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Original Article

The Influence of the Anomalous Coupling on the $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ in the Randall - Sundrum Model

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Abstract: A study to produce a Z boson and a radion through anomalous couplings is considered from muon-muon colliders at the center of mass energy 10 TeV. The leptonic decays of the Z boson and a radion decaying into a pair of photons are considered. The integrated luminosity of muon colliders is 10 ab^{-1}. The production cross-sections depend on the polarization of μ^-, μ^+ initial beams, the center of mass energy $\sqrt{s}$. The minimum integrated luminosity value is shown to correspond to a significance larger than 5σ.

Keywords: radion production, cross-section, anomalous couplings.

1. Introduction

Although the standard model (SM) has been successful to account for all the experimental data so far observed, there remain a few issues to be settled [1]. The post-LHC generation of high-energy physics experiments will most likely be at lepton colliders [2]. The muon colliders, having the potential to achieve center-of-energy in the multi-TeV range with high luminosity, are promising lepton colliders for carrying out accurate studies on the Higgs, gauge boson and Yukawa sectors of the SM as well as probing different beyond Standard model (BSM) scenarios [3-6]. Recently, many possible phenomenological studies beyond the SM have been offered for multi-TeV muon colliders in [7-22]. The International Muon Collider Collaboration (IMCC) has been actively exploring the construction of a muon collider with center-of-mass system energy of 10 TeV or higher and with high luminosity [23-25].

The Randall-Sundrum (RS) model with a curved extra dimension is one of the most attractive extended SMs [26]. In the RS model, there are two 3-branes located at two orbifold fixed points on the

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coordinate of the fifth dimension [27]. The existence of an additional scalar, radion ϕ, corresponds to the fluctuation of the distance between two 3-branes. As a result of the stabilization of the compactification scale, radion and Higgs boson have the same quantum numbers and can be mixed. Therefore, production and decay channels of radion at the hadron colliders are the same as those of Higgs boson in the SM. The possibility of the radion is consistent with experimental observations from LHC, cosmology and fifth force searches [28]. The radion is considered a dark matter candidate. Due to the trace anomaly, the radion has enhanced couplings to a pair of photons and gluons. The production rate of $\sigma(\phi)\times Br(\phi\rightarrow\gamma\gamma)$ can be enhanced to the SM cross-section [29]. Moreover, the cross-section for Z production in $p\bar{p}$ collisions has been measured by both the ATLAS and CMS collaboration [30-33].

In this work, we present a study of the production of $Z\phi$ through the muon collider using the leptonic decay of Z boson and the decay of radion to $\gamma\gamma$. The layout of this paper is as follows. In Section II, we introduce the theoretical framework and Feynman rules for the anomalous couplings used in our analysis. The influence of the anomalous couplings on the $\mu^-\mu^+\rightarrow Z\phi\rightarrow l^-l^+\gamma\gamma$ is presented in Section III. Finally, we summarize our results and draw conclusions in Section IV.

2. Theoretical Framework

In RS model, due to the same quantum numbers, radion and Higgs boson can be mixed. When the Higgs is localized on the TeV brane, this mixing can be given through an action operator that can be written in the form [34-35]:

$$S_\xi=\xi\int d^4x\sqrt{g_{vis}}R(g_{vis})\hat{H}^+\hat{H}, \tag{1}$$

where ξ is the mixing parameter, $R(g_{vis})$ is the Ricci scalar for the metric induced on the visible brane, $\hat{H}$ is the Higgs field in the 5D context before rescaling to canonical normalization. The physical mass eigenstates h and ϕ are given by through h_0 and ϕ_0 fields as follows:

$$\begin{pmatrix} h_0 \\ \phi_0 \end{pmatrix}=\begin{pmatrix} 1 & -6\xi\gamma/Z \\ 0 & 1/Z \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} h \\ \phi \end{pmatrix}=\begin{pmatrix} d & c \\ b & a \end{pmatrix}\begin{pmatrix} h \\ \phi \end{pmatrix}, \tag{2}$$

where

$$\gamma\equiv\upsilon_0/\Lambda_\phi, Z^2\equiv1-6\xi\gamma^2(1+6\xi)=\beta-36\xi^2\gamma^2, \beta=1-6\xi\gamma^2, \tag{3}$$

$$a=\frac{\cos\theta}{Z}, b=-\frac{\sin\theta}{Z}, c=\sin\theta-\frac{6\xi\gamma}{Z}\cos\theta, d=\cos\theta+\frac{6\xi\gamma}{Z}\sin\theta. \tag{4}$$

