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Nonlinear axisymmetric response of thin FGM shallow spherical shells with ceramic-metal-ceramic layers under uniform external pressure and temperature

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Abstract: To increase the thermal resistance of various structural components in high-temperature environments, research reports focusing nonlinear axisymmetric response of thin FGM shallow spherical shells with ceramic – metal – ceramic layers (S-FGM) under uniform external pressure and temperature. Equilibrium and compatibility equations for shallow spherical shells are derived by using the classical shell theory and specialized for axisymmetric deformation with both geometrical nonlinearity and initial geometrical imperfection. One-term deflection mode is assumed and explicit expressions of buckling loads and load–deflection curves are determined due to Galerkin method. Stability analysis for a clamped spherical shell shows the effects of material and geometric parameters, edge restraint and temperature conditions, and imperfection on the behavior of the shells. The results were compared with the P-FGM spherical shell symmetry axis (ceramic – metal).

Keywords: axisymmetric response, S-FGM ceramic-metal-ceramic, thin shallow spherical shells, external pressure, thermal loads.

1. Introduction

Shallow spherical shells constitute an important portion in many engineering structures. They can find applications in the aircraft, missile and aerospace components. These shell elements also be widely used in other industries such as shipbuilding, underground structures and building constructions. As a result, the problems relating to buckling and postbuckling behaviors bring major importance in the design of this type of shell structure and have attracted attention of many researchers. The problem related to the nonlinear stability of the spherical shell made of composite material and layered orthotropic has been resolved in the study [1-4]. Due to advanced characteristics in comparison with traditional metals and conventional composites, Functionally Graded Materials

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(FGMs) consisting of metal and ceramic constituents have received increasingly attention in structural applications recent years. Smooth and continuous change in material properties enable FGMs to avoid interface problems and unexpected thermal stress concentrations. By high performance heat resistance capacity, FGMs are now chosen to use as structural components exposed to severe temperature conditions such as aircraft, aero- space structures, nuclear plants and other engineering applications. Despite the evident importance in practical applications, it is fact from the open literature that investigations on the buckling and postbuckling behaviors of FGM spherical shells are comparatively scarce. Shahsiah and colleagues [6] extended their previous works for isotropic material to analyze linear stability of FGM shallow spherical shells subjected to three types of thermal loading. Paper [7] is performed on the point of view of small deflection and the existence of type-bifurcation buckling of thermally loaded spherical shells. Recently, the nonlinear axisymmetric dynamic stability of clamped FGM shallow spherical shells has been analyzed by Prakash et al. and Ganapathi [8] using the first order shear deformation theory and finite element method. Recently, the nonlinear axisymmetric dynamic stability of clamped FGM shallow spherical shells has been analyzed by Prakash et al. and Ganapathi using the first order shear deformation theory and finite element method. In the research [9], Bich and Tung have studied the nonlinear axisymmetric response of functionally graded shallow spherical shells under uniform external pressure including temperature effects, but only for the P-FGM spherical shell with 2 layers ceramic – metal or metal - ceramic. To best of authors' knowledge, there is no analytical investigation on the nonlinear stability of S-FGM shallow spherical shells with metal-ceramic-metal or ceramic-metal-ceramic.

In this paper, the nonlinear axisymmetric response of thin FGM shallow spherical shells with ceramic – metal – ceramic layers under uniform external pressure and temperature will be considered. The properties of constituent materials are assumed to be temperature-independent and the effective properties of FGMs are graded in thickness direction according to a Sigmoid law function of thickness coordinate (S-FGM). Equilibrium and compatibility equations of a spherical shell are established by using the classical shell theory. Then these equations are specialized for axisymmetrically deformed shallow spherical shells taking into account geometric nonlinearity and initial geometrical imperfection. One-term approximation of deflection is assumed and explicit expressions of extremum buckling loads and load–deflection curves for a clamped spherical shell are determined by Galerkin method. An analysis is carried out to assess the effects of material, geometric parameters, edge restraint, temperature conditions and initial imperfection on the non-linear response of the shells.

