



Original Article

Scalar Sector Interactions in BDW Model

Tran Minh Hieu^{1,*}, Dinh Quang Sang²¹Hanoi University of Science and Technology, 1 Dai Co Viet, Hanoi, Vietnam²VNU University of Science, 334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam

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Abstract: In the Bélanger-Delaunay-Westhoff model, vectorlike fermions and a new scalar sector charged under an extra Abelian gauge symmetry were introduced to explain the flavor observables, and the muon $g - 2$. In this work, we focussed our study to the scalar sector of this model and the relevant interactions. The interaction terms including the exotic Yukawa, the gauge interactions, and the scalar 3-point and 4-point interactions are derived together with the corresponding Feynman rules. It is shown that the new scalar ϕ_r can decay into a pair of standard-model-like Higgs bosons. This channel is considered by analyzing the decay width and its parameter dependence.

Keywords: Standard model extension, new physics, scalar particle, fundamental interaction, abelian gauge symmetry.

1. Introduction

The standard model (SM) have been the most successful theoretical model for elementary particles and their interactions. However, since the experiments became more precise, we have observed many deviations from the theoretical predictions. For example, the muon anomalous magnetic moment is measured to be different from the SM calculation using the dispersion method at the level of about 5σ [1]. The discrepancies in the flavor observables like $BR(B^+ \rightarrow K^+ \mu^+ \mu^-)$, $BR(B^+ \rightarrow K^+ e^+ e^-)$, and $BR(B_s^0 \rightarrow \phi \mu^+ \mu^-)$ are about 3σ [2]. On the other hand, there are the theoretical problems (such as the gauge hierarchy and the strong CP problems) and cosmological observations (such as the existence of dark matter and dark energy, the baryon asymmetry) that can not be explained by the SM itself. Therefore, the SM is just an effective theory, and a better understanding of the new physics is necessary.

In the model proposed by Bélanger, Delaunay, and Westhoff (BDW) [3, 4], additional vectorlike fermions and scalars charged under an extra Abelian gauge symmetry $U(1)_X$ are introduced. The mass

* Corresponding author.

E-mail address: hieu.tranminh@hust.edu.vn<https://doi.org/10.25073/2588-1124/vnumap.4907>

spectrum of new particles and the gauge interactions of fermions were studied [5]. It was shown [6, 7] that this model is able to resolve the tensions in the flavor observables, the muon $g - 2$, and the CKM anomaly [8-14].

In this work, the scalar sector of the model is investigated. We explicitly derive the interaction terms involving scalar particles such as the exotic Yukawa, the gauge interactions, and the scalar 3-point and 4-point interactions. The Feynman rules corresponding to these terms are obtained. By analyzing these vertices, we show that one of the new scalars has an interesting decay channel where the final production is a pair of the SM-like Higgs boson. The decay width of process is calculated as well as its parameter dependence.

This paper is organized as follows. In Section 2, we briefly describe the model including the particle content and the symmetry. In Section 3, we focus on the scalar particles and derive their interaction terms. In Section 4, we investigate the decay channel of a new scalar to a pair of the 125 GeV Higgs bosons. Finally, the conclusion is presented in Section 5.

2. Model Description

In the considered model, vectorlike pairs of leptons and quarks, and two complex scalars are introduced beside the SM particles. While the new scalars are $SU(2)_L$ singlets, the vectorlike fermions transform as fundamental representations of the $SU(2)_L$ gauge group:

$$L_{L,R} = \begin{pmatrix} N_{L,R} \\ E_{L,R} \end{pmatrix}, \quad Q_{L,R} = \begin{pmatrix} U_{L,R} \\ D_{L,R} \end{pmatrix}. \quad (1)$$

In contrast to the SM particles, these new particles are charged under an extra Abelian gauge symmetry $U(1)_X$. The properties of the new particles are summarized in Table 1.

