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Original Article

Application of the Background Field Method to Cosmological Phase Transitions at Finite Temperature and Chemical Potential

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Abstract: We investigate cosmological phase transitions using the background field method (BFM) within finite-temperature quantum field theory. The one-loop thermal free energy density is calculated at high temperature and nonzero chemical potential in order to analyze spontaneous symmetry breaking in the early Universe. By constructing the effective potential in momentum space, we obtain analytical expressions and perform numerical evaluations. The results demonstrate that a first-order phase transition may occur at sufficiently high temperature and chemical potential, characterized by discontinuities in the effective scalar mass and the free energy density. Furthermore, symmetry non-restoration at high temperature is observed, suggesting a persistent asymmetry in the post-transition Universe. These findings highlight the relevance of gauge-invariant and non-perturbative methods for the study of cosmological phase transitions.

Keywords: Cosmological phase transition; background field method; finite temperature; chemical potential.

1. Introduction

Phase transitions are intrinsically complex physical processes and are generally non-perturbative in nature [1-5]. In cosmology, phase transitions occurring in the early Universe are believed to play a crucial role in shaping its present structure and symmetry properties. Finite-temperature quantum field

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theory provides a natural framework for describing such phenomena, as thermal effects modify the effective potential and may induce symmetry breaking or restoration [6, 7].

The effective action formulated through functional integrals allows one to go beyond the leading perturbative approximation and to incorporate higher-order quantum effects. This approach has proven to be particularly useful in the study of thermal phase transitions and non-equilibrium processes [8- 10]. Considerable attention has therefore been devoted to the behavior of scalar and gauge field theories at high temperature, especially in connection with cosmological applications [11].

However, a well-known difficulty arises in gauge theories: the conventional effective potential generally depends on the gauge-fixing parameter, which complicates its physical interpretation. The background field method provides a systematic way to overcome this problem by preserving explicit gauge invariance at every stage of the calculation.

Recent developments in cosmological observations, particularly the potential detection of stochastic gravitational wave backgrounds by missions like LISA, have renewed interest in first-order phase transitions [12]. In this context, ensuring gauge invariance through methods such as the background field method is crucial for obtaining reliable physical predictions [13].

In this work, we employ the background field method to investigate cosmological phase transitions at finite temperature and nonzero chemical potential [14].

The primary goal of this paper is to apply the background field method at high temperatures to investigate cosmological phase transitions. This work provides numerical computations that complement our previous research. The paper is organized as follows: Section 2 introduces the formalism of effective action and the BFM; Section 3 derives the one-loop free energy density at nonzero temperature and chemical potential; Section 4 presents numerical results and graphical solutions regarding cosmological phase transitions; and Section 5 provides the concluding remarks.

2. Formalism

We consider a gauge theory containing scalar and fermionic fields. The Lagrangian density includes gauge, scalar, fermion, and ghost contributions, as well as a chemical potential associated with a conserved charge. In the background field approach, each field is decomposed into a classical background component and a quantum fluctuation. In this work we focus on a non-Abelian SU(N) gauge theory.

$$\begin{aligned} L_0 = \; & -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\mathbf{Y}}(i\gamma^\mu D_\mu - G_i\Phi_i)\mathbf{Y} \\ & + \left[(D_\mu - i\mu\delta_{\mu 0})\Phi_i\right]^+ \left[(D^\mu - i\mu\delta^{\mu 0})\Phi_i\right] - m^2\Phi_i^+\Phi_i - \lambda(\Phi_i^+\Phi_i)^2 \quad , \qquad (1) \\ & -\frac{1}{2\xi}\left(\partial^\mu A^a_\mu\right)^2 - \partial_\mu\omega_a^*\partial^\mu\omega_a + f_{abc}(\partial_\mu\omega_a^*)A^b_\mu\omega^c \end{aligned}$$

where $\bar{\mathbf{Y}}, \mathbf{Y}$ are multiplet of fermion fields, $\Phi_i (i = 1, 2\ldots n)$ are components of scalar fields, A_μ - gauge fields and ω, ω^* - ghost fields. Here μ is chemical potential, G_i and λ are coupling constants, $\lambda > 0$

