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Quantum Phase Transition in Binary Ultra-cold Bose Gases

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Abstract: The quantum phase transition in binary Bose Gases is studied using the Cornwall-Jackiw-Tomboulis effective potential approach in the double-bubble approximation, which preserves the Goldstone theorem. Its main feature is that the transition is second order occurring at ultra-cold temperatures associated with the type of inverse symmetry-breaking transition occurring when the chemical potential reaches a critical value. However, it cannot simultaneously occur for the two components of a binary mixture of Bose gases.

Keywords: Binary Bose Gases, Cornwall-Jackiw-Tomboulis (CJT), Super-fluidity; Symmetry restoration (SR); Symmetry non-restoration (SNR); Inverse symmetry breaking (ISB), Ultra-cold temperature.

1. Introduction

Bose gases can consist of bosons or pairs of Fermi particles bound to have integer spin, which obey Bose-Einstein statistics. Symmetric wave functions describe their state, and the Pauli principle does not limit the filling number. The system can be a homogeneous or a mixture system, The Bose system can have an arbitrary number of particles in a quantum state.

Phase transitions are still one of the most interesting problems in modern Physics. There have been a lot of works dealing with phase transitions in binary-mixture quantum systems [1-6]. However, in almost works there have been studied thermal phase transition while quantum phase transition can occur in the system [7-9]. Quantum phase transitions occur in the systems as the quantum fluctuations of physical quantities become largest at a certain temperature, in which particle density reaches a maximum value at a critical state [10]. This means the system's quantum state changes from one state to another at a certain temperature when the chemical potential or the coupling constant reaches a critical value. Its scenarios are determined by examining the dependence of order parameters characterizing the system

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on the chemical potential or coupling constant at each temperature value. The order parameters of the system can be of field operators, condensation densities, energy density and thermodynamic potential, etc. In particular, there are three scenarios of the phase transition in the system: the scenario of symmetry restoration (SR) in which the symmetry, broken at zero order parameter, gets restored at higher order parameter; the scenario of symmetry non-restoration (SNR) in which symmetry, broken at zero order parameter, no longer gets restored as order parameter is increased and, lastly, the scenario of inverse symmetry breaking (ISB) where the symmetry turns out to be broken as the order parameter is increased [11].

In theory, quantum phase transition can occur in a quantum system at ultra-cold temperature when the chemical potential (μ) or coupling constant (λ) changes, reaching a critical value. However, experiments have also proved that it is possible to create many types of phase transitions by adjusting parameters such as temperature, the chemical potential, or the coupling constant of the system. Furthermore, Bose gases become quantum fluids when cooled at ultra-cold temperatures and a special property of liquid is superfluidity. A fluid is superfluid if the Landau criteria are satisfied. That means, the speed of sound in the momentum space of the fluid reaches a critical value and follows the formula:

$$v_c = \min\left(\frac{E}{k}\right), \tag{1}$$

where E is the energy of the mode excited sound wave in the system [10]. Specially, BEC occurs when Bose gases are cooled to ultra-cold temperatures, reaching their BEC critical temperature value -T_c (near 0K). Indeed, successful research on BEC of Bose gases mixtures won the Physics Nobel Prize in 2001. That achievement belongs to the research group of Wieman, Cornell and Ketterle of Colorado State University (USA), obtained by cooling Bose gases to a temperature of 170 nK [12]. Therefore, the BEC process of Bose gases is a form of thermal phase transition, and the temperature is below 200 nK called ultra-cold temperature.

In this work, we consider binary ultra-cold Bose Gases to investigate the order and types of their quantum phase transition and superfluidity. Binary Bose gases can be miscible or immiscible mixtures. To have a deeper insight into the quantum phase transition in binary ultra-cold Bose gases, we carry out a study of the dependence of their field condensates on chemical potential at ultra-cold temperatures to find out how many types and the order of quantum phase transition can happen when the chemical potential changed.

This work is organized as follows. In Section 2, we present the research methods of quantum phase transition in binary ultra-cold Bose gases. The results and discussions are presented in Section 3. Conclusions are given in Section 4.

