N,N'}(b)| = \frac{N_{min}!}{N_{max}!} e^{-u} b^{N_{max}-N_{min}} [L_{N_{min}}^{N_{max}-N_{min}}(b)]^2$, with $N_{min} = \min(N, N')$, $N_{max} = \max(N, N')$, $L_M^N(x)$ being the associated Laguerre polynomials, $b = \frac{\hbar}{m_e \omega_H} \frac{q_x^2 + q_y^2}{2}$

The quantum kinetic equation for the average number of electrons $f_{N,n,\vec{k}_y} = \langle a_{N,n,\vec{k}_y}^+ a_{N,n,\vec{k}_y} \rangle$ is:

$$\begin{aligned}
 & \frac{\partial f_{N,n,\vec{k}_y}}{\partial t} + \left(\frac{e \vec{E}_1}{\hbar} + \frac{\omega_H}{\hbar} [\vec{k}_y, \vec{h}] \right) \frac{\partial f_{N,n,\vec{k}_y}}{\partial \vec{k}_y} = \frac{2\pi}{\hbar} \sum_{N',n',\vec{q}} |K_{N,N',n,n'}(\vec{q})| \left[\sum_{s=-\infty}^{+\infty} J_s^2 \left(\frac{\lambda}{\Omega} \right) \times (2N_{\vec{q}} + \right. \\
 & 1) \{ (f_{N',n',\vec{k}_y+\vec{q}_y}(N_{\vec{q}} + 1) - f_{N,n,\vec{k}_y} N_{\vec{q}}) \delta(\varepsilon_{N',n',\vec{k}_y+\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} - \hbar \omega_{\vec{q}} - \hbar \Omega) + (f_{N',n',\vec{k}_y-\vec{q}_y} N_{\vec{q}} - \\
 & \left. f_{N,n,\vec{k}_y}(N_{\vec{q}} + 1)) \delta(\varepsilon_{N',n',\vec{k}_y-\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} + \hbar \omega_{\vec{q}} + \hbar \Omega) \right]
 \end{aligned} \tag{7}$$

2.3. Quantum Nernst Coefficient and Kinetic Tensor

In both sides of Eq. (1), multiplying $\frac{e}{m_e} \vec{k}_y \delta(\varepsilon - \varepsilon_{N,n,\vec{k}_y})$ then summing according to N, n, \vec{k}_y we obtained the equation for the specific current density $\vec{M}(\varepsilon)$

$$\frac{\vec{M}(\varepsilon)}{\tau(\varepsilon)} + \omega_H [\vec{h}, \vec{M}(\varepsilon)] = \vec{Y}(\varepsilon) + \vec{I}(\varepsilon) \tag{8}$$

where $\vec{Y}(\varepsilon) = -\frac{e}{m_e} \sum_{N,n,\vec{k}_y} \vec{k}_y \left(\vec{F}, \frac{\partial n_{N,n,\vec{k}_y,\vec{k}_z}}{\hbar \partial \vec{k}_y} \right) \delta(\varepsilon - \varepsilon_{N,n,\vec{k}_y,\vec{k}_z})$

$$\begin{aligned}
 \vec{I}(\varepsilon) = & \frac{2\pi e}{m_e \hbar} \sum_{N,n,\vec{k}_y} |K_{N,N',n,n'}|^2 N_{\vec{q}} \vec{k}_y \left\{ (f_{N',n',\vec{k}_y+\vec{q}_y} - f_{N',n',\vec{k}_y}) \times \left[\left(1 - \frac{\lambda^2}{e \Omega^2} \right) \delta(\varepsilon_{N',n',\vec{k}_y+\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} - \right. \right. \\
 & \left. \left. \hbar \omega_{\vec{q}} \right) + \frac{\lambda^2}{4 \Omega^4} \delta(\varepsilon_{N',n',\vec{k}_y+\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} - \hbar \omega_{\vec{q}} + \hbar \Omega) + \frac{\lambda^2}{4 \Omega^4} \delta(\varepsilon_{N',n',\vec{k}_y+\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} - \hbar \omega_{\vec{q}} - \hbar \Omega) \right] + \right. \\
 & \left. (f_{N',n',\vec{k}_y-\vec{q}_y} - f_{N',n',\vec{k}_y}) \left[\left(1 - \frac{\lambda^2}{e \Omega^2} \right) \delta(\varepsilon_{N',n',\vec{k}_y-\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} - \hbar \omega_{\vec{q}}) + \frac{\lambda^2}{4 \Omega^4} \delta(\varepsilon_{N',n',\vec{k}_y-\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} - \right. \right. \\
 & \left. \left. \hbar \omega_{\vec{q}} + \hbar \Omega) + \frac{\lambda^2}{4 \Omega^4} \delta(\varepsilon_{N',n',\vec{k}_y-\vec{q}_y} - \varepsilon_{N,n,\vec{k}_y} - \hbar \omega_{\vec{q}} - \hbar \Omega) \right] \right\} \delta(\varepsilon - \varepsilon_{N,n,\vec{k}_y})
 \end{aligned}$$

