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Original Article

Theoretical Study of the Hall Effect in Infinite Semi-parabolic Quantum Wells in the Presence of Electromagnetic Waves by Using Quantum Kinetic Equation

Nguyen Dinh Nam, Bui Thi Dung*, Nguyen Minh Hieu, Do Tuan Long

VNU University of Science, 334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam

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Abstract: Theoretical study of the Hall effect in an infinite semi-parabolic Quantum Well (ISPQW) in the presence of electromagnetic using the quantum kinetic equation in the case of electron-acoustic phonon scattering. With the quantum kinetic equation method, the electron distribution function is constructed, from which analytic expressions for the conductivity tensor and the Hall coefficient are derived. The results reveal that the Hall coefficient exhibits a nonlinear dependence on temperature T, magnetic field B, electromagnetic wave frequency Ω, and confinement frequency ω_z. Numerical calculations are carried out for a GaAs/AlGaAs quantum well, analyzing the influence of external fields and confinement parameters. Notably, the presence of the electromagnetic wave induces quantum oscillations of the Shubnikov–de Haas type and reduces the average value of the Hall coefficient at high magnetic fields. The results contribute to a deeper understanding of the quantum nature of the Hall effect in asymmetric quantum systems and expand the potential applications of low-dimensional semiconductor structures in advanced nanoelectronic and quantum technologies.

Keywords: Hall effect, electron-acoustic phonon scattering, infinite semi-parabolic Quantum Well, electromagnetic wave, quantum kinetic equation.

1. Introduction

The rapid development of science and technology in recent decades has paved the way for the fabrication and study of low-dimensional semiconductor materials and nanostructures, such as superlattices, quantum wells, quantum wires, and quantum dots [1-5]. In these systems, the motion of

* Corresponding author.
E-mail address: buithidung@hus.edu.vn

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charge carriers (electrons and holes) is strongly confined in one or more spatial directions, leading to energy quantization and the emergence of novel physical phenomena not found in bulk semiconductors.

Among the most significant phenomena in low-dimensional systems is the Hall effect, especially under the influence of strong electromagnetic waves and low temperatures [6-8]. In such conditions, the energy levels of charge carriers are quantized into Landau levels, resulting in striking effects such as the Integer and Fractional Quantum Hall Effects-key discoveries in condensed matter physics that were recognized with Nobel Prizes [9]. Continued interest in the Hall effect in asymmetric systems, particularly under strong external fields and interactions with confined phonons [10-12]. While well understood in bulk and simple low-dimensional systems, its behavior in complex quantum wells under external fields remains less explored. In particular, the combined effects of a perpendicular magnetic field, a static electric field, and an intense electromagnetic wave on the Hall effect in ISPQWs have not been thoroughly investigated.

In this work, we use the quantum kinetic equation method to study the Hall effect in ISPQWs, considering electron–acoustic phonon scattering. Analytical expressions for the Hall coefficient are derived, and numerical results show its dependence on magnetic field strength, temperature, electromagnetic wave intensity and frequency, and confinement potential. Remarkably, strong oscillations in the Hall coefficient are observed at low temperatures and magnetic fields, highlighting the distinct transport behavior in ISPQW structures compared to bulk materials.

2. Theoretical Framework

2.1. Energy Spectrum and Wave Function of Electrons in a Quantum Well when Placed in Perpendicular Electric and Magnetic Fields

Consider an electron trapped in a quantum well of the form:

$$V(z) = \begin{cases} \infty, & z < 0 \\ \dfrac{1}{2} m_e \omega_z^2 z^2, & z \geq 0 \end{cases} \tag{1}$$

where m_e is the mass of the electron; ω_z and β_z is a parameter characterizing the confinement.

