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Original Article

On the Intermolecular Forces of Charged AdS Black Holes and Charged AdS White Holes

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Abstract: This paper consists of two parts. In the first part, we attempt to find the intermolecular force of charged AdS black hole (BH). Starting from the fact that the equation of states of BH and the van der Waals (vdW) equation have the same compressibility factors $Z = 3/8$, we determine the intermolecular force of BH. We find that this force can always be written as the sum of the topological force created by the topological charge, and the electrostatic force created by the conducting microsphere charged with the electric charge $Q > 0$. This is the intermolecular force for all systems whose phase transition possesses the same compressibility factor $Z = 3/8$. In part 2 we begin with the equation of state of white hole (WH) whose temperature is negative and find that its compressibility factor is equal to $Z = -3/8$, and, at the same time, we establish the anti- vdW equation with compressibility factor $Z = -3/8$. This is the main factor for us to determine the intermolecular force of WH. This force is composed of two terms. The first term is the repulsive force, created by the topological charge, and the second term exhibits the attractive electrostatic force, created by two quasi- Cooper pairs (similar to Cooper pairs in the superconductors) consisting of two charged spheric molecules with electric charge $Q < 0$. The formation of quasi-Cooper pairs is by BH a quantum effect which was realized in the process of quantum tunneling from BH to WH. At high temperature, the quasi-Cooper pairs are broken, leading to the cancellation of the attractive force, and the repulsive force will push all molecules of WH further and further away. The behaviors

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of BH force and WH force are totally suppoted by the corresponding scalar curvatures of the thermodynamic geometry.

Keywords: Intermolecular force, charged AdS black hole, white hole, quantum tunneling, topological force, phase transition.

1. Introduction

At present, the thermodynamics of charged AdS BHs has been successfully formulated [1, 2] and, as a next step, the microscopic structures of these BHs have been hot topics. They have been investigated in a lot of papers, among them we highly appreciate Refs. [3-5]. For simplicity we focus on the four dimensional RN-AdS BH whose metric reads

$$ds^2 = -fdt^2 + \frac{dr^2}{f} + r^2 d\Omega^2_{2,k}, \tag{1}$$

Where

$$f(r) = k - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}, \tag{2}$$

where L is the Anti-de Sitter (AdS) radius, which is related to the cosmological constant Λ by the relation $L^2 = -3/\Lambda$.

In (1) $d\Omega^2_{2,k}$ is the metric of a two–sphere S^2 of radius $1/\sqrt{k}$, $k>0$. For $k=0$ and $k<0$, $d\Omega^2_{2,k}$ represents respectively the metric of a plane and 2 - dimensional hyperbolic. Therefore k plays the role of a topological parameter, its different values correspond different topological configurations of $d\Omega^2_{2,k}$.

The parameters M and Q are related to the mass and charge of BH by corresponding factors. The horizon is defined as the largest root of the equation

$$f(r_+) = k - \frac{2M}{r_+} + \frac{Q^2}{r_+^2} + \frac{r_+^2}{L^2} = 0. \tag{3}$$

Taking into account (2) and (3) the temperature T and entropy S of BH are respectively given by

$$T = \frac{\hbar}{4\pi k_{B_3}}\left(\frac{df(r)}{dr}\right)_{r=r_+} = \frac{k}{4\pi r_+} - \frac{Q^2}{4\pi r_+^3} + \frac{3r_+}{4\pi L^2}, \tag{4a}$$

$$S = \frac{k_B}{G\hbar}\pi r_+^2, \tag{4b}$$

where G and k_B denote the Newton gravitational constant and the Boltzmann constant, respectively. For simplicity, from now on, we will use the geometric system in which $G = k_B = \hbar = c = 1$.