The mixing angle θ is defined by

$$\tan2\theta=12\gamma\xi Z\frac{m_{h_0}^2}{m_{h_0}^2(Z^2-36\xi^2\gamma^2)-m_{\phi_0}^2}. \tag{5}$$

When $\xi\neq0$, there are four independent parameters that must be specified to fix the state mixing parameters a, b, c, d of Eq. (3) defining the mass eigenstates $\Lambda_\phi, m_h, m_\phi, \xi$.

The interaction Lagrangian before mixing that leads to Feynman rules for Z boson is given by [34]:

$$L \ni h_0 \frac{2m_Z^2}{\upsilon} Z_\mu^+ Z^\mu - \phi_0 \frac{2m_W^2}{\Lambda_\phi} \left[Z_\mu^+ Z^\mu (1-\kappa_Z) + Z_{\mu\nu}^+ Z^{\mu\nu} \frac{1}{4m_Z^2 \left(\frac{1}{2} m_0 b_0\right)} \right] \tag{6}$$

where $\kappa_Z = \frac{3m_Z^2 \left(\frac{1}{2} m_0 b_0\right)}{\Lambda_\phi^2 (m_0 / m_{Pl})^2}$. After mixing, this becomes

$$L \ni h \frac{2m_Z^2}{\upsilon} \left[g_h^Z Z_\mu^+ Z^\mu + g_h^r \frac{1}{4m_Z^2 \left(\frac{1}{2} m_0 b_0\right)} Z_{\mu\nu}^+ Z^{\mu\nu} \right] \equiv h \frac{2m_Z^2}{\upsilon} g_h^Z \left[Z_\mu^+ Z^\mu + \eta_h^Z Z_{\mu\nu}^+ Z^{\mu\nu} \right] \tag{7}$$

Similarly, the result for the radion ϕ becomes

$$L \ni \phi \frac{2m_Z^2}{\upsilon} \left[g_\phi^Z Z_\mu^+ Z^\mu + g_\phi^r \frac{1}{4m_Z^2 \left(\frac{1}{2} m_0 b_0\right)} Z_{\mu\nu}^+ Z^{\mu\nu} \right] \equiv \phi \frac{2m_Z^2}{\upsilon} g_\phi^Z \left[Z_\mu^+ Z^\mu + \eta_\phi^Z Z_{\mu\nu}^+ Z^{\mu\nu} \right] \tag{8}$$

The Feynman rules for the hZZ / ϕZZ couplings are as

$$g_{hZZ}^{\mu\nu} = i\bar{g}_{hZ} \left[\eta^{\mu\nu} - 2g_h^Z \left(k_1 k_2 \eta^{\mu\nu} - k_1^\nu k_2^\mu \right) \right], \tag{9}$$

$$g_{\phi ZZ}^{\mu\nu} = i\bar{g}_{\phi Z} \left[\eta^{\mu\nu} - 2g_\phi^Z \left(k_1 k_2 \eta^{\mu\nu} - k_1^\nu k_2^\mu \right) \right]. \tag{10}$$

The Feynman rules for the $\gamma Zh / \gamma Z\phi$ couplings are showed as follows [35]:

$$\begin{aligned} g_{\gamma Zh}^{\mu\nu} &= iC_{\gamma Zh} \left[k_1 k_2 \eta^{\mu\nu} - k_1^\nu k_2^\mu \right] \\ &= i \frac{\alpha}{2\pi v_0} \left[2g_h^r \left(\frac{b_2}{\tan\theta_W} - b_Y \tan\theta_W \right) - g_h \left(A_F + A_W \right) \right] \left[k_1 k_2 \eta^{\mu\nu} - k_1^\nu k_2^\mu \right], \end{aligned} \tag{11}$$

$$\begin{aligned} g_{\gamma Z\phi}^{\mu\nu} &= iC_{\gamma Z\phi} \left[k_1 k_2 \eta^{\mu\nu} - k_1^\nu k_2^\mu \right] \\ &= i \frac{\alpha}{2\pi v_0} \left[2g_\phi^r \left(\frac{b_2}{\tan\theta_W} - b_Y \tan\theta_W \right) - g_\phi \left(A_F + A_W \right) \right] \left[k_1 k_2 \eta^{\mu\nu} - k_1^\nu k_2^\mu \right]. \end{aligned} \tag{12}$$

Here, $\bar{g}_{eZ}, \bar{g}_{hZ}, \bar{g}_{\phi Z}$ can be found in Refs. [36-38], the triangle loop functions A_F, A_W are given in Ref. [39].