Place the quantum well (1) in a magnetic field $\vec{B} = (0,0,B)$ and an electric field $\vec{E} = (E,0,0)$. Choose the vector potential of the above magnetic field as $\vec{A} = (0, Bx, 0)$ then the single-particle wave function and the corresponding energy of the electron are [13, 14]:

$$\psi(\vec{r}) = \psi_0 \Phi(x - x_0) e^{ik_y y} \phi_n(z) \tag{2}$$

$$\varepsilon_{N,n}(\overrightarrow{k_y}) = \hbar\omega_c \left(N + \frac{1}{2}\right) + \hbar\omega_z \left(2n + \frac{3}{2}\right) + \hbar\nu_d k_y - \frac{1}{2} m_e \nu_d^2 \tag{3}$$

where N is the Landau level index and n is the sublevel index, $\omega_c = \frac{eB}{m_e}$ is the cyclotron frequency, $\nu_d = -\frac{E_1}{B}$ is the velocity of electron drift. $\Phi(x - x_0)$ is the wave function centered at $x_0 = -l^2 \left(k_y - \frac{m_e \nu_d}{\hbar}\right)$, where $l = \sqrt{\frac{\hbar}{eB}}$ is the radius of the cyclotron in the Oxy plane, $\phi_n(z)$ is an eigenfunction and is given by:

$$\phi_n(z) = A_n \cdot z^{2s} \cdot e^{-\frac{z^2}{\alpha_z^2}} \cdot L_n^{\gamma}\left(\frac{z^2}{\alpha_z^2}\right) \tag{4}$$

Where $L_n^{\gamma}(z)$ Laguerre polynomial; $s = \frac{1}{4}\left(1 + \sqrt{1 + 4\beta_z}\right)$; $\gamma = 2s - \frac{1}{2}$;$\alpha_z = \sqrt{\frac{\hbar}{m_e\omega_z}}$.

2.2. Hall Effect in the Infinite Semi-parabolic Quantum Well under the Influence of a Strong Electromagnetic Wave

The Hamiltonian of the electron-phonon network in the second quantized representation is written as:

$$\begin{aligned} H \; &= \sum_{N,n,\overrightarrow{k_y}} \varepsilon_{N,n}\left(\overrightarrow{k_y} - \frac{e}{\hbar c}\vec{A}(t)\right) a^+_{N,n,\overrightarrow{k_y}} a_{N,n,\overrightarrow{k_y}} + \sum_{\vec{q}} \hbar\,\omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} + \\ &+ \sum_{N,N'}\sum_{n,n'}\sum_{\vec{q},\overrightarrow{k_y}} C_{\vec{q}} I_{n,n'}(q_z) J_{N,N'}(u)\, a^+_{N',n',\overrightarrow{k_y}+\vec{q}} a_{N,n,\overrightarrow{k_y}}\left(b_{\vec{q}} + b^+_{-\vec{q}}\right) + \sum_{n,N,\vec{k}} \varphi\left(\vec{k}\right) a^+_{n,N,\overrightarrow{k_y}+\vec{q}}\, a_{n,N,\overrightarrow{k_y}} \end{aligned} \tag{5}$$

In which ${a^+}_{n,,N,\overrightarrow{p_y}}$ and $a_{n,,N,\overrightarrow{p_y}}$ ($b^+_{\vec{k}}$ and $b_{\vec{k}}$) are the creation and annihilation operators of electron (phonon). The vector potential of laser radiation is the electromagnetic wave (EMW) $A(t) = \frac{c}{\Omega}E_0\cos(\Omega t)$. n and n' are the band indices of states $|\, n, N, \overrightarrow{k_y}\rangle$ and $|\, n', N', \overrightarrow{k_y} + \vec{q}\rangle$, respectively $\varepsilon_{N,n}$ is the electron energy spectrum, $\overrightarrow{k_y}$, $\vec{q}$ are the wave vectors of electrons and phonons, respectively. $C(\vec{q})$ is the interaction constant that depends on the type of electron-phonon interaction.

$I_{n,n'}$ is the form factor of the electron and is given by:

$$I_{n,n'} = \frac{11\sqrt{\pi}}{\sqrt{2}\alpha_z} \tag{6}$$

and

$$\left|J_{N,N'}(u)\right|^2 = \frac{N_{min}!}{N_{max}!} e^{-u} u^{N_{max}-N_{min}}\left[L_{N_{min}}^{N_{max}-N_{min}}(u)\right]^2 \tag{7}$$

where $N_{min} = \min\{N, N'\}$, $N_{max} = \max\{N, N'\}$, $u = \frac{l_B^2 q_\perp^2}{2}$, $q_\perp^2 = q_x^2 + q_y^2$, $L_{N_{min}}^{N_{max}-N_{min}}(u)$ is the adjoint Laguerre polynomial. $\varphi\left(\vec{k}\right) = (2\pi i)^3\left(e\vec{E} + \omega_H\left[\vec{k}, \vec{h}\right]\right)\frac{\partial}{\partial\vec{k}}\delta\left(\vec{k}\right)$ is a scalar potential.

