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Spheric Aberration Evaluation of Thin Lens by Ray Transfer Matrix

Bui Xuan Kien*

Electric Power University, 235 Hoang Quoc Viet, Cau Giay Hanoi, Vietnam

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Abstract: Spheric aberration is one of the optical aberrations influencing on the optical quality of the optical system, specially, the objective. In this work, we used the rays transfer matrix to evaluate the spheric aberration of the nonparaxial rays. Basing on the element configuration of the thin lens, we introduced the rays transfer matrix (RTM) describing the relation the object's and image's heigh from the optical axis. The transverse spheric aberration is numerically observed and compared to that obtained by using the paraxial approximation (PA). The obtained results are discussed to evaluate the spheric aberration of the thin lens, particularly, and the objective, generally.

Keywords: Optics, Ray optics, Optical aberration.

1. Introduction

The optical quality of an optical system depends on a lot of facts, including the spheric aberration [1]. The spheric aberration always appears in the optical system as an inherent feature of every optical system, which collects the nonparaxial light rays [2]. To evaluate and correct the spheric aberration there are following methods: collimating the nonparaxial rays to paraxial ones [3]; using the aspheric lens [5-8] or electrically thin flat lens [9]. From our understanding, the rays transfer matrix (RTM) is a method that can be used to evaluate the transverse spheric aberration of the thin lens and then to discuss about application for the complex optical system.

In this work, we present in detail the scheme of rays' traverse pass through the thin lens, and then introduce the rays transfer matrix of this system. Then the transverse spheric aberration is derived and numerically calculated to evaluate and discuss in competition with that obtained by the paraxial approximation.

* Corresponding author.
E-mail address: kienbx@epu.edu.vn

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2. Scheme of Ray Paths Through Thin Lens

Fig. 1a describes the ray paths traveling through a thin lens [3], where n is the lens' refractive index, d_0 is the object distance, d_i is the image distance, h is the distance from the optical axis at which the outermost ray enters the lens, R_1 is the first lens radius, R_2 is the second lens radius, and f is the lens' focal length. The distance h can be understood as a half of the clear aperture. The nonparaxial rays do not meet at the paraxial focus.

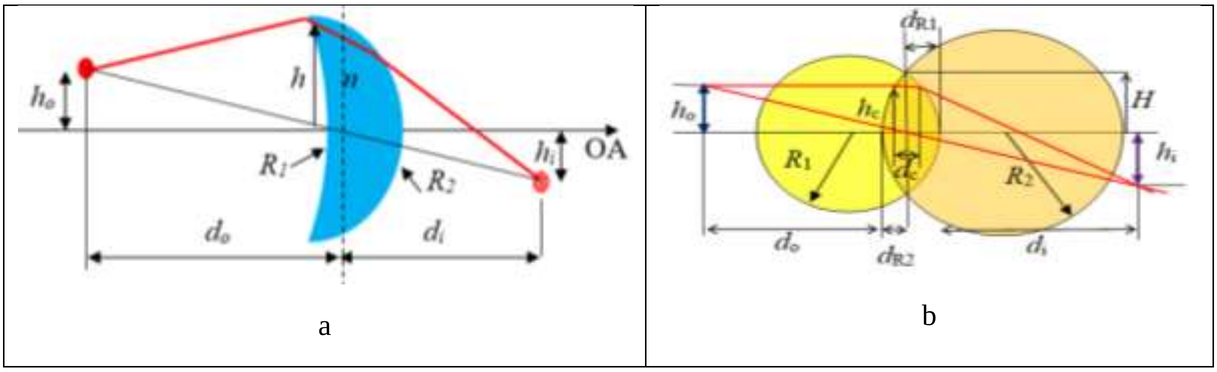

Figure 1. The ray paths travers through thin lens for Paraxial approximation (A) and Rays transfer matrix (B).