2. Functionally graded (S-FGM) shallow spherical shells

Consider a functionally graded shallow spherical shell with radius of curvature *R*, base radius *a* and thickness *h* as shown in Fig.1. The shell is made from a mixture of ceramics and metals with ceramic-metal-ceramic layers, and is defined in coordinate system (φ, ϑ, z) whose origin is located whose origin is located, φ and θ and *z* is perpendicular to the middle surface and points outwards ($-\frac{h}{2} \leq z \leq \frac{h}{2}$).

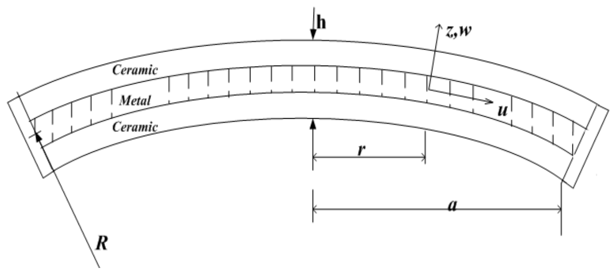

Fig. 1. Configuration and the coordinate system of a S-FGM shallow spherical shell.

Suppose that the material composition of the shell varies smoothly along the thickness in such a way that the both surface is ceramic with metal core by Sigmoid power law in terms of the volume fractions of the constituents (S-FGM) as

$$V_c(z) = \begin{cases} \left(\dfrac{2z+h}{h}\right)^N, & -\dfrac{h}{2} \le z \le 0 \\ \left(\dfrac{-2z+h}{h}\right)^N, & 0 \le z \le \dfrac{h}{2} \end{cases} \qquad (1)$$
$$V_m(z) = 1 - V_c(z)$$

where N (volume fraction index) is a non-negative number that defines the material distribution, subscripts m and c represent the metal and ceramic constituents, respectively.

The effective properties of S-FGM shallow spherical shell such as modulus of elasticity E, the coefficient of thermal expansion α, and the coefficient of thermal conduction K can bedefined as

$$[E(z), \alpha(z), K(z)] = [E_c, \alpha_c, K_c] + [E_{mc}, \alpha_{mc}, K_{mc}] \begin{cases} \left(\frac{2z+h}{h}\right)^N, & -\frac{h}{2} \le z \le 0 \\ \left(\frac{-2z+h}{h}\right)^N, & 0 \le z \le \frac{h}{2} \end{cases} \qquad (2)$$

whereas Poisson ratio ν is assumed to be constant and

$$E_{mc} = E_m - E_c, \quad \alpha_{mc} = \alpha_m - \alpha_c, \quad K_{mc} = K_m - K_c, \quad \nu(z) = const.$$

3. Governing equations

In the present study, the classical shell theory is used to obtain the equilibrium and compatibility equations as well as expressions of buckling loads and nonlinear load–deflection curves of thin S-FGM shallow spherical shells. For a thin shallow spherical shell it is convenient to introduce a variable r, referred to as the radius of parallel circle and defined by $r = R\sin\varphi$. Moreover, due to shallowness of the shell it is approximately assumed that $\cos\varphi = 1$, $R d\varphi = dr$.

The strains across the shell thickness at a distance z from the mid-plane are:

$$\begin{aligned} \varepsilon_r &= \varepsilon_{rm} - z\chi_r \\ \varepsilon_\theta &= \varepsilon_{\theta m} - z\chi_\theta \\ \gamma_{r\theta} &= \gamma_{r\theta m} - z\chi_{r\theta} \end{aligned} \qquad (3)$$

where ε_{rm} and $\varepsilon_{\theta m}$ are the normal strains, $\gamma_{r\theta m}$ is the shear strain at the middle surface of the spherical shell, whereas χ_r ,χ_θ ,$\chi_{r\theta}$ are the change of curvatures and twist.

According to the classical shell theory, the strains at the middle surface and the change of curvatures and twist are related to the displacement components *u, v, w* in the φ, θ,z coordinate directions, respectively, as

$$\begin{aligned} \varepsilon_{rm} &= u_{,r} - \frac{w}{R} + \frac{w_{,r}^2}{2} & \chi_r &= w_{,rr} \\ \varepsilon_{\theta m} &- \frac{v_{,\theta} + u}{r} - \frac{w}{R} + \frac{w_{,\theta}^2}{2r^2} & \chi_\theta &= \frac{w_{,\theta\theta}}{r^2} + \frac{w_{,r}}{r} \\ \gamma_{r\theta m} &= r\left(\frac{v}{r}\right)_{,r} + \frac{u_{,\theta}}{r} + \frac{w_{,r}w_{,\theta}}{r} & \chi_{r\theta} &= \frac{w_{,r\theta}}{r} - \frac{w_{,\theta}}{r^2} \end{aligned} \tag{4}$$

where geometrical nonlinearity in case of small strain and moderately small rotation is accounted for, also, subscript (,) indicates the partial derivative.