Using Hamiltonian (5) in the quantum dynamic equations for the particle number operator in the infinite semi-parabolic quantum well:

$$i\hbar\frac{\partial f_{N,n,\overrightarrow{k_y}}(t)}{\partial t} = <\left[a^+_{N,n,\overrightarrow{k_y}} a_{N,n,\overrightarrow{k_y}}, H\right]>_t \tag{8}$$

The quantum kinetic equation for the electron distribution has now become:

$$\begin{aligned} &-\left(e\overrightarrow{E_1} + \hbar\omega_c\left[\overrightarrow{k_y}, \vec{h}\right]\right)\frac{\partial f_{N,n,\overrightarrow{k_y}}}{\hbar\,\partial\overrightarrow{k_y}} + \frac{h\overrightarrow{k_y}}{m_e}\frac{\partial f_{N,n,\overrightarrow{k_y}}}{\partial\vec{r}} = -\frac{f_{N,n,\overrightarrow{k_y}} - f_0}{\tau} \\ &+\frac{2\pi}{\hbar}\sum_{N',n',\vec{q}} |C(\vec{q})|^2\left|I_{n,n'}\right|^2\left|J_{N,N'}(u)\right|^2\sum_{s=-\infty}^{+\infty} J_s^2\left(\frac{\lambda}{\Omega}\right)\{\left[f_{N',n',\overrightarrow{k_y}+\overrightarrow{q_y}}\left(N_{\vec{q}} + 1\right) - f_{N,n,\overrightarrow{k_y}}N_{\vec{q}}\right], \\ &\times\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y} + \overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) - \hbar\omega_{\vec{q}} - s\hbar\Omega\right) + [f_{N',n',\overrightarrow{k_y}-\overrightarrow{q_y}}N_{\vec{q}} \\ &-f_{N,n,\overrightarrow{k_y}}\left(N_{\vec{q}} + 1\right)]\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y} - \overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) + \hbar\omega_{\vec{q}} - s\hbar\Omega\right)\} \end{aligned} \tag{9}$$

where $\vec{h} = \frac{\vec{B}}{B}$ is the unit vector along the magnetic field, f_0 is the Fermi-Dirac distribution, τ is the momentum relaxation time of the electron ($\tau \sim \varepsilon^{-\frac{1}{2}}$), $f_{N,n,\overrightarrow{k_y}}$ is the electron distribution function perturbed by an external field and $\lambda = \frac{e\overrightarrow{E_0}\overrightarrow{q_y}}{m_e\Omega}$.

In (10), we limit the problem to the case of $s = -1,0,1$ so that we get the expression for the specific current density:

$$\begin{aligned}\vec{R}(\varepsilon) \ &= \frac{\tau}{1+\omega_c^2\tau^2}\{\left(\vec{Q}(\varepsilon)+\vec{S}(\varepsilon)\right) - \omega_c\tau\left([\vec{h},\vec{Q}(\varepsilon)] + [\vec{h},\vec{S}(\varepsilon)]\right) \\ &+\omega_c^2\tau^2\left(\vec{Q}(\varepsilon)\vec{h} + \vec{S}(\varepsilon)\vec{h}\right)\vec{h}\}\end{aligned} \tag{10}$$

where

$$\vec{Q}(\varepsilon) = -\frac{e}{m_e}\sum_{N,n,\overrightarrow{k_y}} \overrightarrow{k_y}\left(\vec{F}\frac{\partial f_{N,n,\overrightarrow{k_y}}}{\partial \overrightarrow{k_y}}\right)\delta\left(\varepsilon - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right)\right) \tag{11}$$