Table 1. New particles and their properties

Particles	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
L_L, L_R	1	2	$-1/2$	1
Q_L, Q_R	3	2	$1/6$	-2
χ	1	1	0	-1
ϕ	1	1	0	2

The Lagrangian of the model includes two separated parts corresponding to the SM and the new physics:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{NP}. \quad (2)$$

The new physics Lagrangian consists of the exotic Yukawa interactions, the mass term of vectorlike fermions, the interactions between the SM-like Higgs and the new scalars, and the potential of $\{\chi, \phi\}$:

$$\begin{aligned} \mathcal{L}_{NP} \supset & -[y\bar{\ell}_L L_R \chi + w\bar{q}_L Q_R \phi + M_L \bar{L}_L L_R + M_Q \bar{Q}_L Q_R + h.c.] \\ & -\lambda_{\phi H} |\phi|^2 |H|^2 - \lambda_{\chi H} |\chi|^2 |H|^2 - V_0(\chi, \phi). \end{aligned} \quad (3)$$

Here, $\ell_L^i = \begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}_i$ and $q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_i$ with $i = \{1, 2, 3\}$ are the doublets of the SM left-handed leptons and quarks. The scalar potential V_0 reads

$$V_0(\chi, \phi) = \lambda_\phi |\phi|^4 + m_\phi^2 |\phi|^2 + \lambda_\chi |\chi|^4 + m_\chi^2 |\chi|^2 + \lambda_{\chi\phi} |\phi|^2 |\chi|^2 + (r\phi\chi^2 + h.c.). \quad (4)$$

After the SM-like Higgs field gets a vacuum expectation value (VEV), $\langle H \rangle$, the mass terms in Eq. (4) will be modified due to contributions of the first two terms in the second line of Eq. (3):

$$\lambda_{\phi H} |\phi|^2 \langle H \rangle^2 + \lambda_{\chi H} |\chi|^2 \langle H \rangle^2.$$

The new potential of ϕ and χ can be expressed as

$$V(\chi, \phi) = \lambda_\phi |\phi|^4 + m_\phi'^2 |\phi|^2 + \lambda_\chi |\chi|^4 + m_\chi'^2 |\chi|^2 + \lambda_{\chi\phi} |\phi|^2 |\chi|^2 + (r\phi\chi^2 + h.c.), \quad (5)$$

where $m_\phi'^2 = m_\phi^2 + \lambda_{\phi H} \langle H \rangle^2$, and $m_\chi'^2 = m_\chi^2 + \lambda_{\chi H} \langle H \rangle^2$.

For simplicity, we assume that $m_\phi'^2 < 0$ and $m_\chi'^2 > 0$. Therefore, the $U(1)_X$ symmetry is broken by the nonzero VEV of ϕ :

$$\langle \phi \rangle = \sqrt{-\frac{m_\phi'^2}{2\lambda_\phi}}.$$

We redefine the field ϕ as

$$\phi = \langle \phi \rangle + \frac{1}{\sqrt{2}} (\varphi_r + i \varphi_i).$$

Substituting this into Eq. (5), we obtain the masses of the real and imaginary components:

$$m_{\varphi_r} = 2\sqrt{\lambda_\phi \langle \phi \rangle}, \quad m_{\varphi_i} = 0.$$

The massive field φ_r is the physical state, while the Nambu-Goldstone field φ_i is absorbed by the $U(1)_X$ gauge field to be its longitudinal component in the unitary gauge.

After the $U(1)_X$ symmetry is broken, the mass term of χ is shifted again due to $\langle \phi \rangle$, resulting in an induced potential of χ as

$$V_\chi(\chi) = \lambda_\chi |\chi|^4 + m_\chi''^2 |\chi|^2 + (r\langle \phi \rangle \chi^2 + h.c.),$$

where $m_\chi''^2 = m_\chi'^2 + \lambda_{\chi\phi} \langle \phi \rangle^2$. By decomposing the field χ as $\chi = \frac{1}{\sqrt{2}} (\chi_r + i \chi_i)$, we find the mass matrix of (χ_r, χ_i) :

$$M_\chi^2 = \begin{pmatrix} m_\chi''^2 + (r + r^*) \langle \phi \rangle & i(r - r^*) \langle \phi \rangle \\ i(r - r^*) \langle \phi \rangle & m_\chi''^2 - (r + r^*) \langle \phi \rangle \end{pmatrix}.$$

