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Calculation of the Hall Coefficient in Doped Semiconductor Superlattices with a Perpendicular Magnetic Field under the Influence of a Laser Radiation

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Abstract: We consider a model of the Hall effect when a doped semiconductor superlattice (DSSL) with a periodical superlattice potential in the *z*-direction is subjected to a crossed dc electric field (EF) $\vec{E}_1 = \left(E_1, 0, 0\right)$ and magnetic field $\vec{B} = \left(0, 0, B\right)$, in the presence of a laser radiation characterized by electric field $\vec{E} = \left(0, E_0 \sin\left(\Omega t\right), 0\right)$ (where E_0 and Ω are the amplitude and the frequency of the laser radiation, respectively). By using the quantum kinetic equation for electrons and considering the electron - optical phonon interaction, we obtain analytical expressions for the Hall conductivity as well as the Hall coefficient (HC) with a dependence on *B*, E_1, E_0, Ω, the temperature *T* of the system and the characteristic parameters of DSSL. The analytical results are computationally evaluated and graphically plotted for the GaAs:Si/GaAs:Be DSSL. Numerical results show the saturation of the HC as the magnetic field or the laser radiation frequency increases. This behavior is similar to the case of low temperature in two-dimensional electron systems.

Keywords: Hall effect, Quantum kinetic equation, Doped superlattices, Parabolic quantum wells, Electron - phonon interaction.

1. Introduction

It is well-known that the confinement of electrons in low-dimensional systems (nanostructures) makes their properties different considerably in comparison to bulk materials [1, 2, 3], especially, the optical and electrical properties become extremly unusual. This brings a vast possibility in application to design optoelectronics devices. In the past few years, there have been many papers dealing with problems related to the incidence of electromagnetic wave (EMW) in low-dimensional semiconductor systems. The linear absorption of a weak EMW caused by confined electrons in low dimensional

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systems has been investigated by using Kubo - Mori method [4, 5]. Calculations of the nonlinear absorption coefficients of a strong EMW by using the quantum kinetic equation for electrons in bulk semiconductors [6], in compositional semiconductor superlattices [7, 8] and in quantum wires [9] have also been reported. Also, the Hall effect in bulk semiconductors in the presence of EMW has been studied in much details by using quantum kinetic equation method [10 - 14]. In Refs. 10 and 11 the odd magnetoresistance was calculated when the nonlinear semiconductors are subjected to a magnetic field and an EMW with low frequency, the nonlinearity is resulted from the nonparabolicity of distribution functions of carriers. In Refs. 12 and 13, the magnetoresistance was derived in the presence of a strong EMW for two cases: the magnetic field vector and the electric field vector of the EMW are perpendicular [12], and are parallel [13]. The existence of the odd magnetoresistance was explained by the influence of the strong EMW on the probability of collision, i.e., the collision integral depends on the amplitude and frequency of the EMW. This problem is also studied in the presence of both low frequency and high frequency EMW [14]. Moreover, the dependence of magnetoresistance as well as magnetoconductivity on the relative angle of applied fields have been considered carefully [10 - 14]. The behaviors of this effect are much more interesting in low-dimensional systems, especially two-dimensional electron gas (2DEG) system.

To our knowledge, the Hall effect in low-dimensional semiconductor systems in the presence of an EMW remains a problem to study. Therefore, in this work, by using the quantum kinetic equation method we study the Hall effect in a doped semiconductor superlattice (DSSL) with the superlattice potential assumed to be in the *z*-direction, subjected to a crossed dc electric field (EF) $\vec{E}_1 = \left(E_1, 0, 0\right)$ and magnetic field $\vec{B} = \left(0, 0, B\right)$ ($\vec{B}$ is applied perpendicularly to the plane of free motion of electrons - the (*x-y*) plane, so we temporarily call the perpendicular Hall coefficient (PHC) in this study), in the presence of an EMW characterized by electric field $\vec{E} = \left(0, E_0 \sin\left(\Omega t\right), 0\right)$. We only consider the case of high temperatures when the electron - optical phonon interaction is assumed to be dominant and electron gas is nondegenerate. We derive analytical expressions for the Hall conductivity tensor and the PHC taking account of arbitrary transitions between the energy levels. The analytical result is numerically evaluated and graphed for the GaAs:Si/GaAs:Be DSSL to show clearly the dependence of the PHC on above parameters. The present paper is organized as follows. In the next section, we show briefly the analytical results of the calculation. Numerical results and discussion are given in Sec. 3. Finally, remarks and conclusions are shown briefly in Sec. 4.