Hooke law for a spherical shell including temperature effect is defined as:

$$\begin{aligned} (\sigma_r, \sigma_\theta) &= \frac{E}{1-v^2}\left[(\varepsilon_r, \varepsilon_\theta) + v(\varepsilon_\theta, \varepsilon_r) - (1+v)\alpha\Delta T(1,1)\right] \\ \sigma_{r\theta} &= \frac{E}{2(1+v)}\gamma_{r\theta} \end{aligned} \tag{5}$$

where ΔT denotes the change of environment temperature from stress free initial state or temperature difference between the surfaces of an S-FGM spherical shell.

The force and moment resultants of an S-FGM spherical shell are expressed in terms of the stress components through the thickness as:

$$(N_{ij}, M_{ij}) - \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{ij}(1,z)dz \ \ ,ij - r,\theta,r\theta \tag{6}$$

Introduction of Eqs. (5) into (6) gives the constitutive relations as:

$$\begin{aligned} N_r &= \frac{E_1}{1-v^2}(\varepsilon_{rm} + v\varepsilon_{\theta m}) - \frac{E_2}{1-v^2}(\chi_r + v\chi_\theta) - \frac{\Phi_m}{1-v} \\ N_\theta &= \frac{E_1}{1-v^2}(\varepsilon_{\theta m} + v\varepsilon_{rm}) - \frac{E_2}{1-v^2}(\chi_\theta + v\chi_r) - \frac{\Phi_m}{1-v} \\ N_{r\theta} &= \frac{E_1}{2(1+v)}\gamma_{r\theta m} - \frac{E_2}{1+v}\chi_{r\theta} \\ M_r &= \frac{E_2}{1-v^2}(\varepsilon_{rm} + v\varepsilon_{\theta m}) - \frac{E_3}{1-v^2}(\chi_r + v\chi_\theta) - \frac{\Phi_b}{1-v} \\ M_\theta &= \frac{E_2}{1-v^2}(\varepsilon_{\theta m} + v\varepsilon_{rm}) - \frac{E_3}{1-v^2}(\chi_\theta + v\chi_r) - \frac{\Phi_b}{1-v} \\ N_{r\theta} &= \frac{E_2}{2(1+v)}\gamma_{r\theta m} - \frac{E_3}{1+v}\chi_{r\theta} \end{aligned} \tag{7}$$

where: $(E_1, E_2, E_3) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, z^2) E(z) dz$

$$\begin{aligned}
E_1 &= \int_{-\frac{h}{2}}^{0} \left(E_c + E_{mc}\left(\frac{2z+h}{h}\right)^N\right) dz + \int_0^{\frac{h}{2}} \left(E_c + E_{mc}\left(\frac{-2z+h}{h}\right)^N\right) dz = hE_c + \frac{hE_{mc}}{(N+1)}) \\
E_2 &= \int_{-\frac{h}{2}}^{0} z\left(E_c + E_{mc}\left(\frac{2z+h}{h}\right)^N\right) dz + \int_0^{\frac{h}{2}} z\left(E_c + E_{mc}\left(\frac{-2z+h}{h}\right)^N\right) dz = 0 \\
E_3 &= \int_{-\frac{h}{2}}^{0} z^2\left(E_c + E_{mc}\left(\frac{2z+h}{h}\right)^N\right) dz + \int_0^{\frac{h}{2}} z^2\left(E_c + E_{mc}\left(\frac{-2z+h}{h}\right)^N\right) dz \\
&= \frac{h^3 E_{mc}}{12} + \frac{h^3 E_{mc}}{2(N+1)(N+2)(N+3)} \\
(\phi_m, \phi_b) &= \int_{-\frac{h}{2}}^{0} \left(E_c + E_{mc}\left(\frac{2z+h}{h}\right)^N\right)\left(\alpha_c + \alpha_{mc}\left(\frac{2z+h}{h}\right)^N\right)\Delta T(1,z)dz \\
&\quad + \int_0^{\frac{h}{2}} \left(E_c + E_{mc}\left(\frac{-2z+h}{h}\right)^N\right)\left(\alpha_c + \alpha_{mc}\left(\frac{-2z+h}{h}\right)^N\right)\Delta T(1,z)dz
\end{aligned} \tag{8}$$