and

$$\begin{aligned}\vec{S}(\varepsilon) \ &= \frac{2\pi e}{m_e}\sum_{N'.n'}\sum_{N,n}\sum_{\vec{q},\overrightarrow{k_y}} |C(\vec{q})|^2\left|I_{n,n'}\right|^2\left|J_{N,N'}(u)\right|^2 N_{\vec{q}}\overrightarrow{k_y} \\ &\times\{\left[f_{N',n',\overrightarrow{k_y+q_y}} - f_{N,n,\overrightarrow{k_y}}\right][\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}+\overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) - \hbar\omega_{\vec{q}}\right) \\ &+\frac{\lambda^2}{4\Omega^2}\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}+\overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) - \hbar\omega_{\vec{q}} + \hbar\Omega\right) \\ &+\frac{\lambda^2}{4\Omega^2}\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}+\overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) - \hbar\omega_{\vec{q}} - \hbar\Omega\right)] \\ &+\left[f_{N',n',\overrightarrow{k_y-q_y}} - f_{N,n,\overrightarrow{k_y}}\right][\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}-\overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) + \hbar\omega_{\vec{q}}\right) \\ &+\frac{\lambda^2}{4\Omega^2}\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}-\overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) + \hbar\omega_{\vec{q}} + \hbar\Omega\right) \\ &+\frac{\lambda^2}{4\Omega^2}\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}-\overrightarrow{q_y}\right) - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right) + \hbar\omega_{\vec{q}} - \hbar\Omega\right)]\}\delta\left(\varepsilon - \varepsilon_{N,n}\left(\overrightarrow{k_y}\right)\right)\end{aligned} \tag{12}$$

The expression is general and can be applied to different types of phonons. From this expression, we get the expression for the current density according to the formula:

$$\vec{J} = \int_0^\infty \vec{R}(\varepsilon)d\varepsilon \tag{13}$$

At low temperatures, acoustic phonon scattering has an important role. If the temperature is low enough, the electron gas is assumed to be degenerate. Besides, in terms of Acoustic Phonon Scattering, $\hbar\omega_{\vec{q}} = \hbar v_s q$, $N_{\vec{q}} = \frac{k_B T}{\hbar\omega_{\vec{q}}} = \frac{1}{\beta\hbar v_s q}$. If the scattering is elastic, we can ignore the energy of the acoustic phonons in the delta functions in the expression (12). Here, the magnetic field and electromagnetic waves are strong enough that the distance between the Landau levels (cyclotron energy) and the photon energy is much larger than the acoustic phonon energy. So that (12) becomes:

$$\vec{S}(\varepsilon) = \frac{4\pi e}{m_e}\sum_{N,N'}\sum_{n,n'}\sum_{\vec{q},\vec{k_y}}|C(\vec{q})|^2\left|I_{n,n'}\right|^2\left|J_{N,N'}(u)\right|^2 N_{\vec{q}}\overrightarrow{k_y}\left[f_{N',n',\overrightarrow{k_y}+\overrightarrow{q_y}} - f_{N,n,\overrightarrow{k_y}}\right]\times$$

$$\times\left[\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}+\overrightarrow{q_y}\right)-\varepsilon_{N,n}\left(\overrightarrow{k_y}\right)\right)+\frac{\lambda^2}{4\Omega^2}\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}+\overrightarrow{q_y}\right)-\varepsilon_{N,n}\left(\overrightarrow{k_y}\right)+\hbar\Omega\right)+\right.$$

$$\left.+\frac{\lambda^2}{4\Omega^2}\left(\varepsilon_{N',n'}\left(\overrightarrow{k_y}+\overrightarrow{q_y}\right)-\varepsilon_{N,n}\left(\overrightarrow{k_y}\right)-\hbar\Omega\right)\right]\delta\left(\varepsilon-\varepsilon_{N,n}\left(\overrightarrow{k_y}\right)\right) \quad (14)$$