In the case where r is real, the squared masses of χ_r and χ_i are:

$$m_{\chi_r}^2 = m_\chi''^2 + 2r \langle \phi \rangle,$$

$$m_{\chi_i}^2 = m_\chi''^2 - 2r \langle \phi \rangle.$$

3. Interactions of New Scalars

The physical states of new scalars in this model include φ_r , χ_r , and χ_i . In this section, we derive all the interactions terms relevant to these particles.

3.1. Exotic Yukawa Interactions

Let us first consider the exotic Yukawa interactions terms in Eq. (3)

$$-\mathcal{L}_{NP}^{Yukawa} \supset y \bar{\ell}_L L_R \chi + w \bar{q}_L Q_R \phi + h.c.$$

$$\begin{aligned}
&= y\bar{\ell}_L L_R \frac{1}{\sqrt{2}}(\chi_r + i\chi_i) + w\bar{q}_L Q_R \left[\langle\phi\rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right] + h.c. \\
&\supset \frac{y}{\sqrt{2}}\bar{\ell}_L L_R(\chi_r + i\chi_i) + \frac{w}{\sqrt{2}}\bar{q}_L Q_R \varphi_r + h.c.
\end{aligned}$$

Here, the unitary gauge is used so that the φ_i is absorbed by rephasing the vectorlike quark Q_R .

3.2. Gauge Interactions of Scalars

The gauge interactions of scalars can be derived from their kinetic terms

$$\begin{aligned}
\mathcal{L}_{NP}^{kinetic} &\supset (D_\mu\phi)^\dagger D^\mu\phi + (D_\mu\chi)^\dagger D^\mu\chi \\
&= \left\{ (\partial^\mu - ig_X X_\phi Z'^\mu) \left[\langle\phi\rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right] \right\}^\dagger (\partial^\mu - ig_X X_\phi Z'^\mu) \left[\langle\phi\rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right] \\
&\quad + \left[(\partial^\mu - ig_X X_\chi Z'^\mu) \frac{1}{\sqrt{2}}(\chi_r + i\chi_i) \right]^\dagger (\partial^\mu - ig_X X_\chi Z'^\mu) \frac{1}{\sqrt{2}}(\chi_r + i\chi_i) \\
&= \left| \partial^\mu \left(\langle\phi\rangle + \frac{1}{\sqrt{2}}\varphi_r \right) + \frac{1}{\sqrt{2}}g_X X_\phi Z'^\mu \varphi_i + i \left[\frac{1}{\sqrt{2}}\partial^\mu \varphi_i - g_X X_\phi Z'^\mu \left(\langle\phi\rangle + \frac{1}{\sqrt{2}}\varphi_r \right) \right] \right|^2 \\
&\quad + \frac{1}{2} \left| \partial^\mu \chi_r + g_X X_\chi Z'^\mu \chi_i + i(\partial^\mu \chi_i - g_X X_\chi Z'^\mu \chi_r) \right|^2 \\
&= \left(\frac{1}{\sqrt{2}}\partial^\mu \varphi_r + \frac{1}{\sqrt{2}}g_X X_\phi Z'^\mu \varphi_i \right)^2 + \left[\frac{1}{\sqrt{2}}\partial^\mu \varphi_i - g_X X_\phi Z'^\mu \left(\langle\phi\rangle + \frac{1}{\sqrt{2}}\varphi_r \right) \right]^2 + \frac{1}{2}(\partial^\mu \chi_r + g_X X_\chi Z'^\mu \chi_i)^2 \\
&\quad + \frac{1}{2}(\partial^\mu \chi_i - g_X X_\chi Z'^\mu \chi_r)^2 \\
&\supset \left(g_X X_\phi Z'^\mu \frac{1}{\sqrt{2}}\varphi_r \right)^2 + 2(g_X X_\phi Z'^\mu)^2 \langle\phi\rangle \frac{1}{\sqrt{2}}\varphi_r + g_X X_\chi Z'^\mu \chi_i \partial^\mu \chi_r + \frac{1}{2}(g_X X_\chi Z'^\mu \chi_i)^2 - g_X X_\chi Z'^\mu \chi_r \partial^\mu \chi_i \\
&\quad + \frac{1}{2}(g_X X_\chi Z'^\mu \chi_r)^2 \\
&= \frac{1}{2}g_X^2 X_\phi^2 Z'^\mu Z'^\mu \varphi_r^2 + \sqrt{2}\langle\phi\rangle g_X^2 X_\phi^2 Z'^\mu Z'^\mu \varphi_r + \frac{1}{2}g_X^2 X_\chi^2 Z'^\mu Z'^\mu \chi_r^2 + \frac{1}{2}g_X^2 X_\chi^2 Z'^\mu Z'^\mu \chi_i^2 \\
&\quad - g_X X_\chi Z'^\mu (\chi_r \partial^\mu \chi_i - \chi_i \partial^\mu \chi_r)
\end{aligned}$$