2. Research Methods

We investigate the scenarios and the order of quantum phase transition in a binary mixture of Bose gases by using the Cornwall–Jackiw–Tomboulis (CJT) effective potential approach. This approach is considered to be of an adequate and reliable approach for the study of phase transition. However, to determine the CJT effective potential, a certain approximation must be used [13, 14]. Therefore, we use the CJT effective potential approach in the double-bubble approximation which preserves the Goldstone theorem, and apply quantum phase transition field theory to binary Bose gases. The loop expansion stops at the 2-loop approximation and all transfer function matrices of Bose gases components must be transformed to diagonalization to recover the Goldstone theorem. To begin with, we first write the Lagrangian for the binary mixture of Bose gases:

$$L=\phi^*\left(-i\hbar\frac{\partial}{\partial t}-\frac{\hbar^2\nabla^2}{2m_1}\right)\phi+\psi^*\left(-i\hbar\frac{\partial}{\partial t}-\frac{\hbar^2\nabla^2}{2m_{12}}\right)\psi-V(\phi,\psi),$$
$$V(\phi,\psi)=\mu_1\phi^*\phi+\mu_2\psi^*\psi-\frac{\lambda_1}{2}(\phi^*\phi)^2-\frac{\lambda_2}{2}(\psi^*\psi)^2-\frac{\lambda}{2}(\phi^*\phi)(\psi^*\psi)^2, \tag{2}$$

in which μ_1 (μ_2) denotes the chemical potential of the field ϕ (ψ), m_1 (m_2) the mass of the bosons, λ_1, λ_2, and λ the coupling constants which are expressed through s-wave scattering lengths a_1, a_2, and a_{12} of the corresponding collisions 1 + 1, 2 + 2 and 1 + 2 as follows:

$$\lambda_1=\frac{4\pi\hbar^2a_1}{m_1},\lambda_2=\frac{4\pi\hbar^2a_2}{m_2},\lambda=\frac{4\pi\hbar^2a_{12}}{m_{12}},m_{12}=\frac{m_1m_2}{m_1+m_2}, \tag{3}$$

where m_{12} is reduced mass.

Next, to get a simple and easy process in research, we consider the binary mixture of Bose gases consisting of two components, namely. ^{85}Rb, and ^{87}Rb. In addition, to become a miscible binary Bose gas, the coupling constant must not only always be positive but also satisfy the following condition [5]:

$$4\lambda_1\lambda_2-\lambda^2>0\,. \tag{4}$$

Hence, e.g. we choose the values of parameters in the range which can be easily adjusted in the Lab [12] as follows:

$\lambda_1=5.10^{-12}eV^{-2},\lambda_2=0,4.10^{-12}eV^{-2},\lambda=10^{-12}eV^{-2},\mu_1=5.10^{-12}eV$ for the miscible case; (5)

$\lambda_1=5.10^{-12}eV^{-2},\lambda_2=0,4.10^{-12}eV^{-2},\lambda=4.10^{-12}eV^{-2},\mu_1=5.10^{-12}eV$ for the immiscible case. (6)

3. Results and Discussion

3.1. Results

Based on the results in [14] and by restricting ourselves to an improved double-bubble approximation we arrive at the CJT effective potential which preserves the Goldstone theorem,

$$\begin{aligned}V^{CJT}(\phi_0,\psi_0,D,G)&=-\frac{\mu_1}{2}\phi_0^2+\frac{\lambda_1}{8}\phi_0^4-\frac{\mu_2}{2}\psi_0^2+\frac{\lambda_2}{8}\psi_0^4+\frac{\lambda}{8}\phi_0^2\psi_0^2+\\ &\frac{1}{2}\int_\beta Tr\left\{\ln D^{-1}(k)+\ln G^{-1}(k)+\left[D_0^{-1}(k,\phi_0,\psi_0)D+G_0^{-1}(k,\phi_0,\psi_0)G\right]-2\right\}+\\ &\frac{\lambda_1}{8}P_{11}^2+\frac{\lambda_1}{8}P_{22}^2+\frac{3\lambda_1}{4}P_{11}P_{22}+\frac{\lambda_2}{8}Q_{11}^2+\frac{\lambda_2}{8}Q_{22}^2+\frac{3\lambda_2}{4}Q_{11}Q_{22}+\frac{\lambda}{8}P_{11}Q_{11}+\frac{\lambda}{8}P_{11}Q_{22}+\frac{\lambda}{8}P_{22}Q_{11}+\frac{\lambda}{8}P_{22}Q_{22}\end{aligned}, \tag{7}$$