$$\begin{aligned} D_\mu \equiv \; & \partial_\mu - iT^a A^a_\mu, \\ F^a_{\mu\nu} \equiv \; & \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{abc}A^b_\mu A^c_\nu, \end{aligned}$$

where T_a are group generators, f_{abc} are structure constants which satisfy Lie algebra

$$f_{abc}f_{dbc} = \quad g^2 C_A \delta_{ad} \ , \tag{2}$$

$$Tr(T_a T_b) = \quad g^2 C_F \delta_{ab} \ , \tag{3}$$

with C_A is numerical constant of gauge group, $C_A = N$ for $SU(N)$, C_F is representation of this group.

The fields are shifted by

$$A_\mu \to \quad \mathbf{A}_\mu + A'_\mu; \qquad \langle 0 | \mathbf{A}_\mu | 0 \rangle = const, \qquad \langle 0 | A'_\mu | 0 \rangle = 0, \tag{4}$$

$$\Phi_i \to \quad \mathbf{F}_i + \Phi'_i; \qquad \langle 0 | \mathbf{F}_i | 0 \rangle = \phi_{0i}, \qquad \langle 0 | \Phi'_i | 0 \rangle = 0, \tag{5}$$

$$\Psi \to \quad \mathbf{Y} + \Psi'; \qquad \langle 0 | \mathbf{Y} | 0 \rangle = \langle 0 | \mathbf{Y}' | 0 \rangle = 0, \tag{6}$$

$$\omega_a \to \quad \mathbf{w}_a + \omega'_a; \qquad \langle 0 | \mathbf{w}_a | 0 \rangle = \langle 0 | \omega'_a | 0 \rangle = 0, \tag{7}$$

where $\mathbf{A}_\mu, \mathbf{F}, \mathbf{Y}, \mathbf{w}_a$ are the background fields, and $A'_\mu, \Phi', \Psi', \omega'_a$ are the quantum fields, which are variables of integration in the functional integral.

As is well known, the background field method allows one to fix a gauge, thereby computing quantum effects without losing explicit gauge invariance [14].

Now we consider the effective action in background field for which $\mathbf{A}^a_\mu$ and $\mathbf{F}_i$ are constant, and $\mathbf{Y} = \bar{\mathbf{Y}} = \mathbf{w} = \mathbf{w}^* = 0$. The effective action is calculated from the part of the action that is quadratic in quantum fields A'_μ, Φ', Ψ' and ω', ω'^* over which one integrated

$$\begin{aligned}
I_{quad} = \quad & \int dx L_{quad} = \int dx \left[-\frac{1}{4}\left(\bar{D}_\mu A'^a_\nu - \bar{D}_\nu A'^a_\mu\right)^2 - \frac{1}{4} F^a_{\mu\nu} f_{abc} A'^b_\mu A'^c_\nu \right] \\
& - \int dx \bar{\Psi}' \left(\gamma^\mu D_\mu + \mu\gamma^0 + M + g_i \phi'_i\right) \Psi' \\
& + \int dx \left[\frac{1}{2}\left(D_\mu \phi'_i D^\mu \phi'_i - M^2_{ij} \phi'^2_i\right) - \frac{\lambda}{4} \phi'^4_i \right] \\
& - \int dx \left[\frac{1}{2\xi}\left(\bar{D}_\mu A'^a_\nu\right)^2 + \left(\bar{D}_\mu \omega'^*_a\right)\left(\bar{D}_\mu \omega'_a\right) \right]
\end{aligned}$$

$$\begin{aligned}
I_{quad} = \quad & \frac{1}{2}\int dxdy A'^a_\mu(x) \mathrm{D}^{ab}_{\mu\nu}(x,y) A'^b_\nu(y) - \int dxdy \Psi'(x) \mathrm{D}_{ab}(x,y) \Psi'(y) \\
& + \frac{1}{2}\int dxdy \phi'_i(x) \mathrm{D}_{ik}(x,y) \phi'_k(y) - \int dxdy \omega'^*_a(x) \mathrm{D}_{ab}(x,y) \omega'_b(y).
\end{aligned} \tag{8}$$