Two majors breakthroughs in the construction of the BH thermodynamics are as follows

- The cosmological constant Λ has been treated as the thermodynamical pressure [1, 2],

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi L^2}, \tag{5}$$

and

-The topological parameter k as the topological charge [6, 7],

$$\varepsilon = 4\pi k. \tag{6}$$

In this setup M becomes the enthalpy of the system and we then arrive at the extended first law of thermodynamics [8, 9],

$$dM = TdS + \omega d\varepsilon + VdP + \phi dQ, \tag{7}$$

here $\omega = r_+ / 8\pi$ is the conjugate potential of topological charge, $\phi = Q / r_+$ is the conjugate potential of charge Q and the volume $V = 4\pi r_+^3$.

Based on (4), (5) and (6) we obtain the BH equation of state

$$P = \frac{T}{2r_+} - \frac{\varepsilon}{32\pi^2 r_+^2} + \frac{Q^2}{8\pi r_+^4}. \tag{8}$$

The critical point of the BH transition is obtained from

$$\left(\frac{\partial P}{\partial v}\right)_{T,\varepsilon,Q} = 0, \left(\frac{\partial^2 P}{\partial v^2}\right)_{T,\varepsilon,Q} = 0,$$

$$v = 2r_+,$$

which gives

$$T_c = \frac{\sqrt{6}\varepsilon^{3/2}}{96\pi^{3/2}Q}, r_c = \frac{2\sqrt{6\pi}Q}{\sqrt{\varepsilon}}, P_c = \frac{\varepsilon^2}{1536\pi^3 Q^2}, \tag{9}$$

with the corresponding compressibility factor

$$Z = \frac{P_c v_c}{T_c} = \frac{3}{8}. \tag{10}$$

The corresponding law [10] indicates that all systems,which have the same compressibility factor, belong to the same corresponding states. In other words, all physical systems, which undergo the first-order phase transition, are classified by their compressibility factors.

In Fig. 1 is shown the phase diagram of the phase transition from small to large BHs.

It is remembered that v and n in the above equations are determined in Refs. [2, 11] as follows

$$N = \frac{A}{6l_P^2}, n = \frac{N}{V} = \frac{1}{2l_P^2 r_+},$$

here $l_P = \sqrt{\hbar G_N / c^3}$ is the Planck length. In order to determine the intermolecular force of BH, let us begin with the vdW equation of the state of the liquid which was established by vdW a long time ago [12]. It has the form where

$$\left(P + \frac{a}{v^2}\right)(v - b) = T, \tag{11}$$

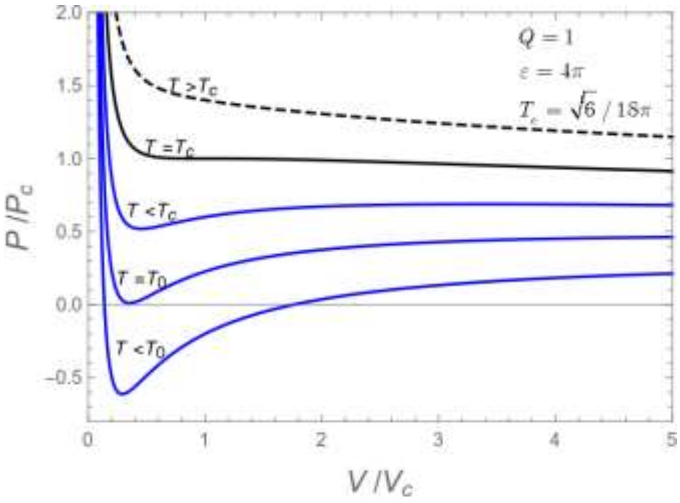


Figure 1. Phase diagram of the phase transition from small to large BH.

$$a = U_0 v_0, b = v_0 = r_0^3. \tag{12}$$

and U_0 and r_0 are the parameters of the Lennard – Jones (LJ) energy potential

$$U(r) = 4U_0 \left[\left(\frac{r_0}{r} \right)^{12} - \left(\frac{r_0}{r} \right)^{6} \right].$$