where ϕ_0, ψ_0 are the corresponding condensates of ϕ, ψ fields which are the non-trivial solutions to the gap equations

$$\frac{\partial V^{CJT}}{\partial\phi_0}=0,\frac{\partial V^{CJT}}{\partial\psi_0}=0 \tag{8}$$

yielding,

$$\frac{\phi_0^2}{4} = \frac{M}{4\lambda_1\lambda_2 - \lambda^2}, M = 2\mu_1\lambda_2 - \mu_2\lambda; \tag{9}$$

$$\frac{\psi_0^2}{4} = \frac{N}{4\lambda_1\lambda_2 - \lambda^2}; N = 2\mu_2\lambda_1 - \mu_1\lambda. \tag{10}$$

In addition, obtained the inverse propagators from the Schwinger-Dyson equations

$$\frac{\partial V^{CJT}}{\partial D} = 0, \frac{\partial V^{CJT}}{\partial G} = 0 \tag{11}$$

giving

$$D^{-1}(k) = \begin{pmatrix} \frac{\vec{k}^2}{2m_1} + c_1 & -\omega_n \\ \omega_n & \frac{\vec{k}^2}{2m_1} \end{pmatrix}; G^{-1}(k) = \begin{pmatrix} \frac{\vec{k}^2}{2m_2} + c_2 & -\omega_n \\ \omega_n & \frac{\vec{k}^2}{2m_2} \end{pmatrix}, \tag{12}$$

in which, (13)

$$\sum\nolimits_1^{\phi} = \frac{\lambda_1}{2}P_{11} + \frac{3\lambda_1}{2}P_{22} + \frac{\lambda}{4}\left(Q_{11} + Q_{22}\right); \sum\nolimits_2^{\psi} = \frac{\lambda_2}{2}Q_{11} + \frac{3\lambda_2}{2}Q_{22} + \frac{\lambda}{4}\left(P_{11} + P_{22}\right), \tag{14}$$

and $P_{ab} = \int_\beta D_{ab}; Q_{ab} = \int_\beta G_{ab}, \int_\beta f(k) = T\sum_{n=-\infty}^{+\infty}\int \frac{dk}{(2\pi)^3} f(\omega_n, k), \omega_n = 2\pi nT$. (15)

We obtain expressions of the Nambu Goldstone energy modes in the momentum space by taking traces of the matrices (12):

$$E_1 = \sqrt{\frac{\vec{k}^2}{2m_1}\left(\frac{\vec{k}^2}{2m_1} + c_1\right)}, E_2 = \sqrt{\frac{\vec{k}^2}{2m_2}\left(\frac{\vec{k}^2}{m_2} + c_2\right)}. \tag{16}$$

At ultra-cold temperatures, k is so small that Eq (16) becomes

$$E_1 = \sqrt{\frac{c_1}{2m_1}}k, E_2 = \sqrt{\frac{c_2}{2m_2}}k. \tag{17}$$

Therefore, the speeds of sound in the binary Bose gases in this case read as follows:

$$v_{s1} = \frac{E_1}{k} = \sqrt{\frac{c_1}{2m_1}}, v_{s2} = \frac{E_2}{k} = \sqrt{\frac{c_2}{2m_2}}. \tag{18}$$

In this work, to perform a numerical study of quantum phase transition scenarios in the system, we used formulas (9) and (10) to draw the dependence of ϕ_0, ψ_0 on the chemical potential μ_2 at a definite chemical potential μ_1 and ultra-cold temperatures. Firstly, we consider the binary Bose gases at T = 150 nK in which (5) is satisfied. This is an example of the miscible ultra-cold binary Bose gases. The results are shown in Fig. 1, which indicates that the symmetry of the first quantum condensate component ϕ_0 is broken at $\mu_2 = 1,07.10^{-12}eV$ or inverse symmetry breaking quantum transition (ISB) occurs when chemical potential reaches this critical value. It also represents another scenario of quantum phase transition in the system for the second component ψ_0: the phase changes to the symmetry restoration phase (SR) when μ_2 reaches the second critical value, $\mu_2 = 1,53.10^{-12}eV$.