3. The Influence of the Anomalous Couplings on the $\mu^-\mu^+ \to Z\phi \to l^- l^+ \gamma\gamma$ Collision

We concentrate proposed runs of a muon collider, in which a center-of-mass energy $\sqrt{s} = 6$ TeV with an integrated luminosity of $\mathcal{L} = 4\ ab^{-1}$ and $\sqrt{s} = 10\ TeV$ with $\mathcal{L} = 10\ ab^{-1}$ [40]. We consider the $\mu^-\mu^+ \to Z\phi \to l^- l^+ \gamma\gamma$ collision process in which the initial state contains muon beams.

$$\mu^-(p_1) + \mu^+(p_2) \to Z(k_1) + \phi(k_2)\,. \tag{13}$$

Here, p_i, k_i (i = 1, 2) stand for the momentums. There are Feynman diagrams contributing to reaction (13), representing the s, u, t-channels exchange depicted in Fig.1.

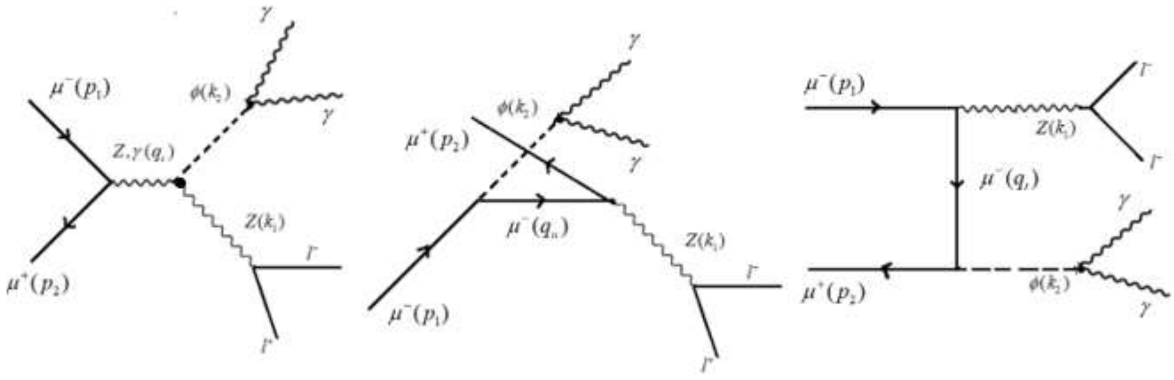

Figure 1. Feynman diagrams for $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ collision, representing the s, u, t-channels, respectively.

The transition amplitude representing the s-channel can be written as

$$M_s = M_{sZ} + M_{s\gamma}\,, \tag{14}$$

here

$$M_{sZ} = \frac{-i\,\overline{g}_{eZ}\; g_{\phi Z}}{q_s^2 - m_Z^2}\,\varepsilon_\nu^*(k_1)\Big[\eta^{\beta\nu} - 2g_\phi^Z\left(\eta^{\beta\nu}\; k_1 q_s - q_s^\nu k_1^\beta\right)\Big]\left(\eta_{\mu\beta} - \frac{q_{s\mu}q_{s\beta}}{m_Z^2}\right)\overline{v}(p_2)\gamma^\mu\left(v_e - a_e\gamma^5\right)u(p_1), \tag{15}$$

$$M_{s\gamma} = -i\,\frac{e\,C_{\gamma Z\phi}}{q_s^2}\,\varepsilon_\nu^*(k_1)\left(\eta^{\beta\nu}k_1 q_s - q_s^\nu k_1^\beta\right)\eta_{\mu\beta}\overline{v}(p_2)\gamma^\mu u(p_1), \tag{16}$$