The acoustic electron-phonon Scattering interaction coefficient is the following:

$$|C(\vec{q})|^2 = \frac{\hbar\xi^2\vec{q}}{2V_0\rho v_s} \quad (15)$$

From the above equations, we finally obtain the expression for the total current density:

$$\vec{J} = \frac{\tau}{1+\omega_c^2\tau^2}\left\{\vec{Q}+\vec{S}-\omega_c\tau\left[h,\vec{Q}+\vec{S}\right]+\omega_c^2\tau^2 h\left(\vec{h},\vec{Q}+\vec{S}\right)\right\} = L\left(\vec{Q}\right)+L\left(\vec{S}\right) \quad (16)$$

with:

$$L_0\left(\vec{Q}(\varepsilon)\right) = \frac{a\tau(\varepsilon)}{1+\omega_c^2\tau^2(\varepsilon)}\delta_{jm}\left[\delta_{ij}-\omega_c\tau(\varepsilon)\varepsilon_{ijk}h_k+\omega_c^2\tau^2 h_i h_j\right]E_{im} \quad (17)$$

$$L_0\left(\vec{S}(\varepsilon)\right) = b\left(\frac{\tau(\varepsilon)}{1+\omega_c^2\tau^2(\varepsilon)}\right)\left[\delta_{ij}S_j-\omega_c\tau(\varepsilon)\varepsilon_{ijk}h_k+\omega_c^2\tau^2(\varepsilon)h_i h_j\right]\delta_{il}\times$$

$$\times\left(\frac{\tau(\varepsilon)}{1+\omega_c^2\tau^2(\varepsilon)}\right)\left\{\delta_{lm}-\omega_c\tau\varepsilon_{lmp}h_p+\omega_c^2\tau^2 h_l h_m\right\}E_{im} \quad (18)$$

The terms a and b are given below:

$$a = \frac{e^2 L_y}{2\pi m_e\hbar^2 v_d}\left(\varepsilon_{N,n}-\varepsilon_F\right) \quad (19)$$

$$b = \frac{\tau}{1+\omega_c^2\tau^2}\frac{4\pi e^2}{m_e^2}(b1+b2+b3+b4) \quad (20)$$

Here:

$$b_1 = \frac{\xi^2 L_y\left|I_{n,n'}\right|^2}{2(2\pi)^2 l_B^2\hbar^4 v_d^2\omega_c\beta\rho v_s^2}\cdot\frac{eB\bar{l}}{\hbar}\left(\varepsilon_{N,n}-\varepsilon_F\right)\left[1+2\sum_{s=1}^{\infty}(-1)^s e^{\left(-\frac{2\pi s\Gamma}{\hbar\omega_c}\right)}\cos(2\pi s\bar{\varepsilon}_1)\right]$$

$$b_2 = \frac{-\xi^2 L_y\left|I_{n,n'}\right|^2}{4(2\pi)^2 l_B^2\hbar^4 v_d^2\omega_c\beta\rho v_s^2}\frac{e^2E_0^2}{m_e^2\Omega^4}\left(\frac{eB\bar{l}}{\hbar}\right)^3\left(\varepsilon_{N,n}-\varepsilon_F\right)\left[1+2\sum_{s=1}^{\infty}(-1)^s e^{\left(-\frac{2\pi s\Gamma}{\hbar\omega_c}\right)}\cos(2\pi s\bar{\varepsilon}_1)\right]$$

$$b_3 = \frac{-\xi^2 L_y\left|I_{n,n'}\right|^2}{8(2\pi)^2 l_B^2\hbar^4 v_d^2\omega_c\beta\rho v_s^2}\frac{e^2E_0^2}{m_e^2\Omega^4}\left(\frac{eB\bar{l}}{\hbar}\right)^3\left(\varepsilon_{N,n}-\varepsilon_F\right)\left[1+2\sum_{s=1}^{\infty}(-1)^s e^{\left(-\frac{2\pi s\Gamma}{\hbar\omega_c}\right)}\cos(2\pi s\bar{\varepsilon}_2)\right]$$

$$b_4 = \frac{-\xi^2 L_y\left|I_{n,n'}\right|^2}{8(2\pi)^2 l_B^2\hbar^4 v_d^2\omega_c\beta\rho v_s^2}\frac{e^2E_0^2}{m_e^2\Omega^4}\left(\frac{eB\bar{l}}{\hbar}\right)^3\left(\varepsilon_{N,n}-\varepsilon_F\right)\left[1+2\sum_{s=1}^{\infty}(-1)^s e^{\left(-\frac{2\pi s\Gamma}{\hbar\omega_c}\right)}\cos(2\pi s\bar{\varepsilon}_3)\right]$$

Where

$$\bar{l} = \left(\sqrt{N+\frac{1}{2}} + \sqrt{N+\frac{3}{2}}\right)\frac{l_B}{2}$$

$$\bar{\varepsilon_1} = \frac{(n-n')\hbar\omega_z + eE_1\bar{l}}{\hbar\omega_c}$$

$$\bar{\varepsilon_2} = \bar{\varepsilon_1} - \frac{\Omega}{\omega_c}, \bar{\varepsilon_3} = \bar{\varepsilon_1} + \frac{\Omega}{\omega_c}$$

The electron distribution function is now non-equilibrium, and the current density is non-linear as a result. Let us consider that the electron gas is nondegenerate, $f_0 = e^{\beta\left(\varepsilon_F - \varepsilon_{N,n\vec{k_y}}\right)}$,$\beta = \frac{1}{k_B T}$ where ε_F is the Fermi level, and k_B is the Boltzmann constant. After some manipulation, the expression for the conductivity tensor is obtained:

$$\begin{aligned}\sigma_{im} &= \frac{\tau}{1+\omega_c^2\tau^2}\left(\delta_{ij} - \omega_c\tau\varepsilon_{ijk}h_k + \omega_c^2\tau^2 h_i h_j\right) \\ &\times\left\{a\delta_{jm} + \frac{be}{m_e}\cdot\frac{\tau}{1+\omega_c^2\tau^2}\delta_{jl}\left(\delta_{lm} - \omega_c\tau\varepsilon_{lmp}h_p + \omega_c^2\tau^2 h_l h_m\right)\right\}\end{aligned} \tag{21}$$

where symbols i, j, k, l, p correspond to the components x, y, z of the Cartesian coordinates, δ_{ij} is the Kronecker delta, and ε_{ijk} is the antisymmetric Levi-Civita tensor.

From the conductivity tensor, the Hall coefficient expression is obtained as follows:

$$R_H = -\frac{1}{B}\frac{\sigma_{yx}}{\sigma_{xx}^2 + \sigma_{yx}^2} \tag{22}$$

Here:

$$\sigma_{xx} = \frac{\tau}{1+\omega_c^2\tau^2}\{a + b[1 - \omega_c^2\tau^2]\} \tag{23}$$

$$\sigma_{yx} = \frac{-\tau}{1+\omega_c^2\tau^2}(a+2b)\omega_c\tau \tag{24}$$

From the expression of the Hall coefficient, we see that the Hall coefficient depends on factors such as magnetic field B, temperature T, electromagnetic wave frequency, and parameters characterizing the material. This result is new and different from the results obtained in the case of other low-dimensional systems (doped superlattices, component superlattices, etc) because the wave function and energy spectrum are different.

3. Numerical Result for Semi-parabolic Quantum Wells with an Infinite Potential

In order to clarify the mechanism for the Hall coefficient in a ISPQW, in this section, we numerically evaluate, plot, and discuss the expression of the Hall coefficient for the quantum well of GaAs/GaAsAl with the parameters: $\rho = 5300\ \mathrm{kg.m^{-3}}$, $m = 0.067m_e$, m_e being the mass of the free electron, $v_s = 5370\ \mathrm{m/s}$, $\tau_0 = 10^{-12}$ s, e=1.60219.10^{-19}, $Lx = Ly = 100\ nm$ and only consider the transitions: $N = 0, N' = 1, n = 0, n' = 1$ (the lowest and the first-excited levels).

3.1. Hall Coefficients Depending on Magnetic Field B at Different Temperatures

Figure 1 shows that the Hall coefficient R_H strongly depends on both magnetic field and temperature. The Hall coefficient decreases with increasing magnetic field, decreases rapidly at low magnetic field values (around 0.5 T to 2 T), and then decreases more slowly. The graph has many fluctuations at different magnetic field values, more pronounced at lower temperatures. This is due to the Shubnikov-de Haas effect, which is common in 2D electronic structures under strong magnetic fields and low temperatures. The Hall coefficient R_H also decreases with increasing temperature, with quantum fluctuations being more pronounced at low temperatures. The graph has many fluctuations at different temperature values, and the fluctuations become smaller as the temperature increases.