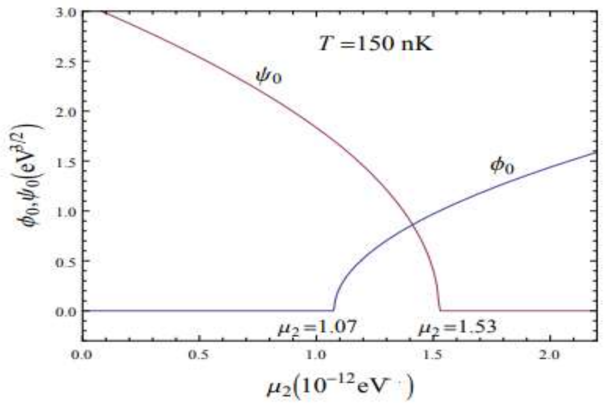

Figure 1. The dependence of order parameter ϕ_0, ψ_0 on μ_2 at $\lambda_1 = 5.0\times10^{-12}$ eV^{-2}, $\lambda_2 = 0,4\times10^{-12}$ eV^{-2}, $\lambda = 10^{-12}$ eV $^{-2}$, $\mu_1 = 5.0\times10^{-12}$ eV and T = 150 nK.

To get better understanding of quantum phase transition in this binary miscible Bose gases satisfied (5), we draw the dependence of order parameter ϕ_0, ψ_0 on μ_2 at the so low temperature, near 0K (*T* = 5 nK, for instance). Its results are shown in Fig. 2. This figure indicates that the symmetry of the second quantum condensate component ψ_0 is broken at $\mu_2 = \mu_{2\psi} = 0,93.10^{-12} eV$ or inverse symmetry breaking quantum transition (ISB) occurs when chemical potential reaches a critical value $\mu_2 = \mu_{2\psi} = 0,93.10^{-12} eV$. Besides, it also shows that the scenario of quantum phase transition in the system for the first component ϕ_0 is the symmetry restoration phase transition (SR) when μ_2 reaches the critical value, $\mu_2 = \mu_{2\varphi} = 2.10^{-12} eV$.

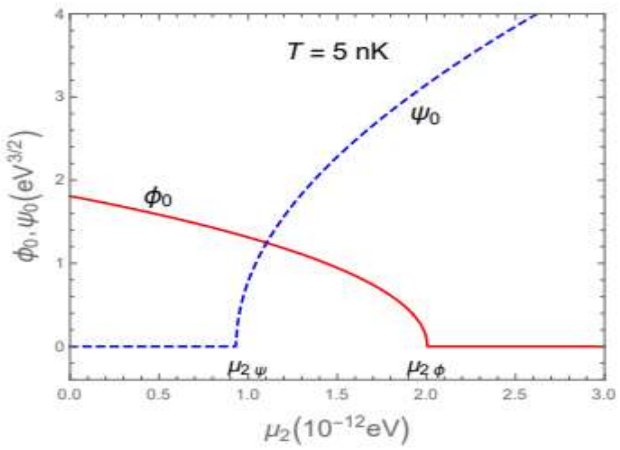

Figure 2. The dependence of order parameter ϕ_0, ψ_0 on μ_2 at $\lambda_1 = 5.10^{-12}$ eV^{-2}, $\lambda_2 = 0,4\times10^{-12}$ eV^{-2}, λ=10^{-12} eV $^{-2}$, $\mu_1 = 5.0\times10^{-12}$ eV and *T* = 5 nK.

A question is that, immediately arises whether or not the scenarios SR and ISB, presented in the foregoing cases, possibly exist in nature. To answer this, one must investigate the *T* dependence of the specific heat at constant volume based on the formula [10-11].

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V . \tag{19}$$

The obtained results are presented in Figs 3 and 4.

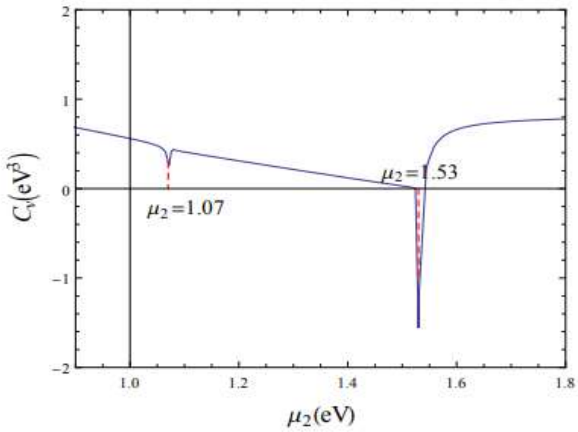

Figure 3. The dependence of the specific heat at constant volume on chemical potential μ_2 at T = 150 nK and $\lambda_1 = 5.0\times10^{-12}$ eV^{-2}, $\lambda_2 = 0,4\times10^{-12}$ eV^{-2}, $\lambda=10^{-12}$ eV $^{-2}$, $\mu_1 = 5.0\times10^{-12}$ eV.

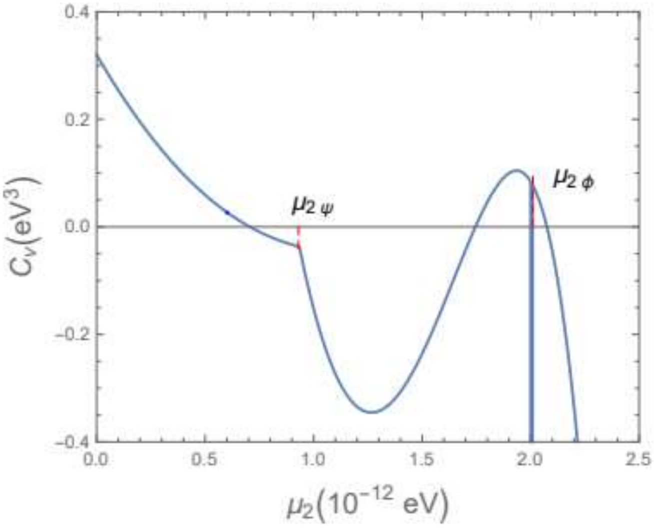

Figure 4. The dependence of the specific heat at constant volume on chemical potential μ_2 at T = 5 nK and $\lambda_1 = 5.0\times10^{-12}$ eV^{-2}, $\lambda_2 = 0,4\times10^{-12}$ eV^{-2}, $\lambda=10^{-12}$ eV $^{-2}$, $\mu_1 = 5.0\times10^{-12}$ eV.

The C_V (T) - graph plotted in Fig. 3 proves that the ISB scenario for the first component possibly exists in nature because its corresponding specific heat is positive, $C_V > 0$. On the other hand, the negative value of specific heat, $C_V < 0$ in Fig. 3 also implies that the SR of the second component is impossible to exist [11]. Besides, Fig. 4 shows that both scenarios SR and ISB can not occur. Moreover, based on Fig. 1, one can infer that the order of quantum phase transition in binary mixtures of Bose gases is second order because of the monotonous variation of the order parameters to zero. This conclusion is confirmed again in Fig. 2, because its heat capacity at constant volume has a singular point at the critical state.

Next, we consider the immiscible case of the binary Bose gases satisfied (6) at ultra-cold temperature, $T = 5$ nK. Taking similar steps as above, the results are shown in Figs. 5 and 6.

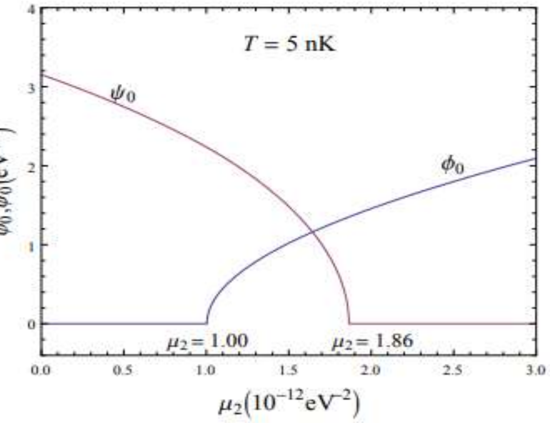

Figure 5. The dependence of order parameter ϕ_0, ψ_0 on μ_2 at $T = 5$ nK, and $\lambda_1 = 5.10^{-12}$ eV^{-2}, $\lambda_2 = 0{,}4\times10^{-12}$ eV^{-2}, $\lambda = 4.0\times10^{-12}$ eV^{-2}, $\mu_1 = 5.0\times10^{-12}$ eV.

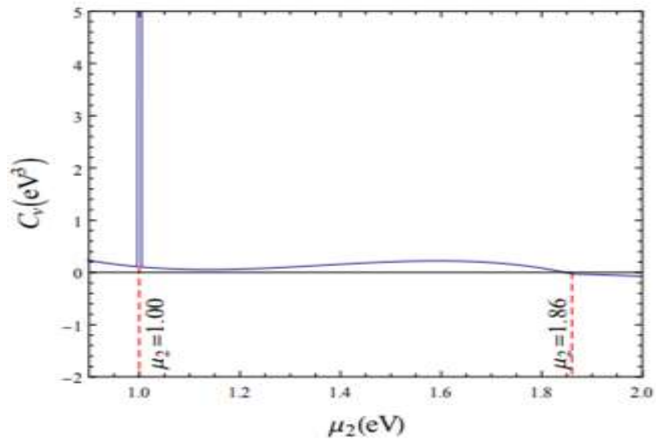

Figure 6. The dependence of the specific heat at constant volume on chemical potential μ_2 at the temperature $T = 5$ nK, and $\lambda_1 = 5.0\times10^{-12}$ eV^{-2}, $\lambda_2 = 0{,}4\times10^{-12}$ eV^{-2}, $\lambda = 4.0\times10^{-12}$ eV^{-2}, $\mu_1 = 5.0\times10^{-12}$ eV.

Fig. 5 shows that the scenario of quantum phase transition is ISB for the first condensate component ϕ_0, and SR for the second component ψ_0. However, Fig. 6 confirms that only the ISB scenario occurred for the first component in the binary ultra-cold Bose gases.

3.2. Discussion

One can see that the Lagrangian (1) is invariant under the Unita U(1)xU(1) phase transformation. Thus, there must exist two massless bosons called Nambu Goldstone bosons according to Goldstone's theorem [15].

Here, Eq (19) is not only expressing the superfluidity condition (1) for the Bose gases but also preserving Goldstone's theorem: Appearing two Nambu Goldstone bosons in the system, Goldstone's theorem is valid. That means the binary mixture of Bose gases at ultra-cold temperature is a superfluidity. It means that the model and method of research in this paper are matched and all results we got in this paper are accurate.

Furthermore, Fig. 4 also proves that BEC occurs in the binary Bose gases at temperature T, near 0K. Because, at $T = 5$ nK, all bosonic particles in the system are in the BEC state. They are at the BEC phase – they are also at the same quantum state. Then, can not occur quantum phase transition in the system for this case. That means the results obtained in this paper are reliable and accurate.

In addition, the type of the transition agrees with the results of the theoretical works. However, experiments can not be carried-outat lower 150 nK. Therefore, the article's results can suggest how to get the expected phases for the experimental process in the Lab.

4. Conclusions

By using the CJT effective potential approach, we focused on the investigation of the quantum phase transition in binary mixture Bose gases at ultra-cold temperatures, the main results we found are as follows:

i) Quantum phase transition in the system is second order when chemical potential changes;

ii) Only the ISB scenario of quantum phase transition in the binary ultra-cold Bose gases can occur and exist in nature for one component;

iii) We confirmed that a Bose gas is superfluid and quantum condensation occurs in the system at ultra-cold temperature (nK) when the chemical potential reaches the critical point. However, experimental results have shown that phase transition in Bose gases is of first order and follows the restoration symmetry scenario. Hence, the first and second results mentioned above can be seen as a discovery of this work.

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