Using the Coddington factors, one can find expressions for shape, *s*, and position, *p* [3]

$$s = \frac{R_1 - R_2}{R_1 + R_2} \tag{1}$$

$$p = \frac{d_i - d_o}{d_i + d_o} \tag{2}$$

and expression for the Longitudinal Spheric Aberration (LSA) as follows:

$$LSA = \frac{1}{8n(n-1)} \frac{h^2 d_i^2}{f^3} \left(\frac{n+2}{n-1} s^2 + 2(2n+2)sp + (3n+2)(n-1)p^2 + \frac{n^3}{n-1} \right) \tag{3}$$

If the focal length, *f*, is much larger than the LSA, an expression for Transverse Spheric Aberration (TSA) corresponding to the diameter of the focal spot can be found as follows:

$$TSA = \frac{h}{d_i} LSA \tag{4}$$

From Eqs. (1) to (4), we have some remarks:

i) The LSA will be only used to evaluate the point object, but not the light beams;

ii) The relation between the distances of the object, image and focal length obeys the following lens equation [10]:

$$\frac{1}{d_i} + \frac{1}{d_0} = \frac{1}{f} \quad ; \tag{5}$$

This relation shows that the LSA is almost neglected.

iii) The relation between radius of curvature and focal length is satisfied the lensmaker's equation:

$$\left(\frac{1}{R_1}-\frac{1}{R_2}\right)(n-1)=\frac{1}{f} \tag{6}$$

From Eq. (6), the focal length may be calculated from the given radius of curvature.

iv) It is not used to evaluate the influence of the incident angle and radial distance of the object on the SA, that means, Eq. (3) is used to evaluate for outermost ray;

v) The height of the object influences not on the Spheric Aberration (SA), that means, the SA is evaluated for all of objects in front of lens.

In other word, Eq. (3) may be used to calculate the SA of the designed optical system, only, but not used to evaluated or reduce the SA of the designing optical system. From five mentions above, it is necessary to modify Eq. (3) to the more flexible equation, which depends on four given parameters: radiuses of curvature R_1, R_2 index n and object's distance d_i (or d_o) and h. However, the modified equation will not used to evaluate the influence of the incident angle and radial distance of the object. There is a main question that, Eq. (3) is introduced using the paraxial approximation, i.e., the lens equation Eq. (5) is always satisfied. Consequence, the SA is neglected for all of light paraxial rays. This mention will be proved by using following rays transfer matrix.

Using the ray transfer matrix [2], and basing on the scheme in Fig.1a, we introduce an output-input relation of the radial distances and incident angles of the light rays:

$$\begin{bmatrix} h_i \\ \alpha_i \end{bmatrix}=\begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}\begin{bmatrix} 1 & d_0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} h_o \\ \alpha_o \end{bmatrix} \tag{7}$$

where $\alpha_o=\text{atan}\left(\frac{h-h_o}{d_o}\right)$. From Eq. (7), we have a height of the image:

$$h_i=h_o+d_0\alpha_o+d_i\alpha_o-d_ih_o/f-d_id_0\alpha_o/f \tag{8}$$

The TSA is evaluated by difference between heights h_i and h_{OA}, which is marked by the ray travels through OA (Optical Axis), given as:

$$h_{OA}=h_o+d_0\alpha_{OA}+d_i\alpha_{OA}-d_ih_o/f-d_id_0\alpha_{OA}/f \tag{9}$$

where $\alpha_{OA}=\text{atan}\left(\frac{-h_o}{d_o}\right)$. An expression for TSA is then found as folows:

$$TSA=h_i-h_{i,OA}=\left(d_0+d_i-d_id_0/f\right)\left(\alpha_o-\alpha_{OA}\right) \tag{10}$$

Because the lens equation (5) is always satified, this leads to , $d_0+d_i-d_id_0/f=0$, and TSA=0.