The nonlinear equilibrium equations of a perfect shallow spherical shell based on the classical shell theory are given by [5, 6, 10]

$$\begin{aligned}
&(rN_r)_{,r} + N_{r\theta,\theta} - N_\theta = 0 \\
&(rN_{r\theta})_{,r} + N_{\theta,\theta} - N_{r\theta} = 0 \\
&(rM_r)_{,rr} + 2(M_{r\theta,r\theta} + \frac{M_{r\theta,\theta}}{r}) + \frac{M_{\theta,\theta\theta}}{r} - M_{\theta,r} + \frac{r(N_r + N_\theta)}{R} + (rN_r w_{,r} + N_{r\theta} w_{,\theta})_{,r} \\
&\qquad + (N_{r\theta} w_{,r} + \frac{N_\theta w_{,\theta}}{r})_{,\theta} + qr = 0
\end{aligned} \tag{9}$$

where q is uniform external pressure positive inwards.

The first two of Eqs. (9) are identically satisfied by introducing a stress function f as:

$$N_r = \frac{f_{,r}}{r} + \frac{f_{,\theta\theta}}{r^2} \;;\; N_\theta = f_{,rr} \;;\; N_{r\theta} = -(\frac{f_{,\theta}}{r})_{,r} \tag{10}$$

Introduction of Eqs. (8), (10) into the third of Eqs. (9) gives the following equation:

$$\begin{aligned}
&D\Delta^2 w - \frac{\Delta f}{R} - \left(\frac{f_{,r}}{r} + \frac{f_{,\theta\theta}}{r^2}\right) w_{,rr} + \frac{2}{r}\left(\frac{f_{,r\theta}}{r} + \frac{f_{,\theta}}{r^2}\right) w_{,r\theta} - \frac{f_{,rr} w_{,\theta\theta}}{r^2} - \frac{f_{,rr} w_{,r}}{r} + \frac{2}{r^2}\left(\frac{f_{,\theta}}{r^2} - \frac{f_{,r\theta}}{r}\right) w_{,\theta} \\
&\quad - q = 0
\end{aligned} \tag{11}$$

where: $D = \frac{E_1 E_3}{E_1(1-\nu^2)}$; $\Delta() = ()_{,rr} + \frac{1}{r}()_{,r} + \frac{1}{r^2}()_{,\theta\theta}$

Eq. (15) is thee quilibriume quation of S-FGM shallow spherical shells in terms of two dependent unknowns, that is deflection of shell w and stress function f. To obtain a second equation relating these two unknowns, the compatibility equation may be used.

The geometrical compatibility equation of a shallow spherical shell is written as [10]

$$\frac{1}{r^2}\varepsilon_{rm,\theta\theta} - \frac{1}{r}\varepsilon_{rm,r} + \frac{1}{r^2}(r^2\varepsilon_{\theta m,r})_{,r} - \frac{1}{r^2}(r\gamma_{r\theta m,r})_{,r\theta} = -\frac{\Delta w}{R} + \chi_{r\theta}^2 - \chi_r\chi_\theta \tag{12}$$

$$(\varepsilon_{rm}, \varepsilon_{\theta m}) = \frac{1}{E_1}[(N_r, N_\theta) - \nu(N_\theta, N_r) + E_2(\chi_r, \chi_\theta) + \Phi_m(1,1)]$$
$$\gamma_{r\theta m} = \frac{2}{E_1}[(1+\nu)N_{r\theta} + E_2\chi_{r\theta}]$$