2. Hall effect in a parabolic quantum well under the influence of a laser radiation

2.1. Quantum kinetic equation for electrons

We consider a simple model of a DSSL (n-i-p-i superlattice) in which electron gas is confined by an additional potential along the *z*-direction and free in the (*x-y*) plane. The motion of an electron is confined in each layer of the system and that its energy spectrum is quantized into discrete levels in the z-direction. If the DSSL is subjected to a crossed dc EF $\vec{E}_1 = (E_1, 0, 0)$ and magnetic field $\vec{B} = (0, 0, B)$, also a laser radiation (strong EMW) is applied in the z-direction with the electric field vector $\vec{E} = (0, E_0 \sin(\Omega t), 0)$, then the Hamiltonian of the electron-optical phonon system in the above mentioned regime in the second quantization representation can be written as

$$H = H_0 + \boldsymbol{U}, \tag{1}$$

$$H_0 = \sum_{N,n,\vec{k}_y} \varepsilon_{N,n}\left(\vec{k}_y - \frac{e}{\hbar c}\vec{A}(t)\right) a^+_{N,n,\vec{k}_y} a_{N,n,\vec{k}_y} + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}}, \tag{2}$$

$$\boldsymbol{U} = \sum_{N,N'} \sum_{n,n'} \sum_{\vec{q},\vec{k}_y} D_{N,n,N',n'}(\vec{q}) a^+_{N',n',\vec{k}_y+\vec{q}_y} a_{N,n,k_y} \left(b_{\vec{q}} + b^+_{-\vec{q}}\right), \tag{3}$$

where $\left|N, n, \vec{k}_y\right\rangle$ and $\left|N', n', \vec{k}_y + \vec{q}_y\right\rangle$ are electron states before and after scattering; $\vec{k}_y = (0, k_y, 0)$; $\hbar\omega_{\vec{q}}$ is the energy of an optical phonon with the wave vector $\vec{q} = (q_x, q_y, q_z)$; $a^+_{N,n,\vec{k}_y}$ and $a_{N,n,\vec{k}_y}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (phonon), respectively; $\vec{A}(t)$ is the vector potential of the EMW; and $D_{N,n,N',n'}(\vec{q})$ is the matrix element of interaction which depends on the initial and final states of electron and the interacting mechanism. In our model, the DSSL potential can be considered as a multiquantum-well structure with the parabolic potential in each well, and if we neglect the overlap between the wave functions of adjacent wells, the single-particle wave function and corresponding eigenenergy of an electron in a single potential well are given by [16 - 18]

$$\left|N, n, k_y\right\rangle = \sqrt{\frac{1}{L_y}} \Phi_N(x - x_0) e^{ik_y y} \Phi_n(z), \tag{4}$$

$$\varepsilon_{N,n}(\vec{k}_y) = \left(N + \frac{1}{2}\right)\hbar\omega_c + \varepsilon_n - \hbar v_d k_y + \frac{1}{2} m v_d^2; \qquad N, n = 0, 1, 2..., \tag{5}$$

where N is the Landau level index and n being the subband index; L_y is the normalization length in the y direction; $\omega_c = eB/m$ being the cyclotron frequency and $v_d = E_1/B$ is the drift velocity of electron. Also, Φ_N represents harmonic oscillator wave functions, centered at $x_0 = -\ell_B^2 (k_y - m v_d/\hbar)$ where $\ell_B = \sqrt{\hbar/(m\omega_c)}$ is the radius of the Landau orbit in the (x-y) plane. $\Phi_n(z)$ and ε_n are the wave functions and the subband energy values due to the superlattice confinement potential in the z-direction, respectively, given by

$$\Phi_n(z) = \sqrt{\frac{1}{2^n n! \sqrt{\pi} \ell_z}} \exp\left(-\frac{z^2}{2\ell_z^2}\right) H_n\left(\frac{z}{\ell_z}\right), \tag{6}$$

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega_p, \tag{7}$$

with $H_n(z)$ being the Hermite polynomial of n^{th} order and $\ell_z = \sqrt{\hbar/(m\omega_p)}$, ω_p is the plasma frequency characterizing for the DSSL confinement in the *z*-direction, given by $\omega_p = \left(4\pi e^2 n_D / \kappa_0 m\right)^{1/2}$, where κ_0 is the electronic constant (vacuum permittivity) and n_D is the doped concentration. The matrix element of interaction, $D_{N,n,N',n'}(\vec{q})$ is given by [16, 17]

$$\left|D_{N,n,N',n'}(\vec{q})\right|^2 = \left|C_{\vec{q}}\right|^2 \left|I_{n,n'}(\pm q_z)\right|^2 \left|J_{N,N'}(u)\right|^2, \tag{8}$$

where $C_{\vec{q}}$ is the electron-phonon interaction constant which depends on scattering mechanism, for electron - optical phonon interaction [6, 7, 16, 17] $\left|C_{\vec{q}}\right|^2 = 2\pi e^2 \hbar\omega_0 \left(\chi_\infty^{-1} - \chi_0^{-1}\right) / \left(\kappa_0 V_0 q^2\right)$, where V_0 is the normalization volume of specimen, χ_0 and χ_∞ are the static and the high-frequency dielectric constants, respectively; $I_{n,n'}(\pm q_z)$ is the form factor of electron, given by

$$I_{n,n'}(\pm q_z) = \sum_{\wp=1}^{s_0} \int_0^d e^{\pm i q_z d} \phi_n(z - \wp d)\phi_{n'}(z - \wp d) dz, \tag{9}$$

with d is the period and s_0 is the number of periods of the DSSL; also $\left|J_{N,N'}(u)\right|^2 = (N'!/N!) e^{-u} u^{N'-N} \left[L_N^{N'-N}(u)\right]^2$ with $L_M^N(x)$ is the associated Laguerre polynomial, $u = \ell_B^2 q_\perp^2$, $q_\perp^2 = q_x^2 + q_y^2$.

By using Hamiltonian (1) and the procedures as in the previous works [10 - 14], we obtain the quantum kinetic equation for electrons in the single (constant) scattering time approximation

$$\begin{aligned} -\left(e\vec{E}_1 + \omega_c\left[\vec{k}_y \wedge \vec{h}\right]\right)\frac{\partial f_{N,n,\vec{k}_y}}{\hbar\partial \vec{k}_y} + \frac{\vec{k}_y}{m}\frac{\partial f_{N,n,\vec{k}_y}}{\partial \vec{r}} = -\frac{f_{N,n,\vec{k}_y} - f_0}{\tau} + \frac{2\pi}{\hbar}\sum_{N',n'}\sum_{\vec{q}}\left|D_{N,n,N',n'}(\vec{q})\right|^2 \sum_{s=-\infty}^{+\infty} J_s^2\left(\frac{\lambda}{\Omega}\right) \\ \times\left\{\left[\bar{f}_{N',n',\vec{k}_y+\vec{q}_y}\left(N_{\vec{q}}+1\right) - \bar{f}_{N,n,\vec{k}_y} N_{\vec{q}}\right]\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y + \vec{q}_y\right) - \varepsilon_{N,n}\left(\vec{k}_y\right) - \hbar\omega_{\vec{q}} - s\hbar\Omega\right)\right. \\ \left. + \left[\bar{f}_{N',n',\vec{k}_y-\vec{q}_y} N_{\vec{q}} - \bar{f}_{N,n,\vec{k}_y}\left(N_{\vec{q}}+1\right)\right]\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y - \vec{q}_y\right) - \varepsilon_{N,n}\left(\vec{k}_y\right) + \hbar\omega_{\vec{q}} - s\hbar\Omega\right)\right\}, \end{aligned} \tag{10}$$

where $\vec{h} = \vec{B}/B$ is the unit vector along the magnetic field; the notation '$\wedge$' represents the cross product (or vector product); f_0 is the equilibrium electron distribution function (Fermi - Dirac distribution); $f_{N,n,\vec{k}_y}$ is an unknown electron distribution function perturbed due to the external fields; τ is the electron momentum relaxation time, which is assumed to be constant; $\bar{f}_{N,n,\vec{k}_y}$ ($N_{\vec{q}}$) is the time-independent component of the distribution function of electrons (phonons); $J_s(x)$ is the s^{th}-order Bessel function of argument *x*; $\delta(...)$ being the Dirac's delta function; and $\lambda = eE_0 q_y/(m\Omega)$.