So, to evaluate the SA of the lens, we use the ray transfer matrix with different approach. Firstly, we consider a thin lens as the series of individual optical systems, in which the medium is sandwitched between two curvature surfaces (see Fig. 1b). From Fig. 1b, the thickness of the lens at radial distance h_c will be given as following:

$$d_c\left(h_c\right)=\tfrac{1}{2}\left(H-h_c\right)\left(1/2R_1+1/2R_2\right) \tag{11}$$

Then, the RTM for light rays traveling through radial distance, h_c described in Fig.1b will be expressessed as:

$$\begin{bmatrix} h_i \\ \alpha_i \end{bmatrix}=\begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix}\begin{bmatrix} 1 & d_c/n \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2/R_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}\begin{bmatrix} h_o \\ \alpha_o \end{bmatrix} \tag{12}$$

where, $\alpha_o=\operatorname{atan}\left(\dfrac{h_c-h_o}{d_o}\right)$. After some arrangements, we arrive at:

$$h_i=\left[\left(h_o+\alpha_o d_o\right)\left(1-\frac{2d_c}{R_1}\right)+d_c\alpha_o\right]\left(1-\frac{2d_i}{R_2}\right)-\frac{2d_i}{R_1}\left(h_o+\alpha_o d_o\right)+d_i\alpha_o \tag{13}$$

SA is calculated from the difference between the image's heights marked by the rays traveling through $h_c \neq 0$ and $h_c = 0$ (two ray paths in Fig.1b). From Eq. (13), the height, h_{OA} of the image marked by the ray traveling through OA is given by:

$$h_{OA}=\left[\left(h_o+\alpha_{OA} d_o\right)\left(1-\frac{2d_{OA}}{R_1}\right)+d_{OA}\alpha_{OA}\right]\left(1-\frac{2d_i}{R_2}\right)-\frac{2d_i}{R_1}\left(h_o+\alpha_{OA} d_o\right)+d_i\alpha_{OA} \tag{14}$$

where, $\alpha_{OA}=\operatorname{atan}\left(\dfrac{-h_o}{d_o}\right)$ and $d_{OA}\left(0\right)=d_{R_1}+d_{R_2}=\tfrac{H}{2}\left(1/2R_1+1/2R_2\right)$. The TSA is given by:

$$TSA_{mod}=h_i-h_{OA} \tag{15}$$

Using Eqs. (11) and (13)-(15), we have carried out numerical calculations for TSA with different radius and different values of the entrance pupil h_c and then have compared the obtained result with that received from Eq. (3).

3. Simulation Results and Discussions

We consider the light rays traveling through an optical system set-up with following parameters: $R_1 = 200\ (\mathrm{cm})$, $R_2 = -100\ (\mathrm{cm})$, $H = 4\ (\mathrm{cm})$, $n = 1.54$, $h_0 = 0.5\ (\mathrm{cm})$, $d_0 = (0 - 4)$ cm. Using these parameters for Eqs. (3) and (15), the transverse SA for the paraxial approximation and Ray transfer matrix are numerically calculated and presented in Fig. 2 and Fig. 3, respectively.

We see that the TSA for the paraxial approximation is very small, meanwhile, for the ray transfer matrix is large. The attitude of the TSA's curve changes from left to right. That mean, the TSA of rays under and upper the optical axis is different. To investigate in detail, TSA observed for the object close to the front surface of the thin lens, ($d_0 = 0.5cm$) is presented in Fig. 4 for two cases of the orientation of the object. It is clear that, in the left TSA of both cases are similar, in opposite, almost different in the right. It means, TSA of both cases can be accepted for the rays laying in the same side of the optical axis.

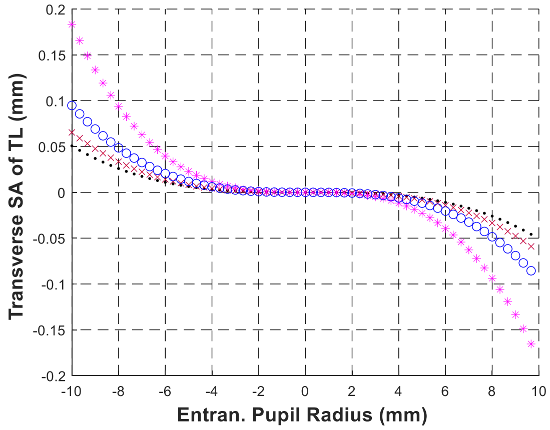

Figure 2. TSA using PA with different distance values of object: 1 cm (magenta-*), 2 cm (Blue-o), 3 cm (red-x), 4 cm (black-.).