Substituting the above equations into Eq. (12), with the aid of Eqs. (5) and (10), leads to the compatibility equation of a perfect S-FGM shallow spherical shell as

$$\frac{1}{E_1}\Delta^2 f = \frac{-\Delta w}{R} + \left(\frac{w_{,r\theta}}{r} - \frac{w_{,\theta}}{r^2}\right)^2 - w_{,rr}\left(\frac{w_{,\theta\theta}}{r^2} + \frac{w_{,r}}{r}\right) \tag{13}$$

Eqs. (12) and (13) are equilibrium and compatibility equations, respectively, of an S-FGM shallow spherical shell in the case of asymmetric deformation. Specialization of these equations for an FGM shallow spherical shell under axisymmetric deformation gives equilibrium equation and compatibility equation

$$D\Delta_s^2 w - \frac{\Delta_s f}{R} - \frac{f' w''}{r} - \frac{f'' w'}{r} - q = 0 \tag{14}$$
$$\frac{1}{E_1}\Delta_s^2 f = -\frac{\Delta_s w}{R} - \frac{w' w''}{r}$$

where $\Delta_s() = ()'' + ()'/r$ and prime indicates differentiation with respect to r, i.e. $()' = d()/dr$.

Let $w_*(r)$ denotes a known small axisymmetric imperfection of the shell. This parameter represents a small initial deviation of the shell surface from a spherical shape. For an imperfect spherical shell, Eq. (14) is modified into form as

$$D\Delta_s^2 w - \frac{\Delta_s f}{R} - \frac{f'(w'' + w_*'')}{r} - \frac{f''(w' + w_*')}{r} - q = 0$$
$$\frac{1}{E_1}\Delta_s^2 f = -\frac{\Delta_s w}{R} - \frac{w' w''}{r} - \frac{w'' w_*'}{r} - \frac{w' w_*''}{r} \tag{15}$$

Eqs. (15) are the basic equations used to investigate the nonlinear axisymmetric stability of Sigmoid functionally graded (S-FGM) shallow imperfect spherical shells. These are nonlinear equations in terms of two dependent unknowns *w* and *f*.

4. Stability analysis

In this section, an analytical approach is used to investigate the nonlinear axisymmetric response of S-FGM shallow spherical shells with ceramic-metal-ceramic under uniform external pressure with and without the effects of temperature. The FGM spherical shells are assumed to be clamped along the periphery and subjected to external pressure uniformly distributed on the outer surface of the shells and, in some cases, exposed to temperature conditions. Depending on the in-plane behavior at the edge, will be considered

$$r = 0, w = W, w' = 0$$
$$r = a, w = w' = 0, \ N_r = N_{r0} \tag{16}$$

Where w is the amplitude of deflection the amplitude of deflection (i.e. radial maximum displacement). In case the edge is clamped and freely movable (FM) in the meridional direction $N_{r0} = 0$, in case the edge is clamped and immovable (IM) N_{r0} is the fictitious compressive edge load rendering the edge immovable.

With the consideration of the boundary conditions (16) the deflection W is approximately assumedas follows

$$w = W\frac{(a^2 - r^2)^2}{a^4}\ ;\ w_* = \mu h\frac{(a^2 - r^2)^2}{a^4};\quad -1 \leq \mu \leq 1 \tag{17}$$

where the imperfections of the shallow spherical shells are assumed to be the same form of the deflection, μ (i.e. $-1 \leq \mu \leq 1$) represents imperfection size.

Introduction of Eqs. (17) into Eq. (15) and integration of the resulting equation give stress function f with

$$f' = \frac{-E_1 W}{a^4 R}\left(\frac{r^5}{6} - \frac{a^2 r^3}{2}\right) - \frac{E_1 W(W + 2\mu h)}{a^8}\left(\frac{r^7}{6} - \frac{2a^2 r^5}{3} + a^4 r^3\right) + \frac{C_1 r}{2}\ln r + \frac{C_2 r}{2} + \frac{C_3 r}{2} \tag{18}$$

where C_1, C_2, C_3 are constants of integration. Due to the finiteness of the strains and resultants at the apex of the shallow spherical shell, i.e. at $r = 0$, the constants C_1 and C_3 must be zero. After determining the constant C_2 from in-plane restraint condition on the boundary, i.e. $N_r|_{r=a} = N_{r0}$, the stress function f is obtained such that

$$f' = \frac{-E_1 W}{a^4 R}\left(\frac{r^5}{6} - \frac{a^2 r^3}{2}\right) - \frac{E_1 W(W + 2\mu h)}{a^8}\left(\frac{r^7}{6} - \frac{2a^2 r^5}{3} + a^4 r^3\right) - \frac{E_1 W}{3R} r + \frac{E_1 W(W + 2\mu h)}{2a^2} r + N_{r0} r \tag{19}$$

where $N_{r0} = 0$ for the spherical shells with movable clamped edge.