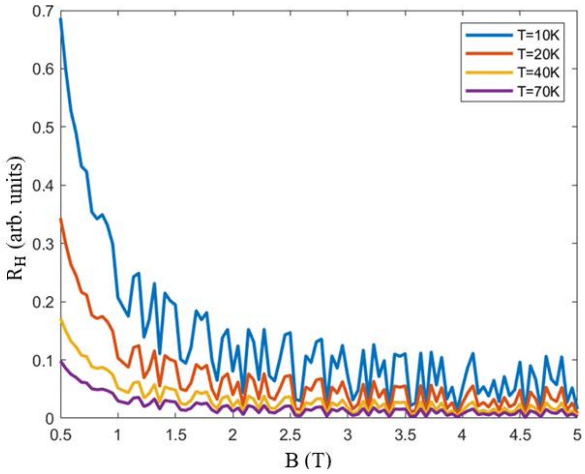

Figure 1. Hall coefficient $\mathbf{R_H}$ depending on magnetic field for various temperatures (T=10,20,40,70 K) with acoustic phonon scattering.

3.2. Hall Coefficients Depending on Temperature at Different Magnetic Fields B

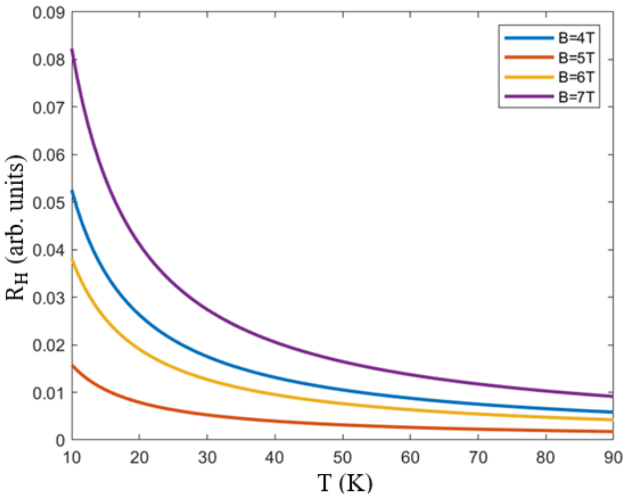

Figure 2. Hall coefficient $\mathbf{R_H}$ depending on temperature for different magnetic field strengths (B=7,9,10,12 T) with acoustic phonon scattering.

In Figure 2, the Hall coefficient decreases with increasing temperature, indicating that the Hall coefficient varies nonlinearly with temperature. The coefficient decreases rapidly in the range of 10-40K and then slows down at higher temperatures. This happens because as the temperature increases, the oscillation intensity of the crystal lattice increases, leading to an increase in the scattering yield of negative electrons and phonons, and a decrease in the Hall coefficient. The Hall coefficient also increases with increasing magnetic field; the curves show a clear dependence on the magnetic field.

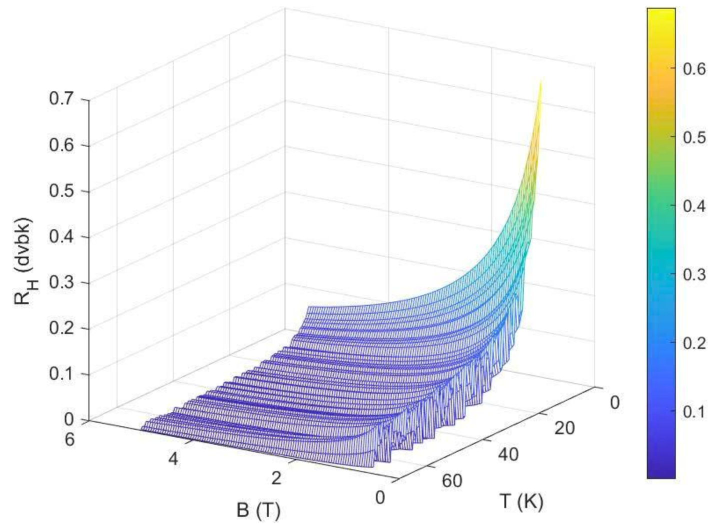

Figure 3. Hall coefficient R_H depending on magnetic field B and temperature T.

