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Original Article

Radio-electric Effect in Semi-parabolic Plus Semi-inverse Squared QuantumWells

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Abstract: Radioelectric field in semi-parabolicplus semi-inverse squared quantum wells has been studied in the presence of a linearly polarized electromagnetic wave. By using quantum kinetic equations for electrons in the case of electrons – optical phonons scattering, the analytical expression for the Radioelectricfield was obtained as a function of the frequency, the amplitude of the linearly polarized electromagnetic field and temperature. Numerical results of specific GaAs/GaAlAs semi-parabolicplus semi-inverse squared quantum wells were also achieved. The results showed that when temperature increases, the Radioelectric field increases nonlinearly.

Keywords: Radio-electric Effect, Semi-parabolic plus Semi-inverse Squared QuantumWells.

1. Introduction

Recently, the study of the low-dimensional semiconductor systems (quantum wells, doping superlattices, compositional superlattices, quantum wires, quantum dots, etc.) has been increasingly interested [1-5]. The kinetic properties of semiconductors, such as the absorption coefficient of electromagnetic waves, the acoustoelectric effect, the Hall effect and the radio-electric effect are studied by quantum kinetic equation [6-9]. The radio-electric effect and the effect of drag of charge carriers by electromagnetic waves have been studied for quantum well [10]. in this work we study radio-electric effect for semi-parabolic plus semi-inverse squared quantum well. Semi-parabolic plus semi-inverse squared quantum well is a quantum well which has a formula [11]:

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$$V(z)=\begin{cases}\infty & when\ \ z\le 0\\ \dfrac{m\omega_z^2 z^2}{2}+\dfrac{\hbar^2\beta}{2mz^2} & when\ z>0\end{cases}$$

where:

$m=0.067m_e$ is effective mass of electron;

ω_z is the frequency of the confined potential energy;

β is the parameter of the quantum well.

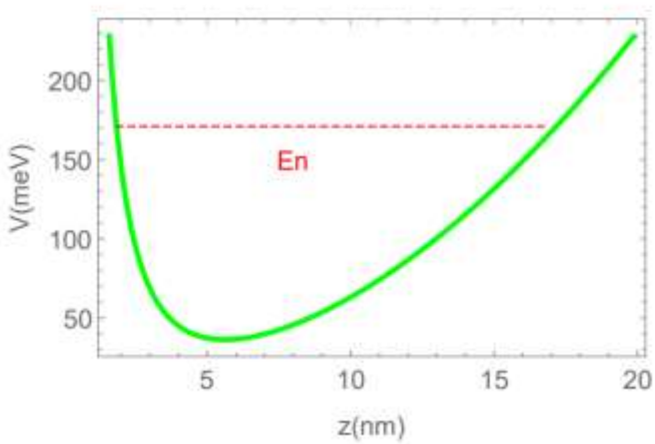

Figure 1. Semi-parabolic Plus Semi-inverse Squared quantum well with $\beta=1$.

As known frm literature, there are some methods to solve problems of kinetic properties, such as the Boltzmann equation, the Kubo – Mori method and the quantum kinetic equation method. The last approach is chosen to use in this work because of its simplicity and accuracy. The article is organized as follows, we briefly describe the radio-electric effect, the Hamiltonian of electrons in semi-parabolic plus semi-inverse squared quantum well and present basic formulae for the calculation in Section 2. Numerical results and discussion are given in Section 3. The final section shows conclusions and remarks.

2. The Radio-electric Field in Semi-parabolic Plus Semi-inverse Squared Quantum Well Under the Influence of Optical Phonons

We examine a simple model of semi-parabolic plus semi-inverse squared quantum well subjected to a linearly polarized electromagnetic field: $\vec{E}(t)=\vec{E}\left(e^{-i\omega t}+e^{i\omega t}\right)$, $\vec{H}(t)=\vec{H}\left(e^{-i\omega t}+e^{i\omega t}\right)$, $\vec{H}\perp\vec{E}$ ($\hbar\omega=\bar{\varepsilon}$, ω is a frequency of the linearly polarized electromagnetic field, $\bar{\varepsilon}$ is an average carrier energy, $\omega\tau\gg 1$, τ is the characteristic relaxation time).

The Hamiltonian of electrons - optical phonons system can be expressed as:

$$H = \sum_{n,\vec{k}} \varepsilon_{n,\vec{k}} a^+_{n,\vec{k}} a_{n,\vec{k}} + \sum_{\vec{q}} \hbar\omega_0 b^+_{\vec{q}} b_{\vec{q}} + \sum_{n,\vec{k}} \sum_{n',\vec{q}} C_{\vec{q}} I_{n,n'} a^+_{n',\vec{k}+\vec{q}} a_{n,\vec{k}} \left(b^+_{-\vec{q}} + b_{\vec{q}}\right) \quad (1)$$