Substituting Eqs.(17), (19) into Eq. (15), and applying Galerkin method for the resulting equation yield

$$q = \left(\frac{64D}{a^4} + \frac{3E_1}{7R^2}\right)W - \frac{976E_1}{693a^2 R}W(W + \mu h) - \frac{409E_1}{693a^2 R}W(W + 2\mu h) + \frac{848E_1}{429a^4}W(W + \mu h)(W + 2\mu h) + \frac{40N_{r0}}{7a^2}(W + \mu h) - \frac{2N_{r0}}{R} \tag{20}$$

Eq.(20) is used to determine the buckling loads and nonlinear equilibrium paths of S-FGM shallow spherical shells under uniform external pressure with and without the effects of temperature conditions..

4.1. Mechanical stability analysis

The clamped S-FGM shallow spherical shell with freely movable edge is assumed tobe subjected to external pressure q (in Pascals) uniformly distributed on the outer surface of the shell in the absence of temperature conditions. In this case $N_{r0} = 0$ and Eq. (20) leads to

$$q=\left(\frac{64\bar{D}}{R_h^4R_a^4}+\frac{3\bar{E_1}}{7R_h^2}\right)\bar{W}-\frac{\bar{E_1}}{693R_h^3R_a^2}\bar{W}(1385\bar{W}+1794\mu)+\frac{848\bar{E_1}}{429R_h^4R_a^4}\bar{W}(\bar{W}+\mu)(\bar{W}+2\mu) \quad (21)$$

Where :

$$R_h=\frac{R}{h}, R_a=\frac{R}{a}, \bar{D}=\frac{D}{h^3}, \bar{E_1}=\frac{E_1}{h}, \bar{W}=\frac{W}{h} \quad (22)$$

For a perfect spherical shell, i.e. $\mu=0$, it is deduced from q that:

$$q=\left(\frac{64D}{R_h^4R_a^4}+\frac{3E_1}{7R_h^2}\right)\bar{W}-\frac{1385E_1}{693R_h^3R_a^2}\bar{W}^2+\frac{848E_1}{429R_h^4R_a^4}\bar{W}^3 \quad (23)$$

For perfect spherical shells, extremum points of $q(\bar{W})$ curves are obtained from condition:

$$\frac{dq}{d\bar{W}}=A-2B\bar{W}+C\bar{W}^2=0 \quad (24)$$

which yields provided $B^2-AC>0$

$$\bar{W}_{1,2}=B+\frac{\sqrt{B^2-AC}}{C} \quad (25)$$

where

$$A-\frac{64\bar{D}}{R_h^4R_a^4}+\frac{3\bar{E_1}}{7R_h^2}, \qquad B-\frac{1385\bar{E_1}}{693R_h^3R_a^2}, \qquad C-\frac{848\bar{E_1}}{429R_h^4R_a^4}$$

It is easy to examine that if condition (36) is satisfied $q(\bar{W})$ curve of perfect shell reaches maximum and minimum with respective load values are

$$q_{upper}=q(W_1)=\frac{1}{3C^2}[B(3AC-2B^2)+2(B^2-AC)^{\frac{3}{2}}]$$
$$q_{lower}=q(\bar{W}_2)=\frac{1}{3C^2}[B(3AC-2B^2)-2(B^2-AC)^{\frac{3}{2}}] \quad (26)$$