By using the Fourier transformation $\mathrm{D}(k) = \int dx e^{ik(x-y)} \mathrm{D}(x-y)$ the matrices in (8) are determined by

$$\begin{aligned}
\mathrm{D}^{ab}_{\mu\nu}(k) = \quad & g_{\mu\nu}[\left(-ik_\rho \delta_{ca} + f_{cda} A^d_\rho\right)\left(-ik^\rho \delta_{cb} + f_{ceb} A^e_\rho\right) \\
& - \left(-ik_\nu \delta_{ca} + f_{cda} A^d_\nu\right)\left(-ik_\mu \delta_{cb} + f_{ceb} A^e_\mu\right) + F^c_{\mu\nu} f_{cab}] \\
& + g_{\mu\nu} \delta_{ij} \Phi_i(k) \Phi_j(k) T^a T^b + \varepsilon terms,
\end{aligned} \tag{9}$$

with $F^a_{\mu\nu} = f_{abc} A^b_\mu A^c_\nu$.

$$\mathrm{D}(k) = (-i\not{k} - iT^a \not{A}^a_\mu + M + \mu\gamma_0) + \varepsilon \text{ terms}, \tag{10}$$

$$\mathsf{D}_{ab}(k)=\left(-ik_{\rho}\delta_{ca}+f_{cda}A_{\rho}^{d}\right)\left(-ik^{\rho}\delta_{cb}+f_{ceb}A^{\rho b}\right)+\varepsilon \text{ terms}, \tag{11}$$

$$\mathsf{D}_{ij}(k)=\left(-ik_{\mu}-i\mu-iT_{a}A_{\mu}^{a}\right)_{i}\left(ik_{\mu}+i\mu+iT_{a}A_{\mu}^{a}\right)_{j}-m_{ij}^{2}-\frac{\lambda}{2}\phi_{i}\phi_{j}+\varepsilon \text{ terms}, \tag{12}$$

In momentum representation, the effective action takes the general form

$$\begin{aligned}\Gamma_{\beta}\left[\psi,\bar{\psi},\phi,A_{\mu},\omega,\omega^{*}\right]&=I\left[\psi,\bar{\psi},\phi,A_{\mu},\omega,\omega^{*}\right]-\frac{i}{2}TrlnG_{\mu\nu}^{ab}(k)\\&+iTrlnS(k)-\frac{i}{2}Trln\Delta_{ij}(k)+iTrlnD_{ab}(k)+\sum_{n=2}^{\infty}\text{n loops1PI}.\end{aligned}, \tag{13}$$

Here the action $I\left[\psi,\bar{\psi},\phi,A_{\mu},\omega,\omega^{*}\right]$ is given in (8) - (12). The Trace, the logarithm is taken in functional sense, and the free propagators are related to the quantum fields via

$$S^{-1}(k)=\not{k}-M-i\varepsilon; \qquad M=gv, \tag{14}$$

$$\Delta_{ij}^{-1}(k)=\delta_{ij}k^{2}-M_{ij}^{2}-i\varepsilon; \quad M_{ij}^{2}=(\mu^{2}-m^{2})\delta_{ij}-\frac{\lambda}{2}\phi_{i}\phi_{j}, \tag{15}$$

$$\left(M_{ab}^{2}-k^{2}\delta_{ab}\right)\left[\frac{k_{\mu}k_{\nu}}{k^{2}}-g_{\mu\nu}\right]+\left[\delta_{ab}\frac{k^{2}}{\xi}-M_{ab}^{2}\right]\frac{k_{\mu}k_{\nu}}{k^{2}}, \tag{16}$$

$$D_{0ab}^{-1}(k)=\delta_{ab}(k^{2}-i\varepsilon), \qquad M_{ab}=\frac{1}{2}\delta_{ab}gv. \tag{17}$$

The background fields are assumed to be constant, while the quantum fields are integrated out in the functional integral. Gauge fixing is performed in a manner consistent with background gauge invariance. Expanding the action to quadratic order in the quantum fields yields the one-loop effective action, which can be expressed in terms of functional determinants of differential operators.

3. One Loop Thermal Free Energy Density

To incorporate finite-temperature effects, we employ the imaginary-time formalism, in which the temporal coordinate is compactified with period β = 1/T. The presence of a nonzero chemical potential is implemented through a shift in the fermionic Matsubara frequencies.

Next we consider the theory at finite temperature by "imaginary time" formalism. For $\langle \mathbf{A}_{\mu}\rangle_{\beta}=\delta_{0\mu}A_{\mu}^{0},\langle \mathbf{F}\rangle=\phi_{0}$ the effective potential is defined by

$$V_{\beta}=-\frac{\Gamma_{\beta}}{\beta\int dx}, \tag{18}$$

where $\beta=T^{-1}$ (we set Boltzmann constant k = 1). It is just thermal free energy density, which concerns with the phase transition at $T=T_{c}$.

In $d=4-2\varepsilon$ dimension, the divergent integrals can be regularized by using the "imaginary time" formalism, where nonzero chemical potential μ is added to the fermionic Matsubara frequencies, i.e.

$$i\int\frac{d^{4}k}{(2\pi)^{4}}f(k)\rightarrow T\sum\!\!\!\!\!\!\int\frac{d^{3}k}{(2\pi)^{3}}\ f(i\omega_{n},\vec{k}),$$

where $i\omega_n = 2\pi nT$ for bosons, $\omega_n = (2n+1)\pi T$ for fermions and $i\omega_n \to i\omega_n + \mu$.

From (8) - (13) and (18) we arrived at the expression for the effective potential

$$\begin{aligned} V_\beta = V_{cl} &- \sum\!\!\!\!\!\!\int \left[ln(k^2+M^2) - ln(k^2+M_{ij}^2) - ln(k^2+M_{ab}^2) \right] \\ &+ ig^2\left(\frac{11}{12}N - \frac{1}{6}N_F + \frac{1}{12}N_B\right) \sum\!\!\!\!\!\!\int \frac{1}{(k^2)^2} \int dx F^a_{\mu\nu}F^{a\mu\nu} \\ &+ \frac{i}{8}g^2 \sum\!\!\!\!\!\!\int \frac{1}{(k^2+m^2)^2} + \frac{i}{2}g_i \sum\!\!\!\!\!\!\int \sum\!\!\!\!\!\!\int \frac{1}{(p^2+M_1^2)(k^2+M_2^2)\left[(k+p)^2+M_{ab}^2\right]} \end{aligned} \quad . \tag{19}$$

Finally, the one loop thermal free energy density is given by

$$\begin{aligned} V_\beta = &-\frac{1}{2}(\mu^2-m^2)\phi^2 + \frac{\lambda}{4}\phi^4 - \frac{1}{2}\delta_{ab}M_{ab}^2A_{0\mu}^2 - \frac{\pi^2T^4}{90}\left(N_B + \frac{7}{8}N_F\right) \\ &+ \frac{T^2}{24}\left\{(\mu^2 - m^2 - \frac{\lambda}{2}\phi^2) + 3TrM_{ab}^2 + \frac{1}{2}Tr\left[\gamma_0(M+\mu\gamma_0)\gamma_0(M+\mu\gamma_0)\right]\right\} \\ &- \frac{T}{12\pi}\left(\mathsf{M}^3 + \delta_{ab}M_{ab}^3\right) - \frac{g^2T^3}{48\times 4\pi}\left(\mathsf{M} + \delta_{ab}M_{ab} + 2M\right) \\ &+ \frac{g^2}{(4\pi)^2}\left(\frac{11}{12}N - \frac{1}{6}N_F + \frac{1}{12}N_B\right)\left(\frac{1}{\varepsilon} - 2ln\frac{\bar{\eta}}{4\pi T} + 2\gamma_E\right)\int dx F^a_{\mu\nu}F^{a\mu\nu}, \end{aligned} \tag{20}$$

where M, M_{ab}, M are thermal masses, e.g the squared scalar mass is

$$\mathsf{M}^2 = (\mu^2 - m^2) + \frac{\lambda}{24}T^2 - \frac{\lambda}{2}\phi^2. \tag{21}$$

The renormalized coupling constant is expressed by

$$g_R = g\left[1 + \frac{g^2}{4\pi^2}\left(\frac{11}{12}N - \frac{1}{6}N_F + \frac{1}{12}N_B\right)\left(\frac{1}{\varepsilon} + 2ln\frac{\bar{\eta}}{4\pi T} + 2\gamma_E\right)\right] + 0(g^4) \tag{22}$$

i.e the physical coupling g_R increases due to quantum corrections in the non - Abelian theory.