The critical point of Eqs.(11, 12) reads

$$T_c = \frac{8a}{27b}, v_c = 3b, P_c = \frac{a}{27b^2}.$$

It is clear that

$$\frac{P_c v_c}{T_c} = \frac{3}{8}.$$

which proves that the charged AdS BH and the vdW equation state of liquids are in the same corresponding states. In order to make clear the role of the LJ potential in the structure of the BH intermolecular force let us connect the critical point of BH equation of state with that of the vdW equation. We find that

$$U_0 = \frac{9a}{10\pi b}, r_0^3 = \frac{b}{\sqrt{2}}, \tag{13}$$

$$a = \frac{3\varepsilon}{16\pi^2}, b = \frac{4\sqrt{6}\pi Q}{3\sqrt{\varepsilon}}.$$

Then, from (13) we get the LJ potential and then the force which controls that BH phase transition

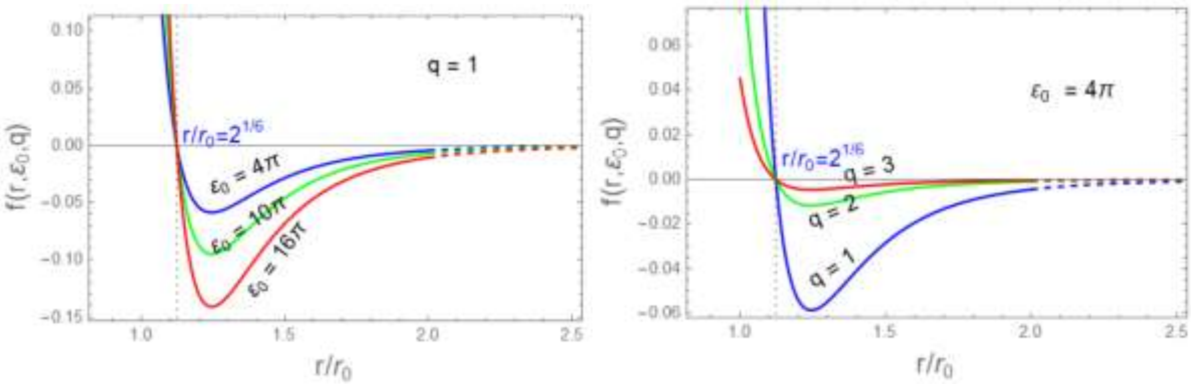
$$F(r) = \frac{24U_0(\varepsilon, Q)}{r} \left[2\left(\frac{r_0(\varepsilon, Q)}{r} \right)^{12} - \left(\frac{r_0(\varepsilon, Q)}{r} \right)^{6} \right] \tag{14}$$

In order that the force (14) becomes the interaction force between two adjacent molecules the topological charge   and the electrical charge Q must be replaced by the corresponding charges of molecules

$$\varepsilon_0 = \frac{\varepsilon}{N}, q = \frac{Q}{N}.$$

It results that the interaction force between the BH molecules reads

$$F_{mol}(r) = \frac{24U_0(\varepsilon_0, q)}{r}\left[2\left(\frac{r_0(\varepsilon_0, q)}{r}\right)^{12} - \left(\frac{r_0(\varepsilon_0, q)}{r}\right)^{6}\right]. \tag{15}$$



(a) (b)

Figure 2. The evolution of $f(r, \varepsilon_0, q)$ versus r/r_0 at several values of ε_0 and q. The solid (dashed) lines correspond to small (large) BH.

Next, we analyze the force (15) in detail. To this end, at first we investigate the quantity

$$f(r, \varepsilon_0, q) = \frac{F_{mol}(r, \varepsilon_0, q)}{b}. \tag{16}$$

Combining (15) and (16) we arrive at the intermolecular force

$$f(r, \varepsilon_0, q) = f_\varepsilon(r, \varepsilon_0) + f_Q(r, q), \tag{17}$$

Where

$$f_\varepsilon(r, \varepsilon_0) = -\frac{27}{40\pi^3}\frac{\varepsilon_0}{r^7} \tag{18}$$

is the attractive force created by the topological charges of molecules and

$$f_Q(r, q) = \frac{36}{5\pi^2}\frac{q^2}{r^{13}}, \tag{19}$$

denotes the repulsive electrostatic force created by two charged spheric molecules with electric charge $q > 0$ [13]. In Fig. 2 the r dependences of $f(r, \varepsilon_0, q)$ are shown at several values of ε_0 and q.