4.2. Thermomechanical stability analysis

A clamped S-FGM shallow spherical shell with immovable edge under simultaneous action of uniform external pressure q (Pascal) and thermal load is considered. The condition expressing the immovability on the boundary edge, i.e. u = 0 on r = a, is fulfilled on the average sense as

$$\int_0^{\pi}\int_0^{a}\frac{\partial u}{\partial r}r\,dr\,d\theta=0$$

From Eqs. (4) and (7) one can obtain the following relation in which Eq. (10), imperfection and axisymmetry have been included

$$\frac{\partial u}{\partial r}=\frac{1}{E_1}\left(\frac{f'}{r}-\nu f''\right)+\frac{E_2}{E_1}w''-\frac{1}{2}(w')^2-w'w_*'+\frac{w}{R}+\frac{\Phi_m}{E_1} \quad (27)$$

Substituting Eqs.(17) and (19) into Eq.(27) and then putting the result into the average sense give

$$N_{r0} = \frac{-\phi_m}{1-v} + [\frac{(5v-7)E_1}{36(1-v)R} - \frac{2E_2}{(1-v)a^2}]W + \frac{(35-13v)E_1}{72(1-v)a^2}W(W+2\mu h) \tag{28}$$

which represents the compressive stress making the edge immovable.

In what follows, specific expressions of thermomechanical load–deflection curves of S-FGM shallow spherical shells under uniform external pressure and two types of thermal loads will be determined.

4.2.1. Uniform temperature rise

Environment temperature is assumed to be uniformly raised from initial value T_i at which the shell is thermal stress free, to final one T_f and temperature change ($\Delta T = T_f - T_i$) is independent to thickness variable. The thermal parameter ϕ_m can be expressed in terms of the ΔT: $\phi_m = Ph\Delta T$. Subsequently, employing this expression ϕ_m in Eq. (8) and then substitution of the resul N_{r0} into Eq. (28) lead to

$$\begin{aligned} q = &\left(\frac{64\overline{D}}{R_h^4R_a^4} + \frac{(103-89v)\overline{E_1}}{126(1-v)R_h^2} + \frac{4\overline{E_2}}{(1-v)R_h^3R_a^2}\right)\overline{W} \\ &+ \left[\frac{(1526v-1746)\overline{E_1}}{693(1-v)R_h^3R_a^2} - \frac{80\overline{E_2}}{(1-v)R_h^4R_a^4}\right]\overline{W}(\overline{W}+\mu) \\ &+ \frac{(2637v-4331)\overline{E_1}}{2772(1-v)R_h^3R_a^2}\overline{W}(\overline{W}+2\mu) \\ &+ \frac{(42833-27103v)\overline{E_1}}{9009(1-v)R_h^4R_a^4}\overline{W}(\overline{W}+\mu)(\overline{W}+2\mu) - \frac{40P\Delta T}{7(1-v)R_h^2R_a^2}\overline{W} + (\frac{2}{R_h} \\ &- \frac{40\mu}{7R_h^2R_a^2})\frac{P\Delta T}{(1-v)} \end{aligned} \tag{29}$$

Where:

$$P = E_c\alpha_c + \frac{E_c\alpha_{mc} + E_{mc}\alpha_c}{N+1} + \frac{E_{mc}\alpha_{mc}}{2N+1}, \qquad \overline{E_2} = \frac{E_2}{h^2} \tag{30}$$

4.2.2. Through the thickness temperature gradient

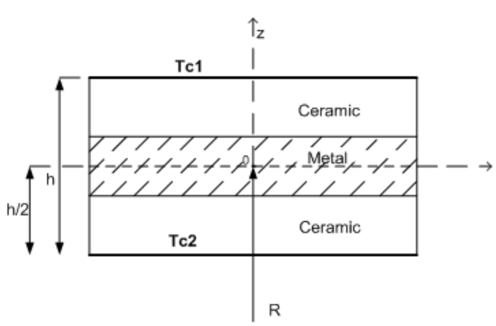

Fig. 2. The layered according to the thickness of the shell

In this case, to consider the temperature through the thickness, can consider the difference in surface temperature at top of the rich-ceramic surface and bottom of the rich-ceramic surface, as shown in the figure.

where T_{c1} and T_{c2} are temperatures top of the rich-ceramic surface and bottom of the rich-ceramic surface, respectively.