Equation (10) is fairly general and can be applied for any mechanism of interaction. It was obtained in both bulk semiconductors and compositional superlattices [10 - 14]. In the following, we will use this expression to derive the Hall conductivity tensor as well as the PHC.

2.2. Expressions for the Hall conductivity and the Hall coefficient

For simplicity, we limit the problem to the cases of $s=-1,0,1$. This means that the processes with more than one photon are ignored. If we multiply both sides of Eq. (10) by $\frac{e}{m}\vec{k}_y\delta\left(\varepsilon-\varepsilon_{N,n}\left(\vec{k}_y\right)\right)$ and carry out the summation over N and $\vec{k}_y$, we have the equation for the partial current density $\vec{j}_{N,n,N',n'}(\varepsilon)$ (the current caused by electrons that have energy of ε):

$$\frac{\vec{j}_{N,n,N',n'}(\varepsilon)}{\tau}+\omega_c\left[\vec{h}\wedge\vec{j}_{N,n,N',n'}(\varepsilon)\right]=\vec{Q}_{N,n}(\varepsilon)+\vec{S}_{N,n,N',n'}(\varepsilon), \tag{11}$$

where

$$\vec{Q}_{N,n}(\varepsilon)=-\frac{e}{m}\sum_{N,n,\vec{k}_y}\vec{k}_y\left(\vec{F}\frac{\partial f_{N,n,\vec{k}_y}}{\hbar\partial\vec{k}_y}\right)\delta(\varepsilon-\varepsilon_{N,n}(\vec{k}_y)) \quad , \quad \vec{F}=e\vec{E}_1 \tag{12}$$

and

$$\begin{aligned}\vec{S}_{N,n,N',n'}(\varepsilon)=&\frac{2\pi e}{m\hbar}\sum_{\vec{k}_y,\vec{q}}\sum_{N',n'}\sum_{N,n}\left|D_{N,n,N',n'}(\vec{q})\right|^2N_{\vec{q}}\vec{k}_y\left\{\left[\bar{f}_{N',n',\vec{k}_y+\vec{q}_y}-\bar{f}_{N,n,\vec{k}_y}\right]\left[\left(1-\frac{\lambda^2}{2\Omega^2}\right)\right.\right.\\&\times\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y+\vec{q}_y\right)-\varepsilon_{N,n}\left(\vec{k}_y\right)-\hbar\omega_{\vec{q}}\right)+\frac{\lambda^2}{4\Omega^2}\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y+\vec{q}_y\right)-\varepsilon_{N,n}\left(\vec{k}_y\right)-\hbar\omega_{\vec{q}}+\hbar\Omega\right)+\frac{\lambda^2}{4\Omega^2}\\&\left.\times\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y+\vec{q}_y\right)-\varepsilon_{N,n}\left(\vec{k}_y\right)-\hbar\omega_{\vec{q}}-\hbar\Omega\right)\right]\\&+\left[\bar{f}_{N',n',\vec{k}_y-\vec{q}_y}-\bar{f}_{N,n,\vec{k}_y}\right]\left[\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y-\vec{q}_y\right)-\varepsilon_{N,n}\left(\vec{k}_y\right)+\hbar\omega_{\vec{q}}\right)\right.\\&\left.\left.+\frac{\lambda^2}{4\Omega^2}\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y-\vec{q}_y\right)-\varepsilon_{N,n}\left(\vec{k}_y\right)+\hbar\omega_{\vec{q}}+\hbar\Omega\right)+\frac{\lambda^2}{4\Omega^2}\delta\left(\varepsilon_{N',n'}\left(\vec{k}_y-\vec{q}_y\right)-\varepsilon_{N,n}\left(\vec{k}_y\right)-\hbar\omega_{\vec{q}}-\hbar\Omega\right)\right]\right\}\\&\times\delta\left(\varepsilon-\varepsilon_{N,n}\left(\vec{k}_y\right)\right)\end{aligned} \tag{13}$$

Solving (11) we have the expression for $\vec{j}_{N,n,N',n'}(\varepsilon)$ as follow

$$\begin{aligned}\vec{j}_{N,n,N',n'}(\varepsilon)=&\frac{\tau}{1+\omega_c^2\tau^2}\left\{\left(\vec{Q}_{N,n}(\varepsilon)+\vec{S}_{N,n,N',n'}(\varepsilon)\right)-\omega_c\tau\left([\vec{h}\wedge\vec{Q}_{N,n}(\varepsilon)]+[\vec{h}\wedge\vec{S}_{N,n,N',n'}(\varepsilon)]\right)\right.\\&\left.+\omega_c^2\tau^2\left(\vec{Q}_{N,n}(\varepsilon)\vec{h}+\vec{S}_{N,n,N',n'}(\varepsilon)\vec{h}\right)\vec{h}\right\}.\end{aligned} \tag{14}$$

The total current density is given by

$$\vec{J} = \int_0^{\infty} \vec{j}_{N,n,N',n'}(\varepsilon) d\varepsilon \text{ or } J_i = \sigma_{im} E_{1m} . \quad (15)$$

Inserting (14) into (15) we obtain the expressions for the Hall current J_i as well as the Hall conductivity σ_{im} after carrying out the analytical calculation. To do this, we consider only the electron-optical phonon interaction at high temperatures, the electrons system is nondegenerate and assumed to obey the Boltzmann distribution function in this case. Also, we assume that phonons are dispersionless, i.e, $\omega_{\vec{q}} \approx \omega_0$, $N_{\vec{q}} \approx N_0 = k_B T / (\hbar\omega_0)$, where ω_0 is the frequency of the longitudinal optical phonons, assumed to be constant, k_B being Boltzmann constant. Otherwise, the summations over $\vec{k}_y$ and $\vec{q}$ are changed into the integrals as follows

$$\sum_{\vec{k}_y} (...) \to \frac{L_y}{2} \int_{-L_x/2\ell_B^2}^{L_x/2\ell_B^2} (...) dk_y , \quad (16)$$

$$\sum_{\vec{q}} (...) \to \frac{V_0}{4\pi^2} \int_0^{+\infty} (...) q_\perp dq_\perp \int_{-\pi/d}^{+\pi/d} dq_z = \frac{V_0}{4\pi^2 \ell_B^2} \int_0^{+\infty} (...) du \int_{-\pi/d}^{+\pi/d} dq_z , \quad (17)$$

here, L_x is the normalization length in the *x*-direction.

After some manipulation, we obtain the expression for the conductivity tensor:

$$\sigma_{im} = \frac{e^2\tau}{\hbar}\left(1+\omega_c^2\tau^2\right)^{-1}\left(\delta_{ij} - \omega_c\tau\varepsilon_{ijk}h_k + \omega_c^2\tau^2 h_i h_j\right)\left\{a\delta_{jm} + b\delta_{j\ell}\left[\delta_{\ell m} - \omega_c\tau\varepsilon_{\ell mp}h_p + \omega_c^2\tau^2 h_\ell h_m\right]\right\}, \quad (18)$$

where δ_{ij} is the Kronecker delta; ε_{ijk} being the antisymmetric Levi - Civita tensor; the Latin symbols i, j, k, l, m, p stand for the components *x, y, z* of the Cartesian coordinates;

$$a = -\frac{\hbar\beta v_d L_y I}{2\pi m} \sum_{N,n} \mathrm{e}^{\beta(\varepsilon_F - \varepsilon_{N,n})} , \quad (19)$$

with ε_F is the Fermi level; and

$$b = \frac{\beta A N_0 L_y I}{8\pi^2 m^2} \frac{\tau}{1+\omega_c^2\tau^2} \sum_{N,n}\sum_{N',n'} I(n,n')\{b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8\}, \quad (20)$$

$$b_1 = \frac{1}{M}\left(\frac{eB\xi}{\hbar}\right) \exp\left[\beta(\varepsilon_F - \varepsilon_{N,n})\right]\left[\frac{(N+M)!}{N!}\right]^2 \delta(X_1),$$

$$b_2 = -\frac{\theta}{2M}\left(\frac{eB\xi}{\hbar}\right)^3 \exp\left[\beta(\varepsilon_F - \varepsilon_{N,n})\right]\left[\frac{(N+M)!}{N!}\right]^2 \delta(X_1),$$

$$b_3 = \frac{\theta}{4M}\left(\frac{eB\xi}{\hbar}\right)^3 \exp\left[\beta(\varepsilon_F - \varepsilon_{N,n})\right]\left[\frac{(N+M)!}{N!}\right]^2 \delta(X_2),$$

$$b_4 = \frac{\theta}{4M}\left(\frac{eB\xi}{\hbar}\right)^3 \exp\left[\beta(\varepsilon_F - \varepsilon_{N,n})\right]\left[\frac{(N+M)!}{N!}\right]^2 \delta(X_3),$$

$$b_5 = \frac{1}{M}\left(\frac{eB\xi}{\hbar}\right)\exp\left[\beta(\varepsilon_{\rm F} - \varepsilon_{N,n})\right]\left[\frac{N!}{(N+M)!}\right]^2 \delta(X_4),$$

$$b_6 = -\frac{\theta}{2M}\left(\frac{eB\xi}{\hbar}\right)^3 \exp\left[\beta(\varepsilon_{\rm F} - \varepsilon_{N,n})\right]\left[\frac{N!}{(N+M)!}\right]^2 \delta(X_4),$$

$$b_7 = \frac{\theta}{4M}\left(\frac{eB\xi}{\hbar}\right)^3 \exp\left[\beta(\varepsilon_{\rm F} - \varepsilon_{N,n})\right]\left[\frac{N!}{(N+M)!}\right]^2 \delta(X_5),$$

$$b_8 = \frac{\theta}{4M}\left(\frac{eB\xi}{\hbar}\right)^3 \exp\left[\beta(\varepsilon_{\rm F} - \varepsilon_{N,n})\right]\left[\frac{N!}{(N+M)!}\right]^2 \delta(X_6),$$

$X_1 = (N'-N)\hbar\omega_{\rm c} + (n'-n)\hbar\omega_{\rm p} - eE_1\xi - \hbar\omega_0$, $X_2 = X_1 + \hbar\Omega$, $X_3 = X_1 - \hbar\Omega$,

$X_4 = (N-N')\hbar\omega_{\rm c} + (n'-n)\hbar\omega_{\rm p} + eE_1\xi + \hbar\omega_0$, $X_5 = X_4 + \hbar\Omega$, $X_6 = X_4 - \hbar\Omega$,

$M = |N-N'| = 1,2,3,...$, $\alpha = \hbar v_{\rm d}$, $\theta = e^2E_0^2/\left(m^2\Omega^4\right)$, $A = 2\pi e^2\hbar\omega_0\left(\chi_\infty^{-1} - \chi_0^{-1}\right)/\kappa_0$, $\xi = \left(\sqrt{N+1/2} + \sqrt{N+1+1/2}\right)\ell_B/2$, $\beta = 1/(k_BT)$, $\varepsilon_{N,n} = \left(N+\frac{1}{2}\right)\hbar\omega_{\rm c} + \left(n+\frac{1}{2}\right)\hbar\omega_{\rm p} + \frac{1}{2}mv_{\rm d}^2$, $I = a_1(\alpha\beta)^{-1}\left[\exp(\alpha\beta a_1) + \exp(-\alpha\beta a_1)\right] - (\alpha\beta)^{-2}\left[\exp(\alpha\beta a_1) - \exp(-\alpha\beta a_1)\right]$, $a_1 = L_x/2\ell_B^2$; where we have set

$$I(n,n') = \int_{-\pi/d}^{+\pi/d} \left|I_{n,n'}(\pm q_z)\right|^2 dq_z \tag{21}$$

which will be numerically evaluated by a computational program. The divergence of delta functions is avoided by replacing them by the Lorentzians as [19]

$$\delta(X) = \frac{1}{\pi}\left(\frac{\Gamma}{X^2+\Gamma^2}\right) \tag{22}$$

where Γ is the damping factor associated with the momentum relaxation time τ by $\Gamma = \hbar/\tau$. The appearance of the parameter ξ is due to the replacement of q_y by $eB\xi/\hbar$, where ξ is a constant of the order of ℓ_B. The purpose is to a simplicity in performing the integral over $q_\perp$. This has been done in [16] and is equivalent to assuming an effective phonon momentum $ev_{\rm d}q_y \approx eE_1\xi$. The PHC is given by the formula [20]

$$R_H = -\frac{1}{B}\frac{\sigma_{yx}}{\sigma_{xx}^2 + \sigma_{yx}^2}, \tag{23}$$

where σ_{yx} and σ_{xx} are given by Eq. (18).

Equations (18) and (23) show the complicated dependences of the Hall conductivity tensor and the PHC on the external fields, including the EMW. It is obtained for arbitrary values of the indices *N, n, N′* and *n′*. However, it contains the term $I(n,n')$ which is difficult to find out the exact analytical result due to the presence of the Hermite polynomials. We will numerically evaluatethis term by the computational method. In the next section, we will give a deeper insight into these results by carrying out a numerical evaluation and a graphic consideration by the computational method.

3. Numerical results and discussion

In this section we present detailed numerical calculations of the Hall conductivity and the PHC in a DSSL subjected to the uniform crossed magnetic and electric fields in the presence of a strong EMW. For the numerical evaluation, we consider the n-i-p-i superlattice of GaAs:Si/GaAs:Be with the parameters [15, 17]: $\varepsilon_F = 50\text{meV}$, $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $\hbar\omega_0 = 36.25\text{meV}$, and $m = 0.067 \times m_0$ (m_0 is mass of a free electron). For the sake of simplicity, we also choose $N = 0, N' = 1, n = 0, n' = 0 \div 1$ (the lowest and the first-excited levels), $\tau = 10^{-12}\text{s}, L_x = L_y = 10^{-9}\text{m}$ and the number of periods used in the computation is 30.

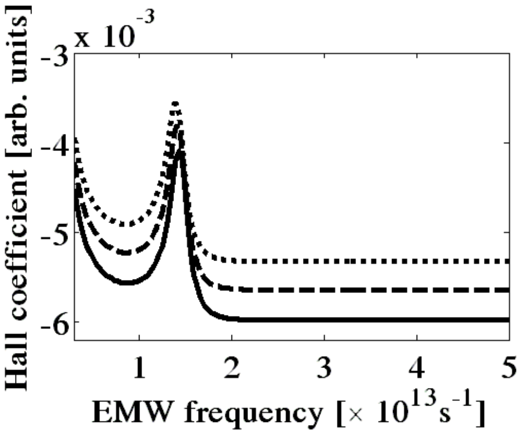

Figure 1. Hall coefficient (arb. Units) as functions of the EMW frequency at different at B = 4.00T (solid line), B = 4.05T (dashed line), and B = 4.10T (dotted line). Here, $E_1 = 5\times10^5 Vm^{-1}$, $E_0 = 10^5 Vm^{-1}$, $\omega_p = 4\times10^{13} s^{-1}$, $d = 15nm$, and T = 270K.

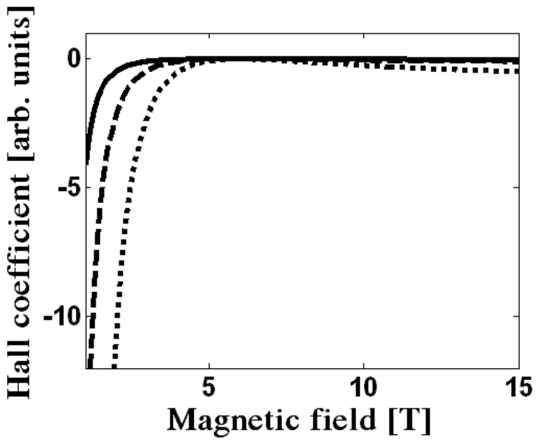

Figure 2. Hall coefficient (arb. Units) as functions of the magnetic field B at different values of the superlattice period: d = 15nm (solid line), d = 16nm (dashed line), and d = 17nm (dotted line). Here, $E_1 = 5\times10^5 Vm^{-1}$, $E_0 = 10^5 Vm^{-1}$, $\omega_p = 4\times10^{13} s^{-1}, \Omega = 5\times10^{13} s^{-1}$, and T = 270K.

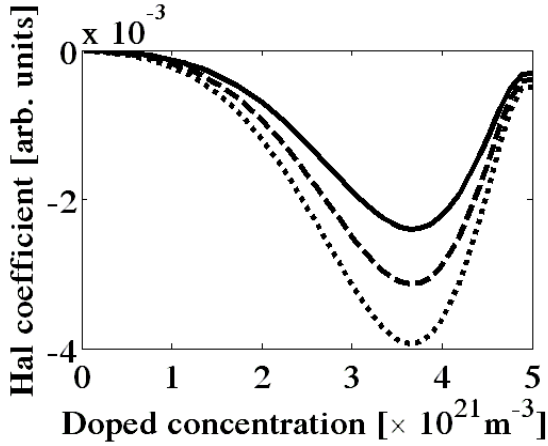

Figure 3. Hall coefficient (arb. Units) as functions of the doped concentration at temperature of 240K (solid line), 270K (dashed line), and 300K (dotted line). Here $B = 4T$, $E_1 = 5\times10^5 Vm^{-1}$, $E_0 = 10^5 Vm^{-1}$, $\omega_p = 4\times10^{13} s^{-1}$, $d = 15nm$, and $\Omega = 5\times10^{13} s^{-1}$.

Figure 1 shows the PHC as a function of the EMW frequency at different values of the magnetic field. In the region $\Omega < 10^{13} s^{-1}$ the PHC decreases quickly with the frequency and has a maximum peak. As the frequency increases continuously the PHC reaches saturation. Moreover, the PHC is always negative and the maximum peak shifts slightly with the change of magnetic field.

In Figure 2, we consider the dependence of the PHC on the magnetic field at different values of the superlattice period. For the chosen parameters, it is seen that the PHC increases very quickly as the magnetic field increases in the region of small values. As the magnetic field increases continuously, the PHC slightly changes and reaches saturations. This behavior is similar to the results obtained previously in some works at low temperatures for both the perpendicular and the in-plane magnetic fields (see Ref.[21] and references therein). It is also seen that the value of the magnetic field at which the saturation reaches, depends on the superlattice period. However, in the region of strong magnetic field, the PHC depends very weakly on the superlattice period. Because when the magnetic field increases, radii of the Landau orbits decrease so the electron density (and followed by the PHC) increases, reaches saturation and hence nearly does not vary with the superlattice period.

The dependence of the PHC on the doped concentration is shown in Figure 3 at the different values of the temperature. We can see that the PHC depends strongly on the doped concentration. As the doped concentration increases, the PHC decreases, reaches a minimum, and then increases. It is also seen that the value of doped concentration at which the PHC reaches minimum does not depend on the temperature. This dependence raises posibility in control of the effect by varying the doped concentration in the fabrication processes as well as we can determine some parameters of the structure from the measurement of the PHC.

4. Conclusion

In this work, we have studied the Hall effect in DSSLs subjected to a crossed dc electric field and magnetic field in the presence of a strong EMW (laser radiation). We obtain the expression of the Hall conductivity and the PHC when the electron - optical phonon interaction is taken into account at high temperature and the electron gas is nondegenerate. The influence of the EMW is interpreted by the dependence of the Hall conductivity and the PHC on the amplitude and the frequency (photon energy) of the EMW besides the dependence on the magnetic and the dc EF as in the ordinary Hall effect. The analytical results are numerically evaluated and plotted for the GaAs:Si/GaAs:Be superlattice to show clearly the dependence of the Hall conductivity and the PHC on the external fields, the temperature and parameters of the system. The most important result is that the PHC reaches saturation as the magnetic field or the EMW frequency increases. This is a typical property of the Hall coefficient in two-dimensional systems. The numerical results also show that the PHC in this calculation is always negative. Experimently, we can control the effect by varying some parameters of structure such as the doped concentration or the period of DSSL, conversely, we can also determine these parameters from the measurement of the PHC.

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