The present paper is organized as follows. In Section II the intermolecular foce of WH will be established and the Section III is devoted to the thermodynamic geometry. Sections and V deal with the WH micromolecule and the Conclusion, respectively.

2. The Intermolecular Force of WH

It is well known that the transition from BH to WH takes place in the process of quantum tunnelling. It is one of many processes which leads to formation of WH. In this paper, for simplicity, we focus on the case when both BH and WH have the same M ,the same horizon event r_+, the same topological charge ε, but the temperature of WH is negative, $T<0$. The systems with negative temperature have been investigated in [14-17]. Relating to the negative temperature it is worth to remember that the negative temperature was observed experimentally in several condensed matter systems [18, 19].

Then, it is easily recognized that the following equation

$$P=-\frac{T}{2r_+}-\frac{\varepsilon}{32\pi^2 r_+^2}+\frac{Q^2}{8\pi r_+^4}, \tag{20}$$

is the equation of state of WH for $Q<0$, and the critical point of the WH transition from small to large WHs reads

$$T_c=\frac{\varepsilon^{3/2}}{24\sqrt{6}\pi^{5/2}Q}<0, r_c=-\frac{2\sqrt{6\pi}Q}{\sqrt{\varepsilon}}>0, P_c=\frac{\varepsilon^2}{1536\pi^3 Q^2}>0, \tag{21}$$

Eqs. (21) satisfy all requirements imposed above if $Q<0$.

The compressibility factor of WH is

$$Z=-\frac{3}{8}. \tag{22}$$

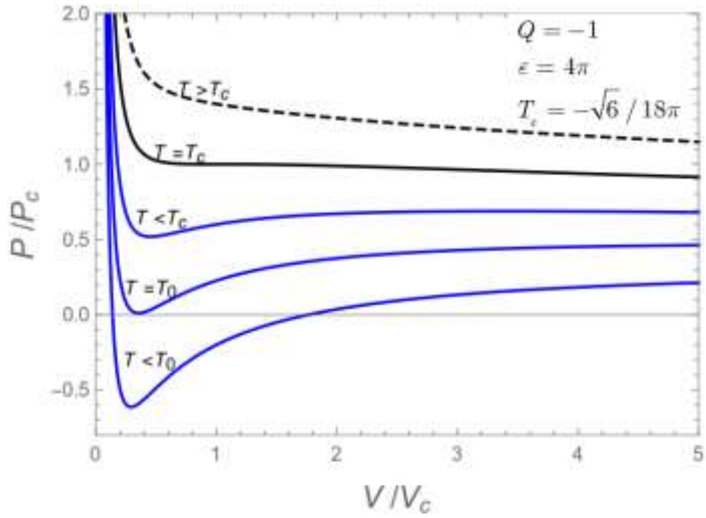


Figure 3. The phase diagram of the transition from the small to large WHs.

$Q=1$

$\varepsilon=4\pi$

$T_c=\sqrt{6}/18\pi$

In Fig. 3 is plotted the phase transition from the small to large WHs.

It is necessary to emphasize that, in principle, the state with \$T < 0\$ is hotter than the state with $T > 0$, since the heat flows from the negative to the positive – tempearture systems. In our case the heat escape fom WH and will be absorbed by BH. On the other hand, we establish the equation, similar to the vdW equation, in which the temperature is negative, $T < 0\ (\mathrm{T}_c < 0)$. It reads

$$\left(P+\frac{c}{v^2}\right)(v-d)=-T, \tag{23}$$

With

$$c = \mathrm{U}_0 v_0, d = v_0. \tag{24}$$

The critical point of eqs. (23, 24) is given by

$$T_c=-\frac{8U_0}{27}<0, v_c=3v_0, P_c=\frac{U_0}{27v_0}>0,$$

and the corresponding compressibility factor reads

$$Z=-\frac{3}{8}. \tag{25}$$

For convenience, from now on, the equation (23, 24) is called the Anti – vdW equation. Eqs. (22) and (25) demonstrate that the WH and the Anti- vdW equation are in the same corresponding state. Connecting their critical points we get

$$U_0=-\frac{3\sqrt{6}\varepsilon^{3/2}}{128\pi^{5/2}Q}, v_0=-\frac{4\sqrt{6\pi}Q}{3\sqrt{\varepsilon}}. \tag{26}$$