3.3. Hall Coefficients Depending on Magnetic Field B at Different Phonon Confinement Frequencies

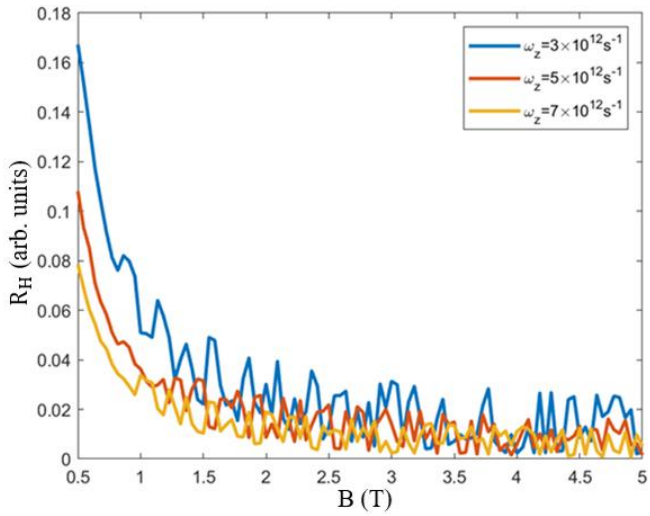

Figure 4. Hall coefficients depending on magnetic field B at different phonon confinement frequencies ($\boldsymbol{\omega_z = 10^{11}, 10^{12}, 10^{13}}$ s^{-1}) with acoustic phonon scattering.

In this Figure, the Hall coefficient also tends to decrease as the magnetic field B increases. The coefficient decreases rapidly at low magnetic field values and slows down at high magnetic fields. Due to the influence of the Shubnikov-de Haas effect, the graph also has many oscillations, especially obvious at low magnetic field values. Although at different confinement frequencies, the graph has in common that the oscillations are denser as the magnetic field increases. In particular, when the phonon confinement value increases, the Hall coefficient becomes smaller.

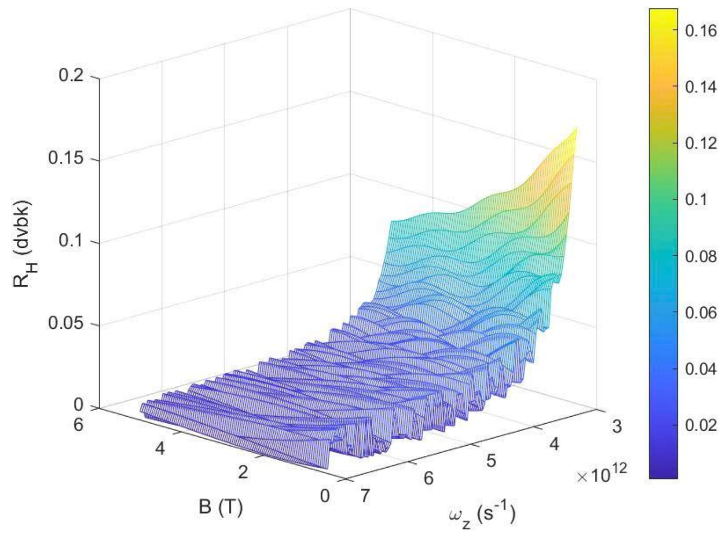

Figure 5. Hall coefficient R_H depending on magnetic field B and phonon confinement frequencie ω_z.

4. Conclusion

In this study, we have developed a theoretical model to investigate the Hall coefficient in an infinite semi-parabolic Quantum Wells in the presence of electromagnetic and in case of acoustic electron–phonon scattering. By using the quantum kinetic equation method, we derived the analytic expression for the Hall coefficient, which reflects the complex interplay between electromagnetic fields, phonon scattering, temperature, and quantum confinement.

Numerical results using a GaAs/AlGaAs quantum well show that the Hall coefficient decreases with increasing magnetic field and temperature, and exhibits clear quantum oscillations at low temperatures, attributed to the Shubnikov–de Haas effect. The presence of the electromagnetic wave not only alters the scattering dynamics but also enhances the modulation of the Hall response, especially in regimes of strong confinement and resonance.

These findings are new and differ significantly from previous studies in other low-dimensional systems due to the unique energy spectrum and wavefunction structure of the asymmetric semi-parabolic potential. This work contributes to the deeper understanding of the quantum Hall effect in complex nanostructures and provides a theoretical foundation for further experimental and technological developments in quantum electronics, photonics, and nanoscale device engineering.

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