The thermal effective potential is identified with the free energy density and is obtained from the one-loop effective action. Thermal masses naturally arise from the finite-temperature corrections and play a central role in determining the phase structure of the theory.

4. Some Numerical Results at Nonzero Temperature and Chemical Potential

The restoration or breaking of symmetry in the early Universe is determined by identifying the global minimum of the effective potential. In this section, we present numerical results for the thermal free energy density and effective scalar mass under varying conditions of temperature (T) and chemical potential (μ).

As illustrated in Fig. 1, when the chemical potential is zero ($\mu = 0$), the free energy density maintains its minimum at $\Phi = 0$ across all temperature ranges, indicating that spontaneous symmetry breaking does not occur. However, the introduction of a nonzero chemical potential significantly alters this landscape. Fig. 2 demonstrates that at high temperatures combined with a finite μ, the symmetry is spontaneously broken, which is a prerequisite for cosmological phase transitions.

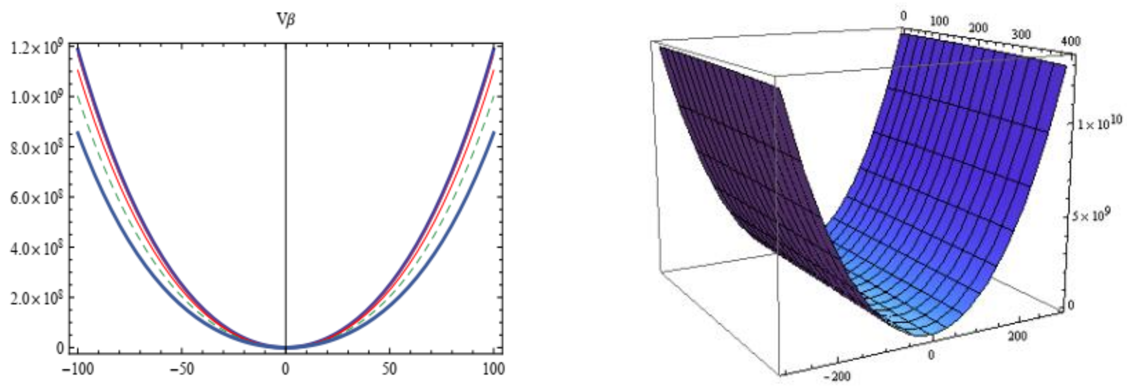

Figure 1. The effective potential V_β as a function of T and Φ in the case $\mu/m = 0.5;1.0$.

a) $\Phi = -100 \div 100$ MeV with $T = 0,100,200,400$ MeV respectively;
b) $T = 0 \div 400$ MeV and $\Phi = -300 \div 300$ MeV.

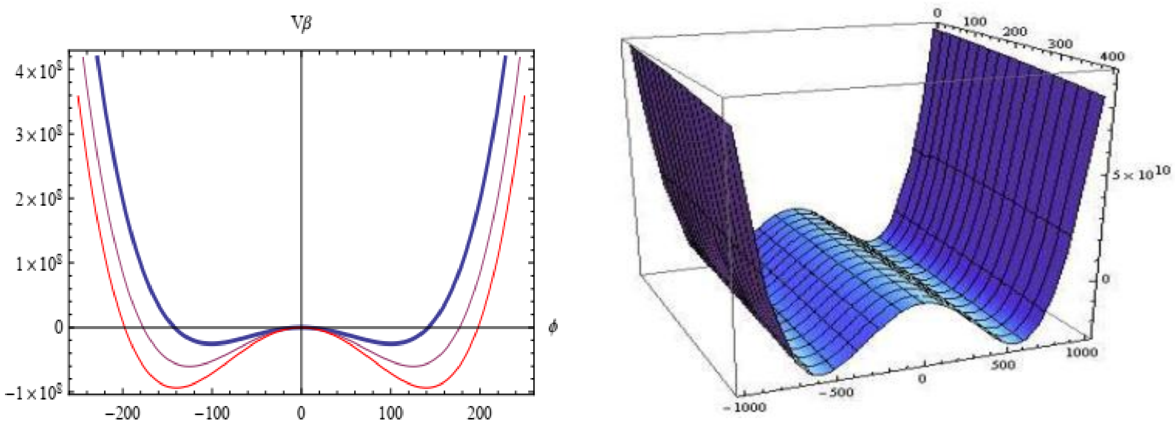

Figure 2. The effective potential V_β as a function of T and Φ in the case $\mu/m = 1.5$.

a) $\Phi = -100 \div 100$ MeV with $T = 0,100,200,400$ MeV, respectively;
b) $T = 0 \div 400$ MeV and $\Phi = -300 \div 300$ MeV.

The behavior of the effective squared scalar mass (M_{eff}^2) as a function of temperature is shown in Fig. 3. At low temperatures, the mass squared is negative, signifying a broken symmetry phase. As the temperature increases, M_{eff}^2 shifts toward positive values, primarily driven by the T^2 corrections in the effective potential. This transition from negative to positive values marks the onset of the symmetry restoration process.

To further investigate the nature of this transition, we analyze the continuity of the free energy density. In Fig. 4, at $\mu = 0$, the free energy is a continuous function of both temperature and the field Φ.

In contrast, Fig. 5 reveals a distinct discontinuity in the free energy density when $\mu \neq 0$. This discontinuity indicates a "buffer zone" between the symmetric and broken phases, which is a hallmark of a first-order cosmological phase transition. Interestingly, our analysis suggests that in certain parameter regimes, the symmetry might not be fully restored even at very high temperatures, leading to a persistent asymmetric state in the post-transition Universe.

Finally, Fig. 6 depicts the effective squared scalar mass as a function of the chemical potential at a fixed temperature. The observed dependence and the abrupt change in M_{eff}^2 further confirm that the chemical potential plays a decisive role in driving the first-order nature of the transition. These numerical findings align with the theoretical predictions of gauge-invariant models using the background field method.

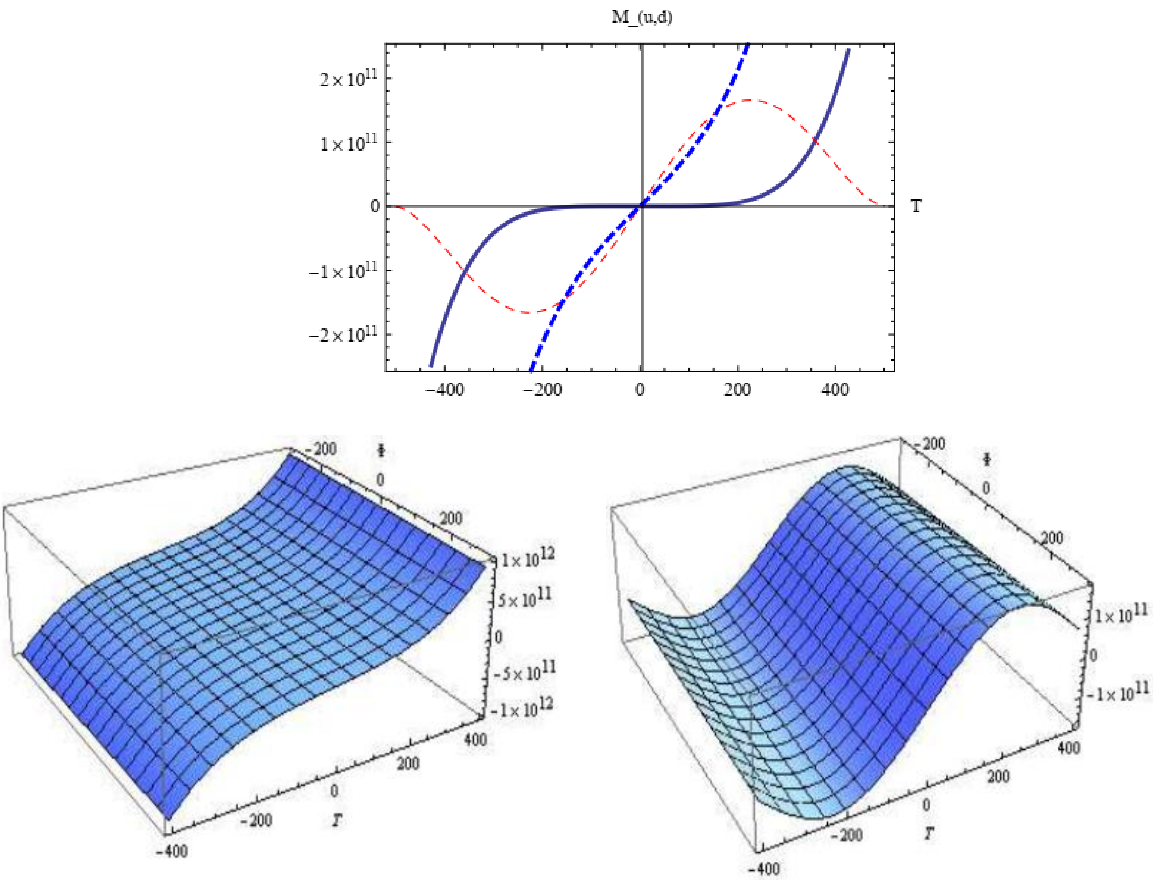

Figure 3. The squared scalar mass M^2 as a function of temperature T.

a) $T = -400 \div 400$ MeV with $\Phi = 100$ MeV in cases $\mu = m, \mu = 1,5m, \mu = 0,5m$.

b,c) $T = -400 \div 400$ MeV and $\Phi = -300 \div 300$ MeV in cases $\mu = 1,2m, \mu = 0,7m$, respectively.

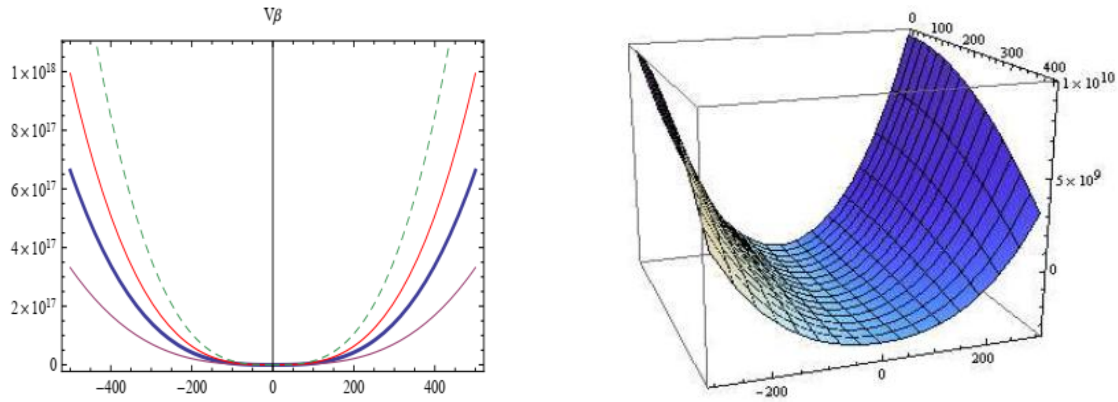

Figure 4. The effective potential V_β as a function of μ and Φ at $T = 400$ MeV.

a) $\mu = -500 \div 500 MeV$ with $\Phi = 0, 100, 200, 400$ MeV, respectively.

b) $\mu = 0 \div 400$ MeV and $\Phi = -300 \div 300$ MeV.

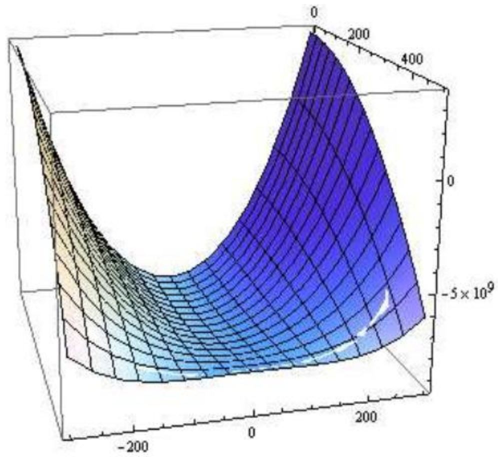

Figure 5. The effective mass as a function of chemical potential. It expresses the first order phase transition. The discontinuity is phase transition region (white region). Plot for $\mu = 0 \div 500$ MeV and $\Phi = -300 \div 300$ MeV.

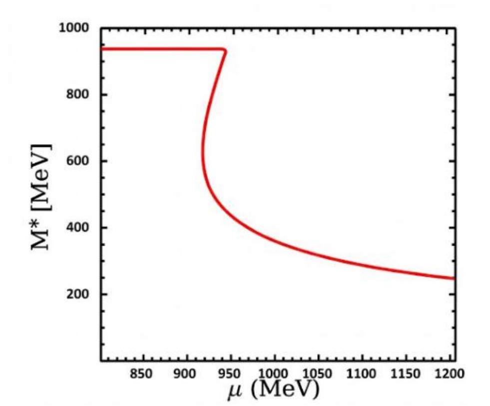

Figure 6. Dependence of the effective squared scalar mass on μ at fixed temperature.
It expresses the first order phase transition. Plot for M^2 with $\mu = 0 \div 1200$ MeV.

The phase structure of the model is analyzed by studying the minima of the effective potential as functions of temperature and chemical potential. When the chemical potential vanishes, the effective potential is minimized at the symmetric point for all temperatures, indicating the absence of spontaneous symmetry breaking.

In contrast, for sufficiently large temperature and nonzero chemical potential, the effective potential develops a nontrivial minimum, signaling spontaneous symmetry breaking. The effective scalar mass exhibits a discontinuous behavior as a function of temperature or chemical potential, which is characteristic of a first-order phase transition.

Our numerical results further show that symmetry restoration does not necessarily occur at high temperature once the symmetry has been broken. This symmetry non-restoration implies that the Universe may remain in an asymmetric phase after the transition.

5. Discussion and Conclusion

In this study, we have employed the BFM within the framework of finite-temperature quantum field theory to investigate the effective potential and cosmological phase transitions at nonzero chemical potentials. By utilizing the imaginary time formalism and momentum representation, we derived the one-loop thermal free energy density for a non-Abelian SU(N) gauge theory. Our numerical results provide compelling evidence that the inclusion of a nonzero chemical potential μ significantly alters the dynamics of symmetry breaking, leading to a robust first-order phase transition characterized by a clear discontinuity in both the effective scalar mass and the free energy density.

The determination of the order of these transitions is of paramount importance in modern cosmology. Recent advancements in multi-messenger astronomy have highlighted that first-order cosmological phase transitions in the early Universe could be a primary source of a stochastic gravitational wave background [12]. The discontinuities and the "buffer zone" identified in our graphical analysis (Fig. 5 and Fig. 6) suggest that the bubbles formed during such a transition could generate detectable gravitational signals for future space-based interferometers like LISA or DECIGO.

Furthermore, the use of the background field method in this work addresses a long-standing challenge in thermal field theory: the gauge-dependence of the effective potential. As emphasized in recent literature [13], obtaining gauge-independent results is crucial for making reliable physical predictions about the Universe's evolution. By explicitly maintaining gauge invariance, our approach ensures that the calculated critical temperatures and phase boundaries are not mere artifacts of the gauge-fixing choice. This is particularly relevant when considering the non-perturbative regime of high-temperature gauge theories, where traditional perturbative expansions often face convergence issues [15].

Another significant finding of our research is the potential for non-restoration of symmetry at high temperatures. The persistence of an asymmetric state suggests that the early Universe might have undergone a more complex thermal history than previously assumed, potentially offering new insights into the origin of baryon asymmetry.In conclusion, the background field method provides a consistent and powerful framework for exploring the thermal properties of gauge fields. Future work could extend this analysis to include higher-loop corrections or apply this formalism to specific extensions of the Standard Model, such as the Two-Higgs-Doublet Model (2HDM), to further refine the predicted gravitational wave spectra and their implications for our understanding of the early Universe.

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