From (26) we get the LJ potential and the the LJ force which controls the WH phase transition

$$F_{WH}(r)=v_0\left[f_\varepsilon(r)+f_Q(r)\right], \tag{27}$$

Where

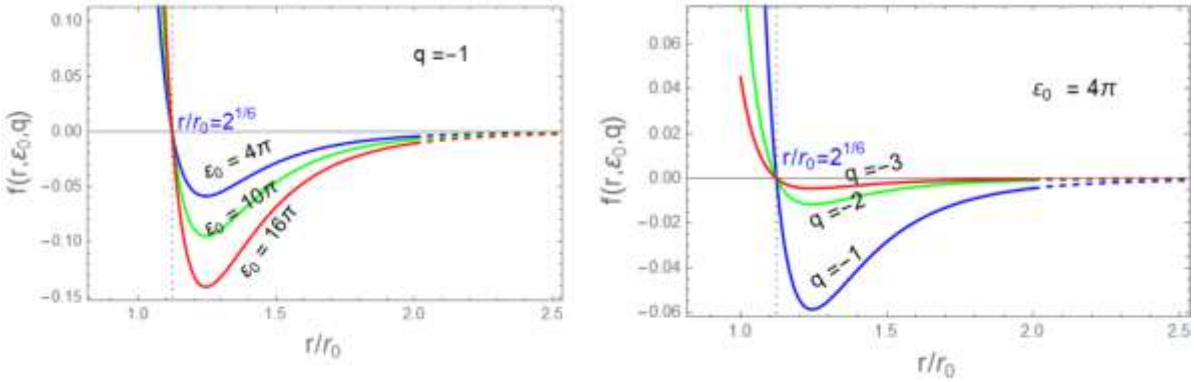
$$f_\varepsilon(r)=\frac{27}{40\pi^3}\frac{\varepsilon}{r^7}, \tag{28}$$

is the repulsive force created by the topological charge and

$$f_Q(r)=-\frac{36}{5\pi^2}\frac{Q^2}{r^{13}}, \tag{29}$$

denotes the attractive electrostatic force created by quasi-Cooper pairs (similar to the Cooper pairs in superconductors) consisting of two charged spheric molecules with electric charge $Q<0$ [13]. It is well known that in superconductors the formation of Cooper pairs is a quantum effect. In this respect, the formation of quasi- Cooper pairs in WH was established in the quantum tunnelling process transition from BH to WH [20]. At high temperature all quasi- Cooper pairs will be annihilated,leading to the cancellation of the attractive force, the repulsive force will then push the WH molecules further and further apart. This fact explains why WHs are unstable.

In Fig. 4 is plotted the evolution of $f(r)=f_\varepsilon(r)+f_Q(r)$ at several values of topological and electric charges.



(a) (b)

Figure 4. The evolution of $f(r)$ at several values of topological and electric charges.

In the next section we shall prove that the previous properties of the WH force (27) will be supported by the Ruppeiner thermodynamic geometry [19].

3. Thermodynamic Geometry

In recent years the thermodynamic geometry [20-23] has been applied to consider the interaction force between micromolecules of charged AdS BH [2-4, 24-26]. However, the application of thermodynamic geometry to WH is unprecedented. To solve this problem let us first list several results obtained for charged AdS black holes. In Refs. [4] and [5] the Riemann space with the metric

$$ds^2 = \frac{\partial^2 M}{\partial x^i \partial x^k} dx^i dx^k,$$

was chosen and the temperature T and the volume v were taken as the fluctuation coordinates. Choosing $x^0 = v$ and $x^1 = P$ we get the reduced scalar curvature R / R_c

$$\frac{R}{R_c} = \frac{1-3r_+^2}{2r_+^5 Q^2},$$

with

$$R_c = \frac{\varepsilon}{48\pi^2 Q^2}.$$

The evolution of reduced thermodynamic curvature versus for small and large BH are plotted in Fig. 5 for reduced curvature and Fig. 6 for curvature, respectively.

It is worth to remark that the explicit expression of the BH interaction force was not obtained in Ref. [4] as a consequence, the authors made use of Fig. 4 to interprete the behavior of this force. In [5] it was indicated that the behaviors of R / R_c and R are in good accordance with the interaction forces between microscopic molecules of Charged BH.

Taking into account the properties of the WH interaction force presented in the Section II let us start from the geometry with metric

$$ds^2 = -\frac{\partial^2 M}{\partial x^i \partial x^k} dx^i dx^k,$$

The temperature $T<0$ is interpreted as the fluctuation coordinate and choosing $x^0 = v$ and $x^1 = P$ we obtain

$$\tilde{R}_R = \frac{-16+12\tilde{\nu}^2}{\tilde{\nu}^5\tau},$$

where

$$R_c = \frac{\varepsilon}{48\pi^2 Q^2}, \tilde{\nu} = \frac{v}{v_c}, \tau = \frac{T}{T_c}, (T_c < 0).$$

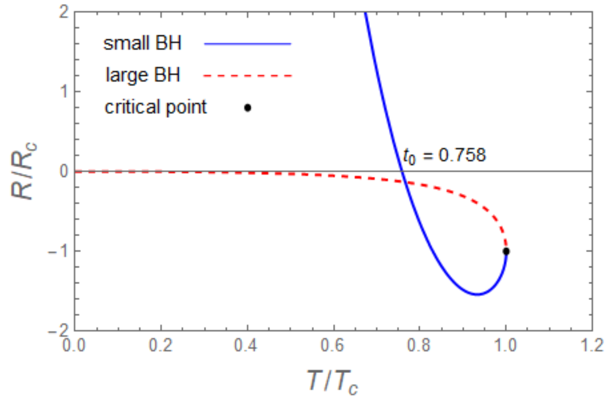
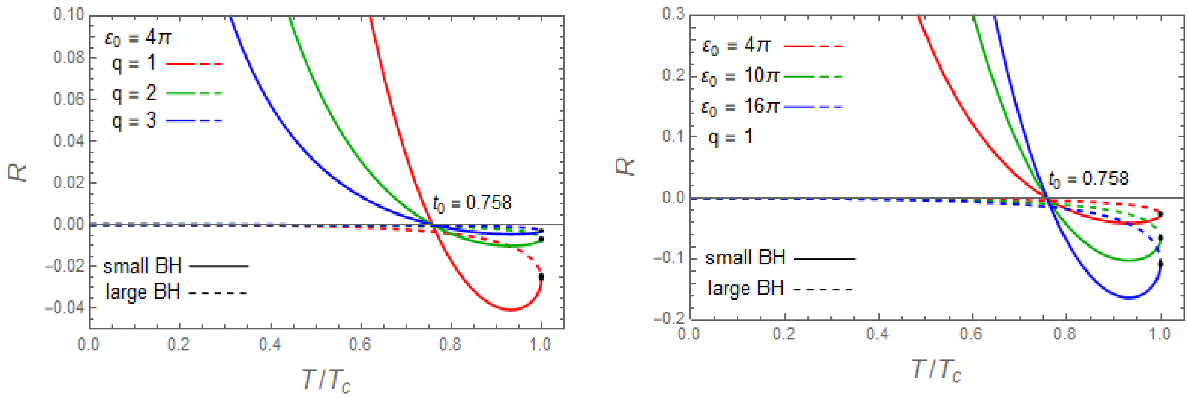


Figure 5. The evolution of R/R_c versus $\tau = T/T_c$. The solid (dashed) lines correspond to small (large) BH.



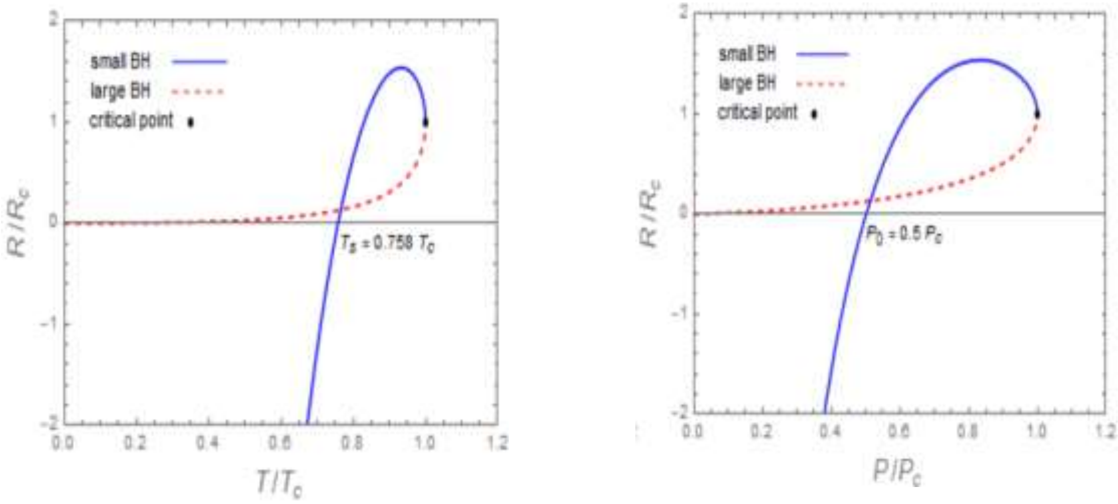
(a) (b)

Figure 6. The evolution of R versus $\tau = T/T_c$ at several values of ε and Q. The solid (dashed) lines correspond to small (large) BH.

The evolutions of R/R_c versus $\tau = T/T_c$ (a) and P/P_c (b) are shown in Fig. 7.

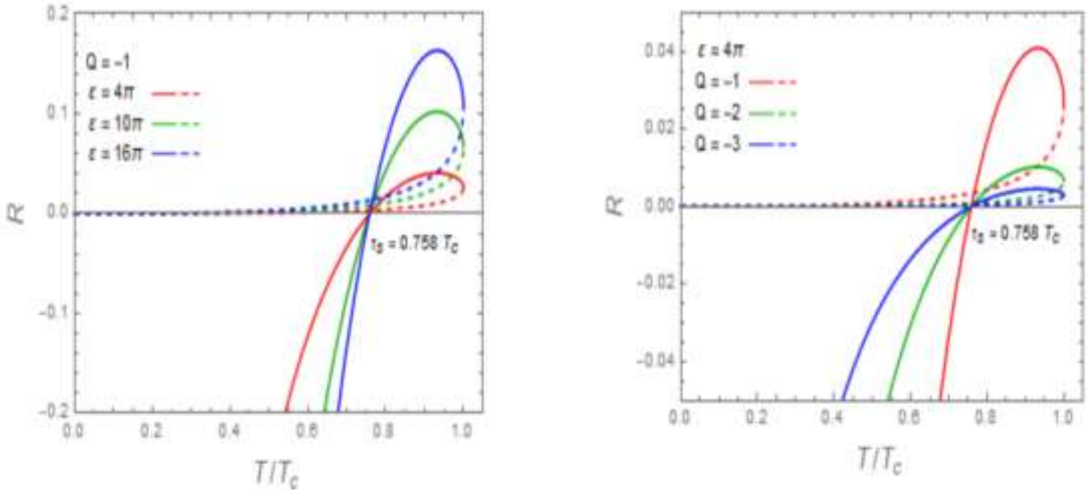
In addition to Fig.7 we plot in Fig. 8 the behaviors of R at different values of the topological charge and electrical charge.

Confronting Fig.5 and Fig. 6 with Fig. 7 and Fig.8 allows us to confirm that the thermodynamic geometry describes rather well the behavior of the intermolecular force (27) of WH.



(a) (b)

Figure 7. The evolutios of R/R_c versus $\tau = T/T_c$ at several values of ε and Q. The solid (dashed) lines correspond to small (large) BH.



(a) (b)

Figure 8. The evolutios of R versus $\tau = T/T_c$ at several values of ε and Q. The solid (dashed) lines correspond to small (large) BH.

4. Micromolecules of BH and WH

The interaction forces (17) and (27) are the milestones to clear the structure of the BH and WH micromolecules.

i) BH molecules have positive electric charge, $Q > 0$, which creates the repulsive electric force, and topological charge ε, which creates the attractive force between two adjacent molecules.

ii) The WH is created by BH in a quantum tunneling process. Consequently, the topological and electric properties of WH are quite different from the precedent properties of BH. Namely the topological force becomes repulsive and the micromolecules of WH bear negative electric charges,they generate the attractive force of quasi- Cooper pairs.

5. Conclusion and Discussion

The main results we have found in the preceding Sections can be summarized as follows.

A-Black hole

1- The interaction force between two adjacent molecules of charged AdS BH is of the form

$$f\left(r,\varepsilon_0,q\right)=-\frac{27}{40\pi^2}\frac{\varepsilon_0}{r^7}+\frac{36}{5\pi^2}\frac{q^2}{r^{13}},$$

which allows us to discover the microscopic structure of BH : they are the charged conducting microspheres with defined radius. Their topological charges create the attractive topological force and their electric charges create the electrostatic force.

2- The properties of $f\left(r,\varepsilon_0,q\right)$ are totally confirmed by the thermodynamic eometry.

3- The phase transition from small to large BHs can be briefly described as ollows:

- At $T<T_c$ the BH behaves like fluid of topological ions.

- At higher $T, T\geq T_c$ there occurs the transition from fluid of topological ions to gas of topological ions.

It is worth to emphasize again that the interaction force $f\left(r,\varepsilon_0,q\right)$ is common for all systems whose compressibility factor equals

$$Z=\frac{P_c v_c}{T_c}=\frac{3}{8},$$

It is not applied for those systems which have value of Z different from 3/8, of course.

B-White Hole

1- The interaction force between two adjacent molecules of charged AdS WH reads

$$f(r,\varepsilon,Q)=f_\varepsilon\left(r,\varepsilon\right)+f_Q\left(r,Q\right),$$

in which

$$f_\varepsilon\left(r,q\right)=\frac{27}{40\pi^3}\frac{\varepsilon_0}{r^7},$$

is the repulsive force created by topological charges of molecules and

$$f_Q\left(r,q\right)=-\frac{36}{5\pi^2}\frac{q^2}{r^{13}},$$

denotes the attractive electrostatic force created by the quasi- Cooper pairs which consist of two charged spheric molecules with electric charge $q<0$. All quasi- Coopers pairs become broken when the temperature T increases, and, as a consequence, only the repulsive force which pushes all molecules of

WH far from each other. The creation of quasi- Cooper pairs in WH is realized in the quantum tunnelling process from BH to WH.

It is important to note that this is the first one investigated systematically the WH phase transition and its intermolecular force.

To end this paper, it is very interesting to compare our BH interaction force with that found in Ref. [27],

$$F_{mol}(r) = \frac{d\phi(r)}{dr},$$

where

$$\phi(r) = 4\phi_0\left[\left(\frac{r_0}{r+r_0}\right)^{12} - \left(\frac{r_0}{r+r_0}\right)^{6}\right],$$

$$\phi_0 \approx \frac{0.0003388}{Q}, r_0 = \frac{495\left(1-4q^2\right)}{248\pi^2\phi_0}.$$

and q is effective specific charge.

Acknowledgments

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