2.3.1. Electron-acoustic Phonon Interaction

From the specific current density expression, we obtain the total current density expression \vec{j} and the thermal flux density \vec{q}_e , respectively

$$\vec{j} = \int_0^\infty \vec{M}(\varepsilon) d\varepsilon = \sigma_{im} \vec{E}_m + \beta_{im} \nabla_m T \quad (8)$$

The quantum Nernst coefficient is given by:

$$NC = -\frac{1}{B} \frac{\sigma_{xx} \beta_{xy} - \alpha_{xy} \beta_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \quad (9)$$

where

$$\begin{aligned} \sigma_{im} = & \alpha \frac{\tau(\varepsilon_F) S}{1 + \omega_H^2 \tau(\varepsilon_F)} + \frac{[A + N] e \tau^2 (X_1 - e E_1 \bar{\Delta} x)}{m_e (1 + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x))^2} T_{1ij} \delta_{jl} T_{1lm} \\ & + L \frac{e \tau^2 (X_1 - e E_1 \bar{\Delta} x - \hbar \Omega)}{m_e (1 + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x - \hbar \Omega))^2} T_{2ij} \delta_{jl} T_{2lm} \\ & + Q \frac{e \tau^2 (X_1 - e E_1 \bar{\Delta} x + \hbar \Omega)}{m_e (1 + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x + \hbar \Omega))^2} T_{3ij} \delta_{jl} T_{3lm} \end{aligned} \quad (10)$$

$$\begin{aligned} \beta_{im} = & -(X_1 - e E_1 \bar{\Delta} x - \varepsilon_F) \frac{[A + N] e \tau^2 (X_1 - e E_1 \bar{\Delta} x)}{m_e T (1 + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x))^2} T_{1ij} \delta_{jl} T_{1lm} \\ & - L (X_1 - e E_1 \bar{\Delta} x - \hbar \Omega - \varepsilon_F) \frac{e \tau^2 (X_1 - e E_1 \bar{\Delta} x - \hbar \Omega)}{m_e T (1 + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x - \hbar \Omega))^2} T_{2ij} \delta_{jl} T_{2lm} \\ & - Q (X_1 - e E_1 \bar{\Delta} x + \hbar \Omega - \varepsilon_F) \frac{e \tau^2 (X_1 - e E_1 \bar{\Delta} x + \hbar \Omega)}{m_e T (1 + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x + \hbar \Omega))^2} T_{3ij} \delta_{jl} T_{3lm} \end{aligned} \quad (11)$$

here τ is the electron relaxation time, δ_{jl} is the Kronecker delta, ε_{ijk} is the antisymmetric Levi – Civita tensor; the Latin symbols i, j, k, m, p, l stands for the x, y, z components of Cartesian coordinates, ε_F is the Fermi level, k_B is the Boltzmann constant

$$\begin{aligned} a &= \frac{e L_y}{e \pi m_e \hbar^2 v} \left(\left(N + \frac{1}{2} \right) \hbar \omega_H + \left(n + \frac{1}{2} \right) \hbar \omega_p + \frac{1}{2} m_e v^2 \right) \\ X_1 &= (N' - N) \hbar \omega_H - \varepsilon_{n', \frac{\pi}{F}} - \varepsilon_{n, 0} \\ S &= [\delta_{ij} - \omega_H \tau(\varepsilon_F) \varepsilon_{ijk} h_k + \omega_H^2 \tau^2(\varepsilon_F) h_i h_j] \\ T_{1xy} &= [\delta_{xy} - \omega_H \tau (X_1 - e E_1 \bar{\Delta} x) \varepsilon_{xyz} h_z + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x) h_x h_y] \\ T_{2xy} &= [\delta_{xy} - \omega_H \tau (X_1 - e E_1 \bar{\Delta} x - \hbar \Omega) \varepsilon_{xyz} h_z + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x - \hbar \Omega) h_x h_y] \\ T_{3xy} &= [\delta_{xy} - \omega_H \tau (X_1 - e E_1 \bar{\Delta} x + \hbar \Omega) \varepsilon_{xyz} h_z + \omega_H^2 \tau^2 (X_1 - e E_1 \bar{\Delta} x + \hbar \Omega) h_x h_y] \\ \bar{\Delta} x &= \left(\sqrt{N + \frac{1}{2}} + \sqrt{N + 1 + \frac{1}{2}} \right) \frac{l_B}{2} \\ A &= \gamma \left(\frac{e B \bar{\Delta} x}{\hbar} \right) \left\{ 1 + 2 \sum_{s=1}^{+\infty} (-1)^s e^{-\frac{2\pi s \Gamma}{\hbar \omega_H}} \cos(2\pi s \bar{f}_1) \right\} \end{aligned}$$

$$\begin{aligned} \bar{f}_1 &= \frac{\varepsilon_{n,0} - \varepsilon_{n'} \frac{\pi}{f} + eE1\bar{\Delta x}}{\hbar\omega_H} \\ N &= -\frac{\gamma\theta}{2} \left(\frac{eB\bar{\Delta x}}{\hbar}\right)^3 \left\{1 + 2 \sum_{s=1}^{+\infty} (-1)^s e^{-\frac{2\pi s\Gamma}{\hbar\omega_H}} \cos(2\pi l\bar{f}_1)\right\} \\ L &= \frac{\gamma\theta}{4} \left(\frac{eB\bar{\Delta x}}{\hbar}\right)^3 \left\{1 + 2 \sum_{s=1}^{+\infty} (-1)^s e^{-\frac{2\pi s\Gamma}{\hbar\omega_H}} \cos(2\pi s\bar{f}_2)\right\} \\ \bar{f}_2 &= \frac{\varepsilon_{n,0} - \varepsilon_{n'} \frac{\pi}{f} + eE1\bar{\Delta x} - \hbar\Omega}{\hbar\omega_H} \\ Q &= \frac{\gamma\theta}{4} \left(\frac{eB\bar{\Delta x}}{\hbar}\right)^3 \left\{1 + 2 \sum_{s=1}^{+\infty} (-1)^s e^{-\frac{2\pi s\Gamma}{\hbar\omega_H}} \cos(2\pi s\bar{f}_3)\right\} \\ \bar{f}_3 &= \frac{\varepsilon_{n,0} - \varepsilon_{n'} \frac{\pi}{f} + eE1\bar{\Delta x} + \hbar\Omega}{\hbar\omega_H} \\ \gamma &= \frac{\zeta^2 L y^l_{n,n'}}{8\pi^3 \beta \hbar^2 \omega_H v_s l_B^2 \alpha^2} \left(\left(N + \frac{1}{2}\right) \hbar\omega_H + \left(\frac{\hbar^2 \pi^2 (n+1)^2}{2m_e f_l^2} + \right. \right. \\ &\quad \left. \left. 4(-1)^n \frac{f_l}{f-f_l} \frac{\hbar^2 \pi^2 (n+1)^2}{2m_e d_l^2} \frac{\exp\left(-\sqrt{\frac{2m_e(f-f_l)^2 W}{\hbar^2}}\right)}{\sqrt{\frac{2m_e(f-f_l)^2 W}{\hbar^2}}} \cos(k_z d) \right) + \frac{1}{2} m_e v^2 \right) \\ \theta &= \frac{e^2 E_0^2}{m^2 \Omega^4}; \Gamma = \frac{\hbar}{\tau} \end{aligned}$$

2.3.2. Electron – optical phonon interaction

$$\begin{aligned} \sigma_{im} &= \alpha \frac{\tau(\varepsilon_F)}{1 + \omega_H^2 \tau(\varepsilon_F)} Q + [g_1 + g_2] \frac{e\tau^2(C1 - eE1\bar{\Delta x})}{m_e(1 + \omega_H^2 \tau^2(C1 - eE1\bar{\Delta x}))^2} A1_{ij} \delta_{jl} A1_{lm} \\ &\quad + g_3 \frac{e\tau^2(C1 - eE1\bar{\Delta x} - \hbar\Omega) A2_{ij} \delta_{jl} A2_{lm}}{m_e(1 + \omega_H^2 \tau^2(C1 - eE1\bar{\Delta x} - \hbar\Omega))^2} + g_4 \frac{e\tau^2(C1 - eE1\bar{\Delta x} + \hbar\Omega) A3_{ij} \delta_{jl} A3_{lm}}{m_e(1 + \omega_H^2 \tau^2(C1 - eE1\bar{\Delta x} + \hbar\Omega))^2} \\ &\quad + [g_5 + g_6] \frac{e\tau^2(C2 - eE1\bar{\Delta x}) A4_{ij} \delta_{jl} A4_{lm}}{m_e(1 + \omega_H^2 \tau^2(C2 - eE1\bar{\Delta x}))^2} + g_7 \frac{e\tau^2(C2 - eE1\bar{\Delta x} - \hbar\Omega) A5_{ij} \delta_{jl} A5_{lm}}{m_e(1 + \omega_H^2 \tau^2(C2 - eE1\bar{\Delta x} - \hbar\Omega))^2} \\ &\quad + g_8 \frac{e\tau^2(C2 - eE1\bar{\Delta x} + \hbar\Omega) A6_{ij} \delta_{jl} A6_{lm}}{m_e(1 + \omega_H^2 \tau^2(C2 - eE1\bar{\Delta x} + \hbar\Omega))^2} \tag{12} \\ \beta_{im} &= -[g_1 + g_2](C1 - eE1\bar{\Delta x} - \varepsilon_F) \frac{e\tau^2(C1 - eE1\bar{\Delta x})}{m_e T(1 + \omega_H^2 \tau^2(C1 - eE1\bar{\Delta x}))^2} A1_{ij} \delta_{jl} A1_{lm} \\ &\quad - g_3(C1 - eE1\bar{\Delta x} - \hbar\Omega - \varepsilon_F) \frac{e\tau^2(C1 - eE1\bar{\Delta x} - \hbar\Omega) A2_{ij} \delta_{jl} A2_{lm}}{m_e T(1 + \omega_H^2 \tau^2(C1 - eE1\bar{\Delta x} - \hbar\Omega))^2} \end{aligned}$$

$$\begin{aligned}
& -g_4(C_1 - eE_1\overline{\Delta x} + \hbar\Omega - \varepsilon_F) \frac{e\tau^2(C_1 - eE_1\overline{\Delta x} + \hbar\Omega)A3_{ij}\delta_{jl}A3_{lm}}{m_e T(1 + \omega_H^2\tau^2(C_1 - eE_1\overline{\Delta x} + \hbar\Omega))^2} \\
& -[g_5 + g_6](C_2 - eE_1\overline{\Delta x} - \varepsilon_F) \frac{e\tau^2(C_2 - eE_1\overline{\Delta x})A4_{ij}\delta_{jl}A4_{lm}}{m_e T(1 + \omega_H^2\tau^2(C_2 - eE_1\overline{\Delta x}))^2} \\
& -g_7(C_2 - eE_1\overline{\Delta x} - \hbar\Omega - \varepsilon_F) \frac{e\tau^2(C_2 - eE_1\overline{\Delta x} - \hbar\Omega)A5_{ij}\delta_{jl}A5_{lm}}{m_e T(1 + \omega_H^2\tau^2(C_2 - eE_1\overline{\Delta x} - \hbar\Omega))^2} \\
& -g_8(C_2 - eE_1\overline{\Delta x} + \hbar\Omega - \varepsilon_F) \frac{e\tau^2(C_2 - eE_1\overline{\Delta x} + \hbar\Omega)A6_{ij}\delta_{jl}A6_{lm}}{m_e T(1 + \omega_H^2\tau^2(C_2 - eE_1\overline{\Delta x} + \hbar\Omega))^2} \quad (13)
\end{aligned}$$

$$Q = [\delta_{ij} - \omega_H\tau(\varepsilon_F)\varepsilon_{ijk}h_k + \omega_H^2\tau^2(\varepsilon_F)h_i h_j]$$

$$A1_{xy} = [\delta_{xy} - \omega_H\tau(C_1 - eE_1\overline{\Delta x})\varepsilon_{xyz}h_z + \omega_H^2\tau^2(C_1 - eE_1\overline{\Delta x})h_x h_y]$$

$$A2_{xy} = [\delta_{xy} - \omega_H\tau(C_1 - eE_1\overline{\Delta x} - \hbar\Omega)\varepsilon_{xyz}h_z + \omega_H^2\tau^2(C_1 - eE_1\overline{\Delta x} - \hbar\Omega)h_x h_y]$$

$$A3_{xy} = [\delta_{xy} - \omega_H\tau(C_1 - eE_1\overline{\Delta x} + \hbar\Omega)\varepsilon_{xyz}h_z + \omega_H^2\tau^2(C_1 - eE_1\overline{\Delta x} + \hbar\Omega)h_x h_y]$$

$$A4_{xy} = [\delta_{xy} - \omega_H\tau(C_2 - eE_1\overline{\Delta x})\varepsilon_{xyz}h_z + \omega_H^2\tau^2(C_2 - eE_1\overline{\Delta x})h_x h_y]$$

$$A5_{xy} = [\delta_{xy} - \omega_H\tau(C_2 - eE_1\overline{\Delta x} - \hbar\Omega)\varepsilon_{xyz}h_z + \omega_H^2\tau^2(C_2 - eE_1\overline{\Delta x} - \hbar\Omega)h_x h_y]$$

$$A6_{xy} = [\delta_{xy} - \omega_H\tau(C_2 - eE_1\overline{\Delta x} + \hbar\Omega)\varepsilon_{xyz}h_z + \omega_H^2\tau^2(C_2 - eE_1\overline{\Delta x} + \hbar\Omega)h_x h_y]$$

$$C_1 = (N' - N)\hbar\omega_H - \varepsilon_{n', \frac{\pi}{f}} - \varepsilon_{n,0} - \hbar\omega_0$$

$$C_2 = (N' - N)\hbar\omega_H - \varepsilon_{n', \frac{\pi}{f}} - \varepsilon_{n,0} + \hbar\omega_0$$

$$g_1 = \left(\frac{eB\overline{\Delta x}}{M\hbar}\right) e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_1); M = |N - N'| = 1, 2, 3, \dots$$

$$T_1 = (N' - N)\hbar\omega_H - \varepsilon_{n', \frac{\pi}{f}} - \varepsilon_{n,0} - \hbar\omega_0 - eE_1\overline{\Delta x}$$

$$g_2 = -\frac{\theta}{2} \left(\frac{eB\overline{\Delta x}}{\hbar}\right)^2 e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_1)$$

$$g_3 = \frac{\theta}{4M} \left(\frac{eB\overline{\Delta x}}{\hbar}\right)^3 e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_1 + \hbar\Omega)$$

$$g_4 = \frac{\theta}{4M} \left(\frac{eB\overline{\Delta x}}{\hbar}\right)^3 e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_1 - \hbar\Omega)$$

$$T_2 = (N' - N)\hbar\omega_H - \varepsilon_{n', \frac{\pi}{f}} - \varepsilon_{n,0} - \hbar\omega_H + eE_1\overline{\Delta x}$$

$$g_5 = \frac{1}{M} \left(\frac{eB\overline{\Delta x}}{\hbar}\right) e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_2)$$

$$g_6 = -\frac{\theta}{2} \left(\frac{eB\overline{\Delta x}}{\hbar}\right)^2 e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_2)$$

$$g_7 = \frac{\theta}{4M} \left(\frac{eB\overline{\Delta x}}{\hbar}\right)^3 e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_2 + \hbar\Omega)$$

$$g_8 = \frac{\theta}{4M} \left(\frac{eB\overline{\Delta x}}{\hbar}\right)^3 e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \left[\frac{(N+M)!}{N!}\right]^2 \delta(T_2 - \hbar\Omega)$$

3. Numerical Results and Discussion for the Two-dimensional Compositional Superlattice GaAs/AlGaAs

In this section, we present the numerical calculations of the Nernst coefficient on temperature and magnetic field for the Compositional superlattice of GaAs/AlGaAs with a parameter used as $m^* = 0.067 m_0$ (m_0 is the rest mass of electron), $n_0 = 3.10^{22} m^{-3}$, $X_\infty = 10.9$, $X_0 = 12.9$, $v_s = 6560 ms^{-1}$, $L_x = L_y = 100 nm$, $E_0 = 10^5 V.m^{-1}$, $E_1 = 10^5 V.m^{-1}$, $\Omega = 10^{14} Hz$, $\Delta Eg = 0.057 eV$

3.1. Electron-acoustic Phonon Interaction

Figure 1 indicates the dependence of Nernst coefficient on the magnetic field with different temperatures. In general, the chart fluctuates strongly from 1T to 5T, and the amplitude of fluctuation decreases significantly. The amplitude of fluctuation falls slowly to 0 from 5T to 15T. We could also see that when the temperature increases, the Nernst coefficient decreases.

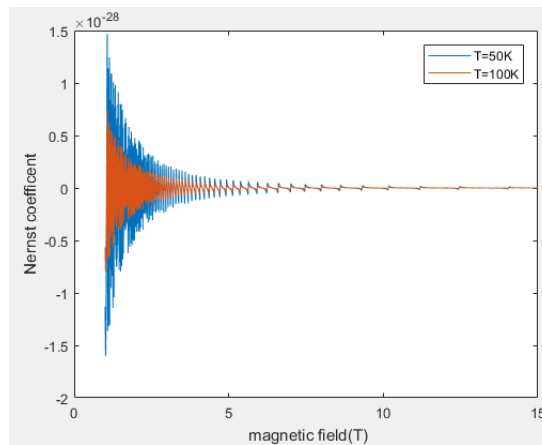


Figure 1. Dependence of Nernst coefficient on the magnetic field for different values of temperature.

According to Figure 2, the Nernst coefficient decreases sharply to 0 when the temperature increase.

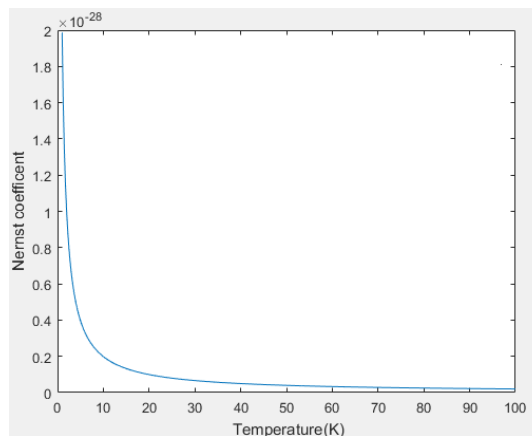


Figure 2. Dependence of Nernst coefficient on temperature.

3.2. Electron – optical Phonon Interaction

Figure 3 indicates the dependence of the Nernst coefficient on the magnetic field with different temperature. Over-all, we can see that the Nernst coefficient reaches a peak of around 2.7T at 50K, and when we decrease temperature, we obtain a higher peak.

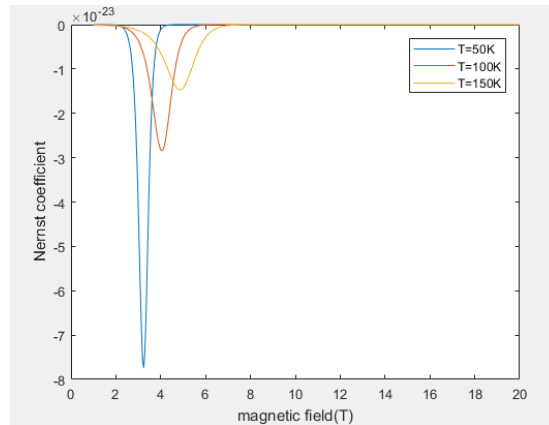


Figure 3. The Dependence of Nernst coefficient on magnetic field for different values of temperature.

Figure 4 shows that when temperature increases, the Nernst coefficient increases sharply and peaks at 180K. After that, the Nernst coefficient decreases and maintain the value from 1000K to higher.

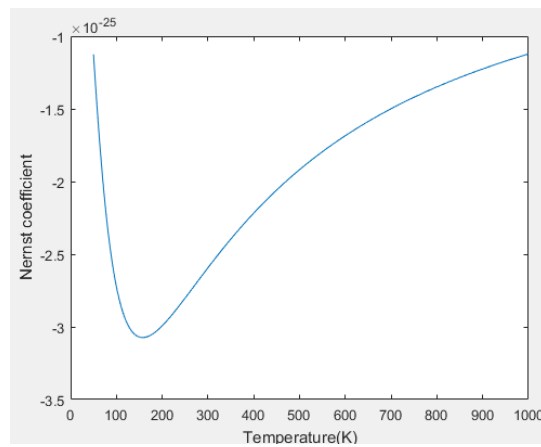


Figure 4. The dependence of the Nernst coefficient on temperature.

4. Conclusion

In summary, we studied the Nernst effect in two-dimensional compositional superlattice using the quantum kinetic equation. We obtain the kinetic tensor for two cases: electron-optical phonon interaction and electron-acoustic phonon interaction and the numerical result for both cases. We examined the dependence of Nernst coefficient on magnetic field and temperature. The result indicates that in first case (electron-acoustic phonon interaction), the Shubnikov-de Hass oscillation appears and the second

case (electron-optical phonon), the photon – phonon resonance peak appears. The result also indicates that the Nernst coefficient depends on elements such as the temperature and magnetic field.

References

- [1] A. V. Ettingshausen, W. Nernst, *Annalen Der Physik*, Vol. 265, 1886, pp. 343-347, <https://doi.org/10.1002/andp.18862651010>.
- [2] E. Sondheimer, *Proceedings of the Royal Society of London Series A, Mathematical and Physical Sciences*, Vol. 193, 1948, pp. 484-512, <https://doi.org/10.1098/rspa.1948.0058>.
- [3] B. V. Paranjape, J. S. Levinger, Theory of the Ettingshausen Effect in Semiconductors, *Physical Review*, Vol. 120, No. 2, 1960, pp. 437, <https://doi.org/10.1103/PhysRev.120.437>.
- [4] Z. Zhu, H. Yang, B. Fauque, Y. Kopelevich, K. Behnia, Nernst Effect and Dimensionality in the Quantum Limit, *Nature Physics*, Vol. 6, 2010, pp. 26-29, <https://doi.org/10.1038/nphys1437>.
- [5] I. Lyapilin, The Nernst Spin Effect in a Two-Dimensional Electron Gas, *Low Temperature Physics*, 39, 2013, pp. 957–960, <https://doi.org/10.1063/1.4830264>.
- [6] S. Figarova, H. Huseynov, V. Figarov, Transverse Nernst–Ettingshausen Effect in Superlattices Upon Electron-Phonon Scattering, *Semiconductors*, Vol. 52, 2018, pp. 853-858, <https://doi.org/10.1134/S1063782618070084>.
- [7] H. Y. Zhou, S. W. Gu, Effect of Magnetopolaron in Cylindrical Quantum Wires, *Solid State Communications*, Vol. 88, 1993, pp. 291-294.
- [8] Y. B. Yu, S. N. Zhu, K. X. Guo, Electron-Phonon Interaction Effect on Optical Absorption in Cylindrical Quantum Wires, *Solid State Communications*, Vol. 139, 2006, pp. 76-79, <https://doi.org/10.1016/j.ssc.2006.04.009>.
- [9] H. V. Ngoc, N. Q. Bau, D. M. Quang, H. T. Hung, One-Dimensional Cylindrical Quantum Wire: The Theoretical Study of Photo-Stimulated Ettingshausen Effect, *International Journal of Modern Physics B*, Vol. 36, No. 01, 2020, pp. 2250009, <https://doi.org/10.1142/S0217979222500096>.
- [10] N. Q. Bau, D. T. Long, Influence of Confined Optical Phonons and Laser Radiation on the Hall Effect in A Compositional Superlattice, *Physica B: Condensed Matter*, Vol. 532, 2018, pp. 149-154, <https://doi.org/10.1016/j.physb.2017.09.127>.
- [11] N.Q. Bau, D.T. Long, Impact of Confined LO-phonons on the Hall Effect in Doped Semiconductor Superlattices, *Journal of Science: Advanced Materials and Devices*, Vol. 1, 2016, pp. 209-213, <https://doi.org/10.1016/j.jsamd.2016.06.010>.
- [12] C. R. Bennett, K. Gu'ven, B. Tanatar, Confined-phonon Effects in the Band-Gap Renormalization of Semiconductor Quantum Wires, *Physical Review B*, Vol. 57, No. 7, 1998, pp. 3994, <https://doi.org/10.1103/PhysRevB.57.3994>.
- [13] N. Q. Bau, N. T. L. Quynh, C. T. V. Ba, L. T. Hung, Doped Two-dimensional Semiconductor Superlattice: Photo-stimulated Quantum Thermo-magnetoelectric Effects under the Influence of a Confined Phonon, *Journal of the Korean Physical Society*, Vol. 77, No. 12, 2020, pp. 1224-1232, <https://doi.org/10.3938/jkps.77.1224>.
- [14] N. Q. Bau, T. T. Dien, N. T. N. Anh, N. D. Nam, Influence of Confined Optical Phonons on the Photo-stimulated Thermo-magnetoelectric Effect in Cylindrical Quantum Wires, *Physica B: Condensed Matter*, Vol. 644, 2022, pp. 414220, <https://doi.org/10.1016/j.physb.2022.414220>.
- [15] É. Épshtein, Theory of Magnetoresistive and Thermomagnetic Effects with Electron Scattering by Optical Phonons in Semiconductors, *Soviet Physics Journal*, Vol. 19, 1976, pp. 226-230, <https://doi.org/10.1007/BF00942873>.
- [16] S. V. Branis, G. Li, K. K. Bajaj, Hydrogenic Impurities in Quantum Wires in the Presence of a Magnetic Field, *Physical Review B*, Vol. 47, Iss. 3, 1993, pp. 1316-1323, <https://journals.aps.org/prb/issues/47/3>.
- [17] M. Abramowitz, I. A. Stegun, *Handbook of Mathematical Functions*, NBS (now NIST), 1965, pp. 537-554.
- [18] N. Q. Bau, T. C. Phong, Parametric Resonance of Acoustic and Optical Phonons in a Quantum Well, *Journal of the Korean Physical Society*, Vol. 42, Iss. 5, 2003, pp. 647-651, <https://www.jkps.or.kr/journal/view.html?uid=5315&vmd=Full> (accessed on: June 1st, 2023).