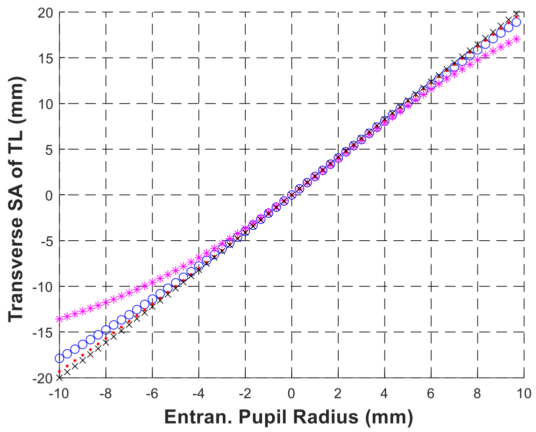

Figure 3. TSA using RTM with different distance values of object: 1 cm (magenta-*), 2 cm (blue-o), 3 cm (red-.), 4 cm (black+).

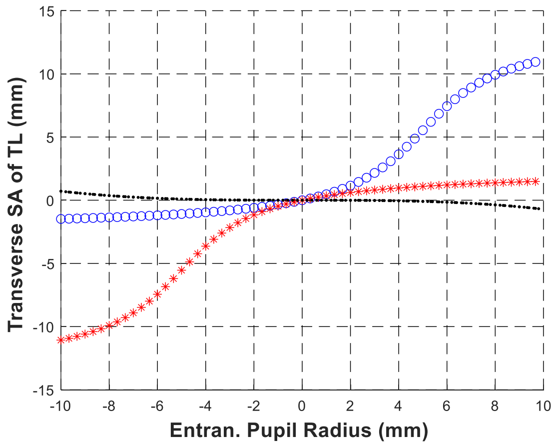

Figure 4. TSA using PA (black-.) and RTM with $h_0 = 0.5\ cm$ (blue-o) and $h_0 = -0.5\ cm$ (red-*).

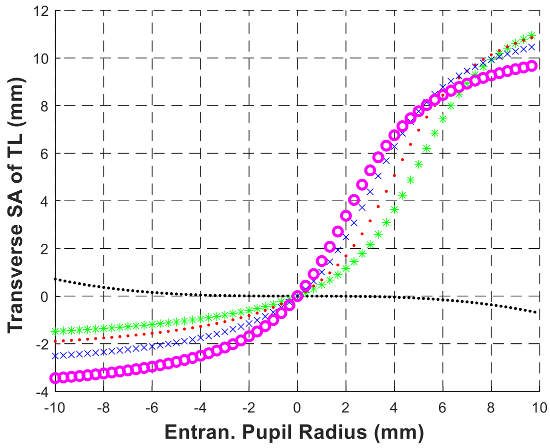

Figure 5. TSA using PA (black -.) and RTM with different object’s heigh: 0.5 cm (green-*), 0.4 cm (red-.), 0.3 cm (blue-x), 0.2 cm (magenta-o).

We are interesting on the influence of the heigh of object on TSA. Plots in Fig. 5 are results obtained by changing the heigh of object. It is shownthat for the paraxial approximation, TSA depends not on the heigh of object, at that time, for the rays transfer matrix it reduces when the heigh increases.

From all of the obtained results, one can conclude that, TSA given in Eq. (3) could be used for the paraxial rays travers through thin lens, only, but not for the nonparaxial rays. To correct TSA of the nonparaxial rays it is necessary to use the rays transfer matrix. Moreover, Eq. (3) will not used to evaluate TSA of the objective, which has wide entrance pupil, to collect light rays from far ways target. To avoid this intricacy, it is necessary to use different method, for example, to use standard formula of the aspheric lens [7] or the rays transfer matrix proposed above. This method satisfies to correct the spheric aberration not only of the wide entrance pupil objective, but also of the wide light beam.

4. Conclussion

We have used the rays transfer matrix to evaluate the spheric aberration of the thin lens. The numerically calculated results showed that using the rays transfer matrix one can evaluate the spheric abarration of both the paraxial and non-paraxial rays. The obtained results suggest an opportunity to use the rays transfer matrix to correct the spheric aberration of the light beam passing through wide entrance.

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