Since $X_\phi = 2$ and $X_\chi = -1$, we have the gauge interaction terms of new physical scalars:

$$\begin{aligned}
\mathcal{L}_{gauge}^{scalar} &\supset 2g_X^2 Z'^\mu Z'^\mu \varphi_r^2 + 4\sqrt{2}\langle\phi\rangle g_X^2 Z'^\mu Z'^\mu \varphi_r + \frac{1}{2}g_X^2 Z'^\mu Z'^\mu \chi_r^2 + \frac{1}{2}g_X^2 Z'^\mu Z'^\mu \chi_i^2 \\
&\quad + g_X Z'^\mu (\chi_r \partial^\mu \chi_i - \chi_i \partial^\mu \chi_r)
\end{aligned}$$

3.2. Scalar Interactions

The scalar interactions are derived from the scalar potential:

$$\begin{aligned}
V(H, \phi, \chi) &\supset \lambda_{\phi H} |\phi|^2 |H|^2 + \lambda_{\chi H} |\chi|^2 |H|^2 + V_0(\chi, \phi) \\
&\supset \{\lambda_{\phi H} |\phi|^2 + \lambda_{\chi H} |\chi|^2\} (\langle H \rangle + h)^2 + V_0(\chi, \phi) \\
&= \{\lambda_{\phi H} |\phi|^2 + \lambda_{\chi H} |\chi|^2\} (\langle H \rangle^2 + 2\langle H \rangle h + h^2) + V_0(\chi, \phi) \\
&= \{\lambda_{\phi H} |\phi|^2 + \lambda_{\chi H} |\chi|^2\} (2\langle H \rangle h + h^2) + V(\chi, \phi) \\
&= \{\lambda_{\phi H} |\phi|^2 + \lambda_{\chi H} |\chi|^2\} (2\langle H \rangle h + h^2) + \lambda_\phi |\phi|^4 + m_\phi'^2 |\phi|^2 + \lambda_\chi |\chi|^4 + m_\chi'^2 |\chi|^2 + \lambda_{\chi\phi} |\phi|^2 |\chi|^2 \\
&\quad + (r\phi\chi^2 + h.c.)
\end{aligned}$$