The transition amplitude representing the u-channel is given by

$$M_u = -i\,\frac{g_{\mu\mu\phi}\overline{g}_{eZ}}{q_u^2 - m_\mu^2}\,\overline{v}(p_2)\left(\hat{q}_u + m_\mu\right)\varepsilon_\nu^*(k_1)\gamma^\nu(v - a\gamma^5)u(p_1), \tag{17}$$

The transition amplitude representing the t-channel is given by

$$M_t = -i\,\frac{g_{\mu\mu\phi}\overline{g}_{\mu Z}}{q_t^2 - m_\mu^2}\,\overline{v}(p_2)\gamma^\nu(v - a\gamma^5)\varepsilon_\nu^*(k_1)\left(\hat{q}_t + m_\mu\right)u(p_1), \tag{18}$$

The cross-section for the whole process can be calculated as follows [41]

$$\sigma = \sigma(\mu^-\mu^+ \to Z\phi) \times Br(Z \to l^-l^+) \times Br(\phi \to \gamma\gamma), \tag{19}$$

where

$$\frac{d\sigma(\mu^-\mu^+ \to Z\phi)}{d(cos\psi)} = \frac{1}{32\pi s}\frac{|\vec{k}_1|}{|\vec{p}_1|}|M_{fi}|^2 \tag{20}$$

is the expressions of the differential cross-section [42]. $\psi = (\vec{p}_1, \vec{k}_1)$ is the scattering angle. For numerical evaluation, we choose a center-of-mass energy of 10 TeV. The vacuum expectation value (VEV) of the radion field was set as $\Lambda_\phi = 5$ TeV. The Higgs mass was set as $m_h = 125$ GeV. The radion mass was set as $m_\phi = 125$ GeV [43].

We give estimates for the cross-sections as follows:

i) In Figure 2, the cross-sections are plotted as the function of P_{μ^-}, P_{μ^+}, which are the polarization coefficients of μ^-, μ^+ beams, respectively. The figure indicates that the cross-section in case of vector propagators (Z boson and photon) achieves the maximum value when $P_{\mu^-} = P_{\mu^+} = \pm 1$ and the minimum value when $P_{\mu^-} = 1, P_{\mu^+} = -1$ or $P_{\mu^-} = -1, P_{\mu^+} = 1$.

ii) In the case of $(P_{\mu^-}, P_{\mu^+}) = (1, -1); (0.8, -0.8); (0.6, -0.6); (0, 0)$, respectively, the cross-sections are measured as the function of the center of mass energy. The cross-sections increase when the collision energy increases in case of $(P_{\mu^-}, P_{\mu^+}) = (0.8, -0.8); (0.6, -0.6); (0, 0)$. However, in case of the left μ^- polarized beam, the right μ^+ polarized beam and vice versa, the cross-section decrease gradually when $\sqrt{s}$ increases.

iii) Some numerical values for cross-section in $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ are shown in Table 1. The cross-section in $l^-l^+\gamma\gamma$ signal for the SM background is 4.355 $\times 10^{-4}$ pb at 14 TeV [44]. We evaluate the minimum integrated luminosity value correspond to $S/\sqrt{S+B} > 5$ as 245.76 fb^{-1}. Therefore, the $l^-l^+\gamma\gamma$ final state production in muon colliders at high energy can be explored with a significance larger than 5σ. By considering not only the interactions of SM particles but also all the other possible contributions within the anomalous couplings, the cross-section in muon colliders is larger than that in electron-positron collider in [45]. Some numerical values for cross-section in case of different VEV of the radion field Λ_ϕ are evaluated in detail in Table 2.

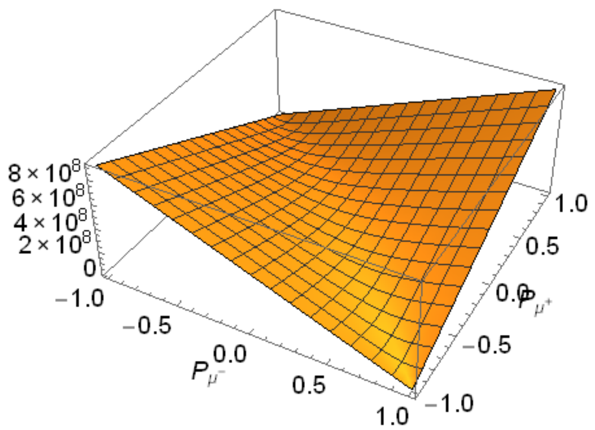

Figure 2. The cross-section as a function of the polarization coefficients (P_{μ^-}, P_{μ^+}) in $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ collision.

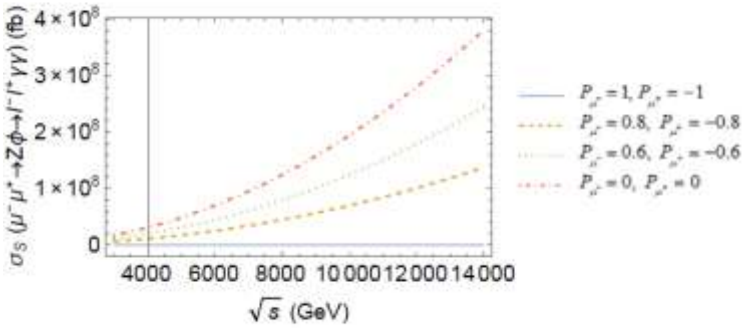

Figure 3. The cross-sections as a function of the collision energy in $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ collision.

Table 1. Typical values for the cross-section in the $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ collision in case of different collision energy with $\Lambda_\phi = 5$ TeV.

$\sqrt{s}$	3 TeV	7 TeV	8 TeV	10 TeV	13 TeV	14 TeV
$\sigma_S(P_{\mu^-}=1, P_{\mu^+}=-1)$ (fb)	0.2698	0.2678	0.2677	0.2676	0.2675	0.2674
$\sigma_S(P_{\mu^-}=0.8, P_{\mu^+}=-0.8)$ (10^8 fb)	0.0626	0.3432	0.4485	0.7011	1.1853	1.3748
$\sigma_S(P_{\mu^-}=0.6, P_{\mu^+}=-0.6)$ (10^8 fb)	0.1113	0.6102	0.7974	1.2464	2.1072	2.4440
$\sigma_S(P_{\mu^-}=0, P_{\mu^+}=0)$ (10^8 fb)	0.1738	0.9535	1.2459	1.9476	3.2925	3.8187

Table 2. Typical values for the cross-section in the $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ collision in case of different VEV of the radion field Λ_ϕ at the center-of-mass energy 10 TeV

Λ_ϕ	1 TeV	3 TeV	5 TeV	10 TeV
$\sigma_S(P_{\mu^-}=1, P_{\mu^+}=-1)$ (fb)	0.4461	0.7662	0.2676	0.6478
$\sigma_S(P_{\mu^-}=0.8, P_{\mu^+}=-0.8)$ (10^8 fb)	1064.16	0.0312	0.7011	2.6637
$\sigma_S(P_{\mu^-}=0.6, P_{\mu^+}=-0.6)$ (10^8 fb)	1891.84	0.0547	1.2464	4.7356
$\sigma_S(P_{\mu^-}=0, P_{\mu^+}=0)$ (10^8 fb)	2955.99	0.0866	1.9476	7.3993

4. Conclusion

In summary, we have evaluated the cross-section in $\mu^-\mu^+ \to Z\phi \to l^-l^+\gamma\gamma$ collision. With the anomalous couplings, the cross-section in muon colliders is larger than that in electron-positron collider. The cross-section is affected by the polarization coefficients of muon beams P_{μ^-}, P_{μ^+}. The cross-section achieves the maximum value when both of muon beams polarize left or right and the minimum value when the μ^- beam polarizes left, the μ^+ beam polarizes right and vice versa. The cross-sections are about 10^8 fb in case of $(P_{\mu^-}, P_{\mu^+}) = (0.8, -0.8); (0.6, -0.6); (0,0)$. In case of the left μ^- polarized beam, the right μ^+ polarized beam and vice versa, the cross-section is about 0.26 fb. The $l^-l^+\gamma\gamma$ final state production in muon colliders at high energy can be explored with a significance larger than 5σ in case the integrated luminosity is larger than 245.76 fb^{-1}.

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