In this case, the temperature through the thickness is governed by the one-dimensional Fourier equation of steady-state heat conduction established in spherical coordinate system whose origin is the center of complete sphere as

$$\frac{d}{d\bar{z}}\left[K(z)\frac{dT}{d\bar{z}}\right]+\frac{2K(\bar{z})dT}{\bar{z}d\bar{z}}=0$$
$$T|_{\bar{z}=R-\frac{h}{2}}=T_m\ ;\ T|_{\bar{z}=R+\frac{h}{2}}=T_c\ ;\ \Delta T=T_{c1}-T_{c2} \tag{31}$$

where $\bar{z}$ is radial coordinate of a point which is distant z from the shell middle surface with respect to the center of sphere, i.e., $\bar{z}=R+z$ và $R-h/2\le\bar{z}\le R+h/2$.

The solution of Eq. (31) can be obtained as follows

$$T(\bar{z}) \;=\; T_{c2}+\frac{\Delta T}{\int_{R-\frac{h}{2}}^{R+\frac{h}{2}}\frac{d\bar{z}}{z^2K(z)}}\int_{R-\frac{h}{2}}^{\bar{z}}\frac{d\varsigma}{\varsigma^2K(\varsigma)} \tag{32}$$

Where (this section only considers linear distribution of metal and ceramic constituents, i.e. N=1)

$$K(\bar{z})=K_c+K_{mc}\begin{cases}\dfrac{2\bar{z}+h}{h} & ,R-\dfrac{h}{2}\le\bar{z}\le R\\ \dfrac{-2\bar{z}+h}{h} & ,R\le z\le R+\dfrac{h}{2}\end{cases}$$

$$I_1=\int_{R-\frac{h}{2}}^{R+\frac{h}{2}}\frac{d\bar{z}}{\bar{z}^2K(\bar{z})}=\int_{R-\frac{h}{2}}^{R}\frac{d\bar{z}}{\bar{z}^2(K_c+K_{mc}\frac{2\bar{z}+h}{h})}+\int_{R}^{R+\frac{h}{2}}\frac{d\bar{z}}{\bar{z}^2(K_c+K_{mc}\frac{-2\bar{z}+h}{h})} \tag{33}$$
$$I_2=\int_{R-\frac{h}{2}}^{\bar{z}}\frac{d\varsigma}{\varsigma^2K(\varsigma)}=\int_{R-\frac{h}{2}}^{R}\frac{d\varsigma}{\varsigma^2(K_c+K_{mc}\frac{2\varsigma+h}{h})}+\int_{R}^{R+z}\frac{d\varsigma}{\varsigma^2(K_c+K_{mc}\frac{-2\varsigma+h}{h})}$$

gives temperature distribution across the shell thickness $T(\bar{z})$ as

$$T(z)=T_{c1}+\frac{\Delta T}{I_1}I_2 \tag{34}$$

Assuming bottom of the rich-ceramic surface temperature as reference temperature and substituting Eq. (34) into Eq. (8) give Φ_m

$$\Phi_m=\int_{-\frac{h}{2}}^{0}\left(E_c+E_{mc}(\frac{2z+h}{h})\right)\left(\alpha_c+\alpha_{mc}(\frac{2z+h}{h})\right)T(z)dz$$
$$+\int_{0}^{\frac{h}{2}}\left(E_c+E_{mc}(\frac{-2z+h}{h})\right)\left(\alpha_c+\alpha_{mc}(\frac{-2z+h}{h})\right)T(z)dz \tag{35}$$

5. Results and discussion

In this section, the nonlinear response of the axisymmetrically deformed S-FGM shallow spherical shells is analyzed. The shell is assumed to be clamped along boundary edge and, unless otherwise specified, edge is freely movable. In characterizing the behavior of the spherical shell, deformations in which the central region of a shell moves toward the plane that contains the periphery of the shell are referred to as inward deflections (positive deflections). Deformations in the opposite direction are referred to as outward deflection (negative deflections).

To evaluate the results of the research, we compare our results with results obtained by Bich and Tung for a thin spherical shell with ceramic-metal layers in [9].

The following properties of the S- FGM shell are chosen:

$$E_m = 70GPa;\ E_c = 380GPa;\ \alpha_m = 23x10^6\ {}^oC^{-1};$$

$$\alpha_c = 7{,}4x10^6\ {}^oC^{-1};\ K_c = 204W/mK;\ K_m = 10{,}4W/mK$$

Where Poisson's ratio is chