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Original Article

The Production and Decay of Saxion in the $\gamma^* e^-$ Collision in the Supersymmetric DFSZ Axion Model

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Abstract: The production and decay of the saxion in the $\gamma^* e^-$ collision in the SUSY DFSZ axion model were considered. The production cross-sections were dependent on the polarization of the initial and final electron beams, the center of mass energy $\sqrt{s}$, and the saxion mass m_σ. We analyzed the decay rate of saxion in the case where the saxion comes to dominate the energy density of the universe before being thermalized for n = 1 and n = 2.

Keywords: saxion production, $\gamma^* e^-$ collision, SUSY DFSZ axion model.

1. Introduction

The standard model (SM) of particle physics has successfully explained many experimental facts. Still, there exist several problems as the gauge hierarchy problem, the strong charge-parity (CP) problem, and the cosmological problems. Therefore, the appearance of the framework beyond the SM (BSM) is inevitable. One of the best motivated candidates which extends the SM to solve the hierarchy problem and CP problem is supersymmetry (SUSY) [1-8]. The most popular solution to the strong CP problem is based on the Peccei-Quinn (PQ) symmetry [3]. As a consequence of the breaking of PQ symmetry, the existence of a pseudo-Nambu-Goldstone boson, axion (a), is predicted [2, 9-10]. There are three general types of quantum chromodynamics (QCD) axion models: the Peccei-Quinn-Weinberg-Wilczek (PQWW) axion model which introduces one additional complex scalar field only, tied to the EW Higgs sector; the Kim-Shifman-Vainshtein-Zakharov (KSVZ) axion model which introduces heavy quarks as well as the PQ scalar; the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axion model which

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introduces an additional Higgs field as well as the PQ scalar [11]. The PQ fields contain the axion (a), the scalar superpartner of the axion called saxion (σ), and the fermionic superpartner of the axion called axino $(\tilde{a})$ [4].

The saxion plays a vital role in the cosmological evolution of the universe and thus makes a great impact on the dark matter abundance. The saxion is naturally expected to have a large initial amplitude to reduce the Planck scale during inflation and start coherent oscillation after inflation. As a result, the saxion dominates the universe due to the energy density of the saxion in the total energy density after reheating. The saxion decays at a later epoch and reheats the universe again releasing huge entropy [2]. The study on the saxion has mainly focussed on the decay rate and the decay temperature. [8, 12, 15-19]. The SUSY DFSZ axion model is compelling in that it contains the SUSY solution to the gauge hierarchy problem, the PQ solution to the strong CP problem, and the Kim-Nilles solution to the SUSY μ problem [1]. In high-scale lepton-axiogenesis, the saxion mass is large, at 10-100 TeV [12-14].

In this work, we evaluated the production and decay of saxion in the SUSY DFSZ axion model at the $\gamma^* e^-$ subprocess. This paper is laid out as follows. In section II, we briefly introduce the SUSY DFSZ axion model. In section III, we consider the $\gamma^* e^-$ subprocess and calculate the cross-section of the saxion production at the ILC. In section IV, we discuss the saxion decays. Finally, we draw some conclusions in section V.

2. The Review of the SUSY DFSZ Axion Model

The SUSY axion model naturally causes hybrid inflation and axion becomes the dominant componet of dark matter. The PQ fields contain the axion (a), the scalar partner of the axion called saxion (s), and the fermionic superpartner of the axion called axino $(\tilde{a})$. The super potential for the SUSY axion model is given by [3]

$$\mathrm{W} = \kappa S(\psi\bar{\psi} - f_a^2) + \lambda\psi X\bar{X}, \tag{1}$$

where S is a gauge singlet superfield and has a zero PQ charge, ψ and $\bar{\psi}$ are the PQ superfields that are gauge singlets and have +1 and -1 PQ charges, respectively. f_a is the PQ symmetry-breaking scale and κ is a dimensionless coupling constant assumed to be real and positive. X is the superfield interacting with a PQ field at tree level and has some PQ charges as well as gauge charges through which it interacts with the minimal supersymmertry SM (MSSM) fields.

In particular, for the SUSY KSVZ axion model, X and $\bar{X}$ are additional heavy quarks, denoted by Q and $\bar{Q}$, that have color charges. For the SUSY DFSZ axion model, X and $\bar{X}$ are MSSM Higgses, H_u and H_d. The axion a and the saxion σ are related to the PQ fields as

$$\psi = \upsilon \exp\left(\frac{\sigma + ia}{\sqrt{2}F_a}\right), \ \bar{\psi} = \bar{\upsilon} \exp\left(-\frac{\sigma + ia}{\sqrt{2}F_a}\right) \tag{2}$$

where

$$F_a = \sqrt{\upsilon^2 + \bar{\upsilon}^2}, \upsilon ; \left(\frac{c_2}{c_1}\right)^{1/4} f_a, \ \bar{\upsilon} ; \left(\frac{c_1}{c_2}\right)^{1/4} f_a. \tag{3}$$

c_1 and c_2 are real-valued constants that are positive and order unities. In a minimal setup where the SUSY-breaking masses for ψ and $\bar{\psi}$ only come from the supergravity effects, c_1 is equal to c_2 at tree level.

For the saxion-photon system, a Lagrangian density is given by [20]

$$L_{\sigma\gamma\gamma} = \frac{\alpha_c}{8\pi F_a} \sigma F_{\mu\nu} F^{\mu\nu}, \tag{4}$$

where α_c is the colour constant, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor. A saxion – photon – photon coupling is given by [21]

$$V_\mu^\nu(s,\gamma,\gamma) = \frac{-i\alpha_c}{4\pi F_a}[2(q.k)\eta_\mu^\nu - q^\nu k_\mu - q_\mu k^\nu]. \tag{5}$$

3. The Production of the Saxion on the $e^-\gamma^* \to e^-\sigma$ Subprocess at ILC

We consider the $e^-\gamma^* \to e^-\sigma$ subprocess in which the initial state contains electron beam and photon beam

$$e^-(p_1) + \gamma^*(p_2) \to e^-(k_1) + \sigma(k_2). \tag{6}$$

Here, p_i, k_i (i = 1, 2) stand for the momentums. There is Feynman diagram contributing to reaction (6), representing the t-channel exchange depicted in Fig. 1.

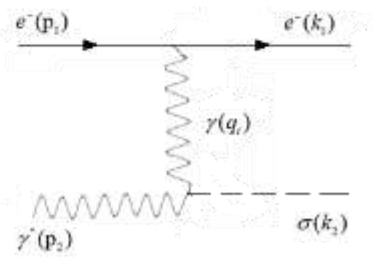

Figure 1. Feynman diagrams for $e^-\gamma^* \to e^-\sigma$ collision, representing the t-channel.

The transition amplitude representing the t-channel is given by

$$M_t = -ie\frac{\alpha_c}{4\pi F_a q_t^2}\left[2(p_2 q_t)\eta^{\mu\nu} - q_t^\mu p_2^\nu - q_t^\nu p_2^\mu\right]\varepsilon_\mu(p_2)\eta_{\sigma\nu}\,\bar{u}(k_1)\gamma^\sigma u(p_1), \tag{7}$$

The effective cross-section $\sigma(s)$ for the subprocess $e^-\gamma^* \to e^-\sigma$ at the ILC experiment can be written as follows

$$\sigma(s) = \int_{\frac{(m_e+m_\sigma)^2}{s}}^{\frac{\zeta}{1+\zeta}} dx\, f_{\gamma/e}(x) \int_{(\cos\theta)_{\min}}^{(\cos\theta)_{\max}} d\cos\theta \frac{d\hat{\sigma}(\hat{s})}{d\cos\theta}, \tag{8}$$

Here $\theta = (\vec{p}_1, \vec{k}_1)$ is the scattering angle. The photon distribution function $f_{\gamma/e}$ is written as

$$f_{\gamma/e} = \frac{1}{D(\zeta)}\left[(1-x) + \frac{1}{1-x} - \frac{4x}{\zeta(1-x)} + \frac{4x^2}{\zeta^2(1-x)^2}\right] \quad (9)$$

with

$$D(\zeta) = \left(1 - \frac{4}{\zeta} - \frac{8}{\zeta^2}\right)\ln(1+\zeta) + \frac{1}{2} + \frac{8}{\zeta} - \frac{1}{2(1+\zeta)^2} \quad (10)$$

ζ should be less than 4.8 in order to avoid producing e^-e^+ collisions by the interaction of the incident and backscattered photons [22]. For $\zeta = 4.8$, there is $x_{max} = 0.83$. For numerical evaluation, we choose a center-of-mass energy of 500 GeV. For a domain wall number; the lower bound weakens for larger domain wall numbers, the saxion mass was set as $m_\sigma = 30$ TeV [14]. For $\alpha_c = 0.1$, $F_a = 10^{11}$GeV, we give estimates for the cross-sections as follows:

i) In Fig. 2, the cross-sections are plotted as the function of P_1, P_2, which are the polarization coefficients of initial and final e^- beams, respectively. The figure indicates that the cross-section achieves the maximum value when $P_1 = P_2 = \pm 1$ and the minimum value when $P_1 = 1, P_2 = -1$ or $P_1 = -1, P_2 = 1$.

ii) To make the scattered particles be detected, the scattering angle is cut by $10^o < \theta < 170^o$ [22]. In the case of $(P_1, P_2) = (1, 1); (0.8, -0.8); (0.6, -0.6); (0, 0)$, respectively, the Fig. 3 shows that the differential cross-sections increase when the cosθ increases. The differential cross-sections reach the maximum value when the scattering angle is about 170^o.

iii) The polarization coefficients (P_1, P_2) are chosen as (1, 1); (0.8, -0.8); (0.6, -0.6); (0, 0); (1, 0); (0.8, 0); (0.6, 0), respectively. In Table 1, we evaluate some the cross-section values for the center-of-mass energy in region 500 GeV $\le \sqrt{s} \le$ 1000 GeV. In Table 2, we calculate the cross-sections in the different mass values of saxion at $\sqrt{s} = 500$ GeV. When the final electron beam is unpolarised, the cross-section values are independent on the polarization of the initial electron beam. The cross-section reaches maximum value $\sigma = 7.05526 \times 10^{-9}$ fb when $(P_1, P_2) = (1, 1)$, $\sqrt{s} = 500$ GeV.

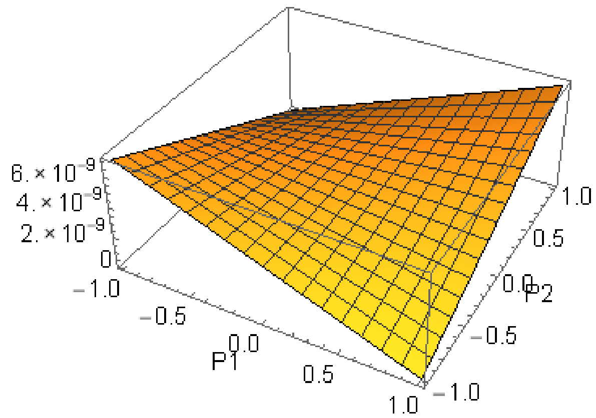

Figure 2. The cross-section as a function of the polarization coefficients (P_1, P_2) in $e^-\gamma^* \to e^-\sigma$ collision.

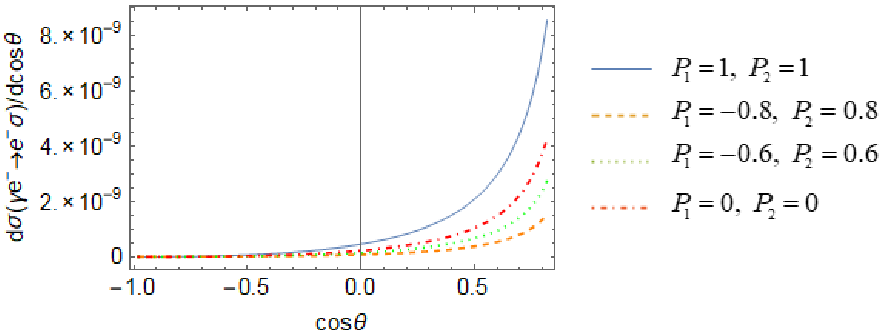

Figure 3. The differential cross-sections as a function of $\cos\theta$ in $e^-\gamma^* \to e^-\sigma$ collision.

Table 1. Typical values for the cross-section in the $e^-\gamma^* \to e^-\sigma$ collision in case of different collision energy

$\sqrt{s}$ (GeV)	500	600	700	800	900	1000
$\sigma(P_1 = 1, P_2 = 1)$ $(10^{-9}$ fb)	7.05526	3.62834	1.28301	1.35156	4.70382	1.28486
$\sigma(P_1 = 0.8, P_2 = -0.8)$ $(10^{-9}$ fb)	1.26995	0.65310	2.30942	0.24328	0.84668	0.23127
$\sigma(P_1 = 0.6, P_2 = -0.6)$ $(10^{-9}$ fb)	2.25768	1.16107	4.10563	0.43249	1.50522	0.41115
$\sigma(P_1 = 0, P_2 = 0)$ $(10^{-9}$ fb)	3.52763	1.81417	6.41505	0.67578	2.35191	0.64243
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$\sigma(P_1 = 0.6, P_2 = 0)$ $(10^{-9}$ fb)	3.52763	1.81417	6.41505	0.67578	2.35191	0.64243

Table 2. Typical values for the cross-section in the $e^-\gamma^* \to e^-\sigma$ collision in case of different saxion mass m_σ at the center-of-mass energy 500 GeV

m_σ (TeV)	10	30	50	70	90	100
$\sigma(P_1 = 1, P_2 = 1)$ $(10^{-9}$ fb)	16.653	7.055	1896.9	409.992	350.322	1407.98
$\sigma(P_1 = 0.8, P_2 = -0.8)$ $(10^{-9}$ fb)	0.0299	1.270	341.42	73.798	63.057	253.436
$\sigma(P_1 = 0.6, P_2 = -0.6)$ $(10^{-9}$ fb)	0.0533	2.258	607.01	131.197	112.103	450.554
$\sigma(P_1 = 0, P_2 = 0)$ $(10^{-9}$ fb)	0.0833	3.527	948.45	204.996	175.161	703.990
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$\sigma(P_1 = 0.6, P_2 = 0)$ $(10^{-9}$ fb)	0.0833	3.527	948.45	204.996	175.161	703.990

4. The Decay of Saxion in SUSY DFSZ Axion Model

The saxion has two possible decay channels [19]. Kahler potential interactions induce saxion decays to two axions with the rate

$$\Gamma_{\sigma}(\sigma \rightarrow aa) = \frac{\kappa^2 m_{\sigma}^3}{64\pi V_{PQ}^2}. \tag{11}$$

In general, κ is a free parameter. Superpotential and SUSY breaking interactions are responsible for visible saxion decays to SM final states. The rate of WW, ZZ, hh mode is given by [23]

$$\Gamma_{\sigma}(\sigma \rightarrow W^{+}W^{-}, ZZ, hh) = \frac{q_{\mu}^2 \mu^4}{4\pi m_{\sigma} V_{PQ}^2}. \tag{12}$$

Here, the PQ charges q_{μ} is chosen as $q_{\mu} = 2$ as in the minimal supersymmetric DFSZ model and the Higgs mass parameter $\mu = 200$ GeV. In SUSY model, PQ symmetry is broken at the scale V_{PQ} to solve the strong CP problem. The limit $V_{PQ} = f_a N_{DW} / \sqrt{2} \leq N_{DW} \times 10^{12}$ GeV is given by the misalignment mechanism. The saxion mass is varied over the range of $10\,TeV < m_{\sigma} < 100\,TeV$ [14]. In SUSY DFSZ model, the saxion mainly decays into a pair of W, Z or Higgs bosons with $m_{\sigma} > 2m_{W}$. Parameter space for the SUSY DFSZ scenario, $N_{DW} = 3n$, the value of decay constant f_a is chosen as 10^8 GeV $\leq f_a \leq 10^{12}$ GeV [11]. From the Fig. 4, we can show that the decay rate of saxion in the case of $n = 1$ is larger than $n = 2$. In the case of $n = 2$, the minimum value of the decay rate is about 10000 sec^{-1}, so the lifetime of saxion is about 10^{-4} sec. The lifetime of saxion is quite short for visible saxion decays to SM final states.

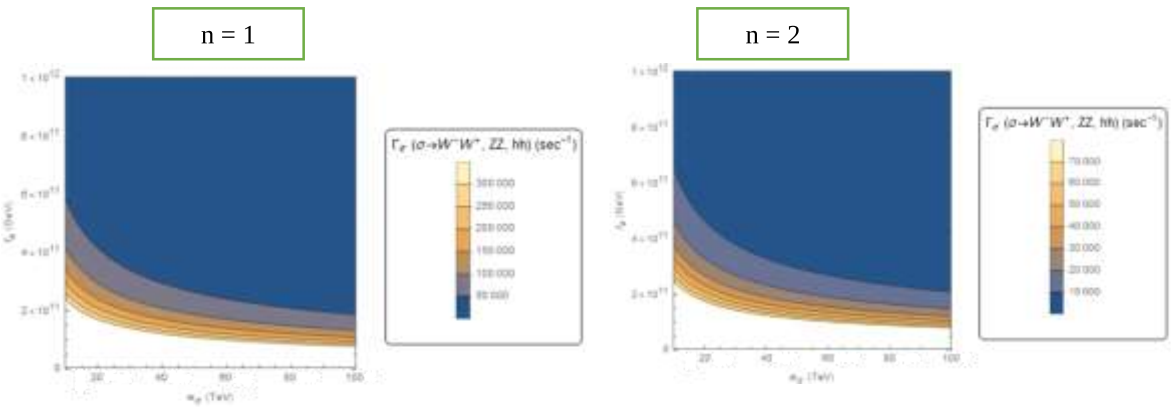

Figure 4. The decay rate of saxion to WW, ZZ, hh mode. *n* = 1 (*n* = 2) for the left (right) panel.

5. Conclusion

In our work, we have evaluated the cross-section in $e^{-}\gamma^{*} \rightarrow e^{-}\sigma$ subprocess. The cross-section is affected by the polarization coefficients of the initial and final electron beams. The cross-section achieves a maximum value when both the initial and final e^{-} beams polarize left or right and the minimum value when the initial beam polarizes left, the final beam polarizes right and vice versa. The decay of saxion into SM final states was evaluated in detail. The saxion comes to dominate the energy density of the universe before being thermalized for n = 1 and n = 2. The lifetime of saxion can reach at 10^{-4} sec.

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