where $a^+_{n,\vec{k}}$ and $a_{n,\vec{k}}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (phonon) respectively; $\varepsilon_{n,\vec{k}} = \left(2n+1+\frac{\sqrt{1+4\beta}}{2}\right)\hbar\omega_z + \frac{\hbar^2 k^2}{2m}$ is the electron energy in semi-parabolic plus semi-inverse squared quantum well; $\hbar\omega_0$ is the optical phonon energy; $\left|C_{\vec{q}}\right|^2 = \frac{2\pi e^2 \hbar\omega_0}{\varepsilon_0 V}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\frac{1}{q^2}$ is the electron-optical phonon interaction constant, V is the normalized volume, ε_0 is the permittivity, χ_0 is the staticdielectric constant, χ_∞ is the high frequency dielectric constant, ω_0 is the frequency of an optical phonon, q is a phonon wave-vector,

$$I_{n,n'} = \alpha_z^{1+4s}\left[\frac{3}{2}+2s\right]\left(HF1\left[\frac{1}{2}+2s,\ \frac{1}{2}, -\frac{q^2\alpha_z^2}{4}\right] - HF1\left[\frac{3}{2}+2s,\ \frac{1}{2},\ -\frac{q^2\alpha_z^2}{4}\right]\right)$$

is electron form factor in semi-parabolic plus semi-inverse squared quantum well, $HF1(x,y,z)$ is the Hypergeometric function at *x*, *y* and *z*, $\alpha_z = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$, $s = \frac{1}{4}\left(1+\sqrt{1+4\beta}\right)$.

Using the Heisenberg equation of motion, we establish the quantum kinetic equation for electrons distribution function $f_{n,\vec{k}}(t) = a^+_{n,\vec{k}} a_{n,\vec{k}}$ in an electric field E_0 and a linearly polarized electromagnetic field *E(t)*:

$$\frac{\partial f_{n,\vec{k}}(t)}{\partial t} + \left(e\vec{E}(t) + e\vec{E}_0 + \hbar\omega_c\left[\vec{k},\vec{h}(t)\right],\ \frac{1}{\hbar}\frac{\partial f_{n,\vec{k}}(t)}{\partial \vec{k}}\right) =$$

$$= \sum_{n',\vec{q}} \frac{2\pi}{\hbar^2}\left|C_{\vec{q}}\right|^2 \left|I_{n,n'}\right|^2 N_{\vec{q}}\left[f_{n',\vec{k}+\vec{q}}(t) - f_{n,\vec{k}}(t)\right]\left[\delta\left(\varepsilon_{n',\vec{k}+\vec{q}} - \varepsilon_{n,\vec{k}} - \hbar\omega_{\vec{q}}\right) + \delta\left(\varepsilon_{n',\vec{k}+\vec{q}} - \varepsilon_{n,\vec{k}} + \hbar\omega_{\vec{q}}\right)\right] \quad (2)$$

here $\vec{h}(t) = \frac{\vec{H}(t)}{H}$, $\omega_c = \frac{eH}{m}$ is the cyclotron frequency. The electrons distribution function is:

$$f_{n,\vec{k}}(t) = f_0\left(\varepsilon_{n,\vec{k}}\right) + \hat{k}\,\vec{\chi}(t)\frac{\partial f_0\left(\varepsilon_{n,\vec{k}}\right)}{\partial \varepsilon_{n,\vec{k}}} \quad (3)$$

where $\vec{\chi}_{n,\vec{k}}(t) = \vec{\chi}_0\left(\varepsilon_{n,\vec{k}}\right) + \vec{\chi}\left(\varepsilon_{n,\vec{k}}\right)e^{-i\omega t} + \vec{\chi}^*\left(\varepsilon_{n,\vec{k}}\right)e^{i\omega t}$ is the equilibrium electron distribution function, $\hat{k}$ is the unit vector point in the direction of $\vec{k}$.

By substituting (3) into (2) and one can obtain:

$$\vec{\chi}_0\left(\varepsilon_{n,\vec{k}}\right) = e\vec{E}_0\frac{\hbar}{m}\tau\left(\varepsilon_{n,\vec{k}}\right)$$

$$\vec{\chi}\left(\varepsilon_{n,\vec{k}}\right) = e\vec{E}\frac{\hbar}{m}\frac{\tau\left(\varepsilon_{n,\vec{k}}\right)}{1-i\omega\tau\left(\varepsilon_{n,\vec{k}}\right)}$$

$$\vec{\chi}^*\left(\varepsilon_{n,\vec{k}}\right) = e\vec{E}\frac{\hbar}{m}\frac{\tau\left(\varepsilon_{n,\vec{k}}\right)}{1+i\omega\tau\left(\varepsilon_{n,\vec{k}}\right)}$$

The total current density is presented in the following form:

$$\vec{J}_{tot} = \int_0^{+\infty}\vec{R}(\varepsilon)d\varepsilon \tag{4}$$

in which

$$\vec{R}(\varepsilon) = \sum_{n,\vec{k}}\frac{e\hbar}{m}\vec{k}f_{n,\vec{k}}(t)\delta\left(\varepsilon-\varepsilon_{n,\vec{k}}\right) \tag{5}$$

Multiplying both sides of the equation (2) by $\frac{e\hbar}{m}\vec{k}\delta(\varepsilon-\varepsilon_{n,\vec{k}})$ and sum over $n, \vec{k}$. Then, putting the root $\vec{R}(\varepsilon)$ of the resulting equation into (4) and assuming the sample in all direction is opened, regarding the electron gas is completely degenerate $f_0(\varepsilon_{n,\vec{k}}) = \theta(\varepsilon_F - \varepsilon_{n,\vec{k}})$; ε_F is Fermi energy level. We obtain the expression of the Radio-electric field after some manipulation:

$$E_{0y} = \frac{u}{r+s_0}\left(E_x h_z - E_z h_x\right) \tag{6}$$

where:

E_x (E_z) is a *x*-component (*z*-component) of the linearly polarized electromagnetic field,

$$u = \sum_n \frac{2e^2\omega_0}{\pi\hbar^2\omega^2}e^{\frac{\varepsilon_F}{k_BT}}\left\{k_BT - \varepsilon_n + \frac{\gamma^2}{k_bT}\left[\varepsilon_n I_1(\gamma,\mu)\right] - I_2(\gamma,\mu)\right\} \tag{7}$$

$$r = \sum_n \frac{e^2\tau(\varepsilon_F)k_BT(2k_BT-\varepsilon_n)}{\pi\hbar^3}e^{\frac{\varepsilon_F}{k_BT}} \tag{8}$$

$$s_0 = \sum_{\alpha=\pm1}\sum_{\beta=0,\pm1}\left(2-|\beta|\right)(-1)^\beta\sum_{n,n'}\frac{e^6\omega_0E^2}{4m\varepsilon_0\hbar^4\omega^4k_BT}\frac{\tau^2(\varepsilon_F)}{\varepsilon_F^2}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right)$$
$$\times\frac{1}{e^{\frac{\hbar\omega_0}{k_BT}}-1}\left|I_{n,n'}\right|^2 2\left(k_BT\right)\left(3k_BT-\varepsilon_n\right)e^{\frac{\varepsilon_F}{k_BT}} \tag{9}$$

$$\gamma = \frac{\varepsilon_F}{\omega\tau(\varepsilon_F)},\ \mu = \frac{1}{k_BT}$$

$$I_1(\gamma,\mu) = \frac{1}{\gamma}\left[ci(\gamma\mu)\sin(\gamma\mu) - \sin(\gamma\mu)\cos(\gamma\mu) + \frac{\pi}{2}\cos(\gamma\mu)\right]$$

$$I_2(\gamma,\mu) = -ci(\gamma\mu)\cos(\gamma\mu) - \sin(\gamma\mu)\sin(\gamma\mu) + \frac{\pi}{2}\sin(\gamma\mu),$$

where $\sin(x)$ is the sine integral function at *x*, $ci(x)$ is the cosine integral function at *x*, k_B is a Boltzmann constant.

From the expression of the radio-electric field in Eq. (6), it is shown that the radio-electric field depends on the following parameters of the system: the frequency (ω) of the linearly polarized electromagnetic wave and the temperature (T).

3. Numerical Results and Discussions

In order to clarify the influence of optical phonons, we estimate numerical values of the radio-electric field and graph in cases of unconfined phonons. The parameters used in this calculation are as follows:

$m = 0.067m_e$ (m_e is the mass of a free electron),

$\chi_\infty = 10.9, \chi_0 = 12.9, \omega_0 = 5 \times 10^{13}\ Hz, \tau(\varepsilon_F) = 10^{-12}$s,

$\omega \approx \omega_c$.

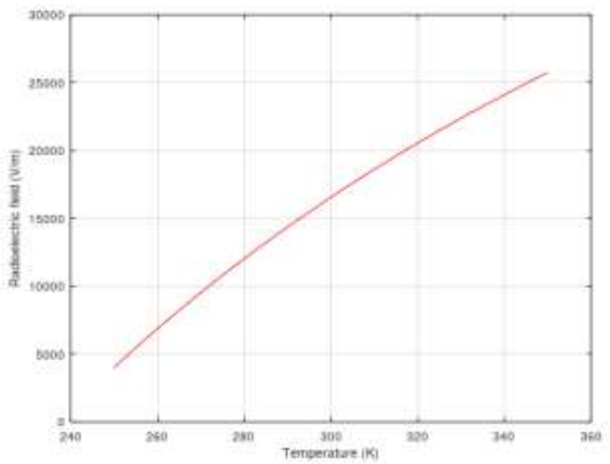

Figure 2. The dependence of the Radio-electric field on the temperature.

Here $E = 5.10^5$ *V/m*, $H = 6$ *A/m*.

The influence of temperature on the Radio-electric field is shown in Fig.1, one can see that with the increase of T, the radio-electric field increases nonlinearly.

Conclusion

In summary, the influence of optical phonons on the radio-electric field in semi-parabolic plus semi-inverse squared quantum well has been theoretically studied. We calculated the analytical expression for the radio-electric field. Both the numerical calculations and the theoretical results for the semi-parabolic plus semi-inverse square quantum wells are indicated. The obtained results show that the radio-electric field increases nonlinearly with the increase of temperature.

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