$$\begin{aligned}
 &= \left\{ \lambda_{\phi H} \left| \langle \phi \rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right|^2 + \lambda_{\chi H} |\chi|^2 \right\} (2\langle H \rangle h + h^2) + \lambda_{\phi} \left| \langle \phi \rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right|^4 \\
 &\quad + m'_{\phi}{}^2 \left| \langle \phi \rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right|^2 + \lambda_{\chi} |\chi|^4 + m'_{\chi}{}^2 |\chi|^2 + \lambda_{\chi\phi} \left| \langle \phi \rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right|^2 |\chi|^2 \\
 &\quad + \left\{ r \left[\langle \phi \rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right] \chi^2 + h.c. \right\} \\
 &= \left\{ \lambda_{\phi H} \left[\left(\langle \phi \rangle + \frac{\varphi_r}{\sqrt{2}} \right)^2 + \frac{\varphi_i^2}{2} \right] + \lambda_{\chi H} |\chi|^2 \right\} (2\langle H \rangle h + h^2) + \lambda_{\phi} \left[\left(\langle \phi \rangle + \frac{\varphi_r}{\sqrt{2}} \right)^2 + \frac{\varphi_i^2}{2} \right]^2 \\
 &\quad + m'_{\phi}{}^2 \left[\left(\langle \phi \rangle + \frac{\varphi_r}{\sqrt{2}} \right)^2 + \frac{\varphi_i^2}{2} \right] + \lambda_{\chi} |\chi|^4 + m'_{\chi}{}^2 |\chi|^2 + \lambda_{\chi\phi} \left[\left(\langle \phi \rangle + \frac{\varphi_r}{\sqrt{2}} \right)^2 + \frac{\varphi_i^2}{2} \right] |\chi|^2 \\
 &\quad + \left\{ r \left[\langle \phi \rangle + \frac{1}{\sqrt{2}}(\varphi_r + i\varphi_i) \right] \chi^2 + h.c. \right\} \\
 &\supset \left\{ \lambda_{\phi H} \left(\sqrt{2} \langle \phi \rangle \varphi_r + \frac{\varphi_r^2}{2} \right) + \lambda_{\chi H} |\chi|^2 \right\} (2\langle H \rangle h + h^2) + \lambda_{\phi} \left(\langle \phi \rangle^2 + \sqrt{2} \langle \phi \rangle \varphi_r + \frac{\varphi_r^2}{2} \right)^2 \\
 &\quad + m'_{\phi}{}^2 \left(\langle \phi \rangle^2 + \sqrt{2} \langle \phi \rangle \varphi_r + \frac{\varphi_r^2}{2} \right) + \lambda_{\chi} |\chi|^4 + m'_{\chi}{}^2 |\chi|^2 + \lambda_{\chi\phi} \left(\langle \phi \rangle^2 + \sqrt{2} \langle \phi \rangle \varphi_r + \frac{\varphi_r^2}{2} \right) |\chi|^2 \\
 &\quad + \left\{ r \left(\langle \phi \rangle + \frac{\varphi_r}{\sqrt{2}} \right) \chi^2 + h.c. \right\} \\
 &\supset \lambda_{\phi H} \left(\sqrt{2} \langle \phi \rangle \varphi_r h^2 + \langle H \rangle \varphi_r^2 h + \frac{1}{2} \varphi_r^2 h^2 \right) + \lambda_{\chi H} |\chi|^2 (2\langle H \rangle h + h^2) + \lambda_{\phi} \left(\frac{\varphi_r^4}{4} + \sqrt{2} \langle \phi \rangle \varphi_r^3 \right) + \lambda_{\chi} |\chi|^4 \\
 &\quad + m'_{\chi}{}^2 |\chi|^2 + \lambda_{\chi\phi} \left(\sqrt{2} \langle \phi \rangle \varphi_r + \frac{\varphi_r^2}{2} \right) |\chi|^2 + \left\{ r \frac{\varphi_r}{\sqrt{2}} \chi^2 + h.c. \right\}
 \end{aligned}$$

Here, $m''_{\chi}{}^2 = m'_{\chi}{}^2 + \lambda_{\chi\phi} \langle \phi \rangle^2$. In the above expression, we have used the correlation between $\langle \phi \rangle$, λ_{ϕ} , and $m'_{\phi}{}^2$. Using the expansion of χ , we have:

$$\begin{aligned}
 V(H, \phi, \chi) &\supset \lambda_{\phi H} \left(\sqrt{2} \langle \phi \rangle \varphi_r h^2 + \langle H \rangle \varphi_r^2 h + \frac{1}{2} \varphi_r^2 h^2 \right) + \lambda_{\phi} \left(\frac{\varphi_r^4}{4} + \sqrt{2} \langle \phi \rangle \varphi_r^3 \right) \\
 &\quad + \lambda_{\chi H} \left| \frac{1}{\sqrt{2}}(\chi_r + i\chi_i) \right|^2 (2\langle H \rangle h + h^2) + \lambda_{\chi} \left| \frac{1}{\sqrt{2}}(\chi_r + i\chi_i) \right|^4 + m''_{\chi}{}^2 \left| \frac{1}{\sqrt{2}}(\chi_r + i\chi_i) \right|^2 \\
 &\quad + \lambda_{\chi\phi} \left(\sqrt{2} \langle \phi \rangle \varphi_r + \frac{\varphi_r^2}{2} \right) \left| \frac{1}{\sqrt{2}}(\chi_r + i\chi_i) \right|^2 + \left\{ r \frac{\varphi_r}{\sqrt{2}} \left[\frac{1}{\sqrt{2}}(\chi_r + i\chi_i) \right]^2 + h.c. \right\} \\
 &\supset \lambda_{\phi H} \left(\sqrt{2} \langle \phi \rangle \varphi_r h^2 + \langle H \rangle \varphi_r^2 h + \frac{1}{2} \varphi_r^2 h^2 \right) + \lambda_{\phi} \left(\frac{\varphi_r^4}{4} + \sqrt{2} \langle \phi \rangle \varphi_r^3 \right) + \frac{\lambda_{\chi H}}{2} (\chi_r^2 + \chi_i^2) (2\langle H \rangle h + h^2) \\
 &\quad + \frac{\lambda_{\chi}}{4} (\chi_r^2 + \chi_i^2)^2 + \frac{m''_{\chi}}{2} (\chi_r^2 + \chi_i^2) + \lambda_{\chi\phi} \left(\sqrt{2} \langle \phi \rangle \varphi_r + \frac{\varphi_r^2}{2} \right) \frac{(\chi_r^2 + \chi_i^2)}{2} \\
 &\quad + \left\{ r \frac{\varphi_r}{\sqrt{2}} (\chi_r^2 - \chi_i^2) \right\} \\
 &\supset \lambda_{\phi H} \left(\sqrt{2} \langle \phi \rangle \varphi_r h^2 + \langle H \rangle \varphi_r^2 h + \frac{1}{2} \varphi_r^2 h^2 \right) + \lambda_{\phi} \left(\frac{\varphi_r^4}{4} + \sqrt{2} \langle \phi \rangle \varphi_r^3 \right) + \frac{\lambda_{\chi H}}{2} (\chi_r^2 + \chi_i^2) (2\langle H \rangle h + h^2) \\
 &\quad + \frac{\lambda_{\chi}}{4} (\chi_r^4 + \chi_i^4 + 2\chi_r^2 \chi_i^2) + \frac{1}{\sqrt{2}} (\lambda_{\chi\phi} \langle \phi \rangle + r) \varphi_r \chi_r^2 + \frac{\lambda_{\chi\phi}}{4} (\varphi_r^2 \chi_r^2 + \varphi_r^2 \chi_i^2) \\
 &\quad + \frac{1}{\sqrt{2}} (\lambda_{\chi\phi} \langle \phi \rangle - r) \varphi_r \chi_i^2.
 \end{aligned}$$

From the above expression, we obtain the new scalar 3-point interaction terms as

$$\begin{aligned}
 -\mathcal{L}_{3\text{-point}}^{\text{scalar}} \supset & \sqrt{2}\lambda_\phi\langle\phi\rangle\varphi_r^3 + \sqrt{2}\lambda_{\phi H}\langle\phi\rangle\varphi_r h^2 + \lambda_{\phi H}\langle H\rangle\varphi_r^2 h + \lambda_{\chi H}\langle H\rangle(\chi_r^2 h + \chi_i^2 h) \\
 & + \frac{1}{\sqrt{2}}(\lambda_{\chi\phi}\langle\phi\rangle + r)\varphi_r\chi_r^2 + \frac{1}{\sqrt{2}}(\lambda_{\chi\phi}\langle\phi\rangle - r)\varphi_r\chi_i^2,
 \end{aligned}$$

and the new scalar 4-point interaction terms as

$$\begin{aligned}
 -\mathcal{L}_{4\text{-point}}^{\text{scalar}} \supset & \frac{\lambda_{\phi H}}{2}\varphi_r^2 h^2 + \frac{\lambda_\phi}{4}\varphi_r^4 + \frac{\lambda_{\chi H}}{2}(\chi_r^2 h^2 + \chi_i^2 h^2) + \frac{\lambda_\chi}{4}(\chi_r^4 + \chi_i^4 + 2\chi_r^2\chi_i^2) \\
 & + \frac{\lambda_{\chi\phi}}{4}(\varphi_r^2\chi_r^2 + \varphi_r^2\chi_i^2).
 \end{aligned}$$

In Table 2, we summary the result of our derivation for the interactions terms involving new scalars. The types of interactions and the vertices are shown in the first and the second columns. The corresponding Feynman rules that are useful for studying the physics of these particles are extracted and given in the third column of this table.

Table 2. Interactions of new scalars and their Feynman rules

Interaction types	Vertices	Feynman rules
Exotic Yukawa interactions	$\{\chi_r, \nu_L, N_R\}, \{\chi_r, e_L, E_R\}$	$-i\frac{y}{\sqrt{2}}$
	$\{\chi_i, \nu_L, N_R\}, \{\chi_i, e_L, E_R\}$	$\frac{y}{\sqrt{2}}$
	$\{\varphi_r, u_L, U_R\}, \{\varphi_r, d_L, D_R\}$	$-i\frac{w}{\sqrt{2}}$
Gauge interactions	$\{Z', Z', \varphi_r\}$	$4\sqrt{2}i\langle\phi\rangle g_X^2$
	$\{Z', Z', \varphi_r, \varphi_r\}$	$2ig_X^2$
	$\{Z', \chi_r, \chi_i\}$	$-g_X(p_i - p_r)^\mu$
	$\{Z', Z', \chi_r, \chi_r\},$ $\{Z', Z', \chi_i, \chi_i\}$	$\frac{i}{2}g_X^2$
Scalar 3-point interactions	$\{\varphi_r, \varphi_r, \varphi_r\}$	$-i\sqrt{2}\lambda_\phi\langle\phi\rangle$
	$\{\varphi_r, h, h\}$	$-i\sqrt{2}\lambda_{\phi H}\langle\phi\rangle$
	$\{\varphi_r, \varphi_r, h\}$	$-i\lambda_{\phi H}\langle H\rangle$
	$\{\chi_r, \chi_r, h\}, \{\chi_i, \chi_i, h\}$	$-i\lambda_{\chi H}\langle H\rangle$
	$\{\chi_r, \chi_r, \varphi_r\}$	$\frac{-i}{\sqrt{2}}(\lambda_{\chi\phi}\langle\phi\rangle + r)$
	$\{\chi_i, \chi_i, \varphi_r\}$	$\frac{-i}{\sqrt{2}}(\lambda_{\chi\phi}\langle\phi\rangle - r)$
Scalar 4-point interactions	$\{\varphi_r, \varphi_r, h, h\}$	$-i\frac{\lambda_{\phi H}}{2}$
	$\{\varphi_r, \varphi_r, \varphi_r, \varphi_r\}$	$-i\frac{\lambda_\phi}{4}$
	$\{\chi_r, \chi_r, h, h\}, \{\chi_i, \chi_i, h, h\}$	$-i\frac{\lambda_{\chi H}}{2}$
	$\{\chi_r, \chi_r, \chi_r, \chi_r\}, \{\chi_i, \chi_i, \chi_i, \chi_i\}$	$-i\frac{\lambda_\chi}{4}$
	$\{\chi_r, \chi_r, \chi_i, \chi_i\}$	$-i\frac{\lambda_\chi}{2}$
	$\{\chi_r, \chi_r, \varphi_r, \varphi_r\}, \{\chi_i, \chi_i, \varphi_r, \varphi_r\}$	$-i\frac{\lambda_{\chi\phi}}{4}$

4. Decay of New Scalar Into A Pair of SM-like Higgs Bosons

From the above scalar 3-point interactions, we observe that the new scalar (χ_r, χ_i) always appear in pair since the original field (χ) does not break the $U(1)_X$ symmetry. The situation for the other new scalar (φ_r) is different¹. Therefore, once the particle φ_r is produced, it can decay into a pair of the SM-like Higgs bosons via the vertex $\{\varphi_r, h, h\}$:

$$\mathcal{L}_{3\text{-point}}^{scalar} \supset -\sqrt{2}\lambda_{\phi H}\langle\phi\rangle\varphi_r h^2.$$

The condition for this decay channel is that φ_r must be at least twice heavier than h , namely $m_{\varphi_r} \geq 2m_h$. The rate of this decay process was calculated, it is equal:

$$\Gamma(\varphi_r \rightarrow hh) = \frac{\lambda_{\phi H}^2 \langle\phi\rangle^2}{4\pi m_{\varphi_r}^2} \sqrt{\frac{m_{\varphi_r}^2}{4} - m_h^2},$$

where m_{φ_r} is the mass of φ_r , and $m_h = 125.25$ GeV is the SM-like Higgs boson mass.

In the numerical analysis, we choose $\langle\phi\rangle = 177$ GeV and $m_{\varphi_r} = 612$ GeV that were shown to comply with constraints from other experiment measurements [6]. The dependence of the decay width on the parameter $\lambda_{\phi H}$ is depicted in Fig. 1. Since we use the logarithmic scale on both horizontal and vertical axes of this figure, the quadratic dependence is shown as a straight line. From this plot, we observe that when $\lambda_{\phi H} \sim \mathcal{O}(1)$, the decay rate become $\mathcal{O}(1)$ GeV, implying that this decay channel becomes very efficient.

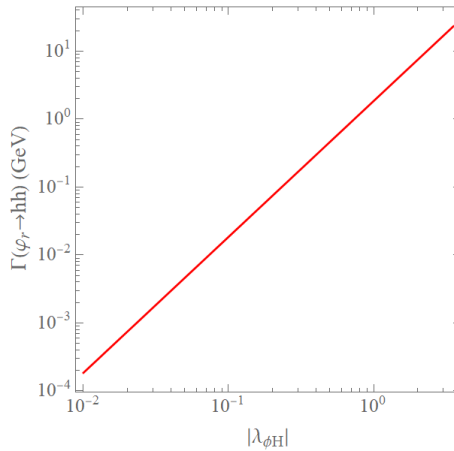


Figure 1. The decay width of φ_r into a pair of SM-like Higgs bosons as a function of $\lambda_{\phi H}$.

5. Conclusion

In summary, we have considered a new physics model where a scalar sector and vectorlike fermions are introduced beside the SM particle content. These new particles are charged under an extra $U(1)_X$ symmetry while the SM particles are neutral. We have derived explicitly all the interaction terms involving the scalar particles including the exotic Yukawa, the gauge interactions, the scalar 3-point and

¹ Note that the vertex $\{\varphi_r, \varphi_r, h\}$ does not lead to a two-body decay of φ_r at the tree level since the process $\varphi_r \rightarrow \varphi_r + h$ is kinematically forbidden.

4-point interactions. The obtained results are important contributions in understanding the physics relevant to the scalar sector. By analyzing the scalar 3-point interaction, we have shown that, once the particle φ_r is produced, it can decay into a pair of the SM-like Higgs bosons, namely $\varphi_r \rightarrow hh$. The decay width of this process has been calculated analytically. For the numerical analysis, the dependence of the decay width on the parameter $\lambda_{\phi H}$ is obtained, allowing one to determine the parameter region where this decay channel becomes efficient.

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