- [19] N. C. Constantinou, B. K. Ridley, Interaction of Electrons with the Confined LO Phonons of a Free-standing GaAs Quantum Wire, *Physical Review B*, Vol. 41, 1990, pp. 10622-10626, <https://doi.org/10.1103/PhysRevB.41.10622>.
- [20] X. F. Wang, X. L. Lei, Polar-optic Phonons and High-field Electron Transport in Cylindrical GaAs/AlAs Quantum Wires, *Physical Review B*, Vol. 49, 1994, pp. 4780-4789, <https://doi.org/10.1103/PhysRevB.49.4780>.
- [21] A. Zou, H. Xie, Effects of Confined LO and SO Phonon Modes on Polaron in Freestanding Cylindrical Quantum Wire with Parabolic Confinement, *Modern Physics Letters B*, Vol. 23, 2009, pp. 3515-3523, <https://doi.org/10.1142/S0217984909021570>.
- [22] H. D. Trien, N. Q. Bau, The Nonlinear Absorption Coefficient of Strong Electromagnetic Waves caused by Electrons Confined in Quantum Wires, *Journal of the Korean Physical Society*, Vol. 56, 2010, pp. 120-127, <https://doi.org/10.3938/jkps.56.120>.
- [23] W. R. Frank, A. O. Govorov, J. P. Kotthaus, C. Steinebach, V. Gudmundsson, W. Hansen, M. Holland, *Phys. Rev. B*, Vol. 55, 1997, pp. 6719-6722, <https://doi.org/10.1103/PhysRevB.55.R1950>.
- [24] J.O. Ryu, R. O'connell, Magnetophonon Resonances in Quasi-one-dimensional Quantum Wires, *Phys Rev B*, Vol. 48, 1993, pp. 9126-9129, <https://doi.org/10.1103/PhysRevB.48.9126>.
- [25] H. V. Phuc, L. T. M. Hue, L. Dinh, T. C. Phong, LO-phonon-assisted Cyclotron Resonance linewidth Via Multiphoton Absorption Process in Cylindrical Quantum Wire, Superlattices and Microstructures, Vol. 60, 2013, pp. 508-515, <https://doi.org/10.1016/j.spmi.2013.05.038>.
- [26] N. Q. Bau, D. T. Hang, D. T. Long, Study of the Quantum Magneto-Thermoelectric Effect in the Two-Dimensional Compositional Superlattice GaAs/AlGaAs under the Influence of an Electromagnetic Wave by Using the Quantum Kinetic Equation, *Journal of the Korean Physical Society*, Vol. 75, No. 12, 2019, pp. 1004-1016, <https://doi.org/10.3938/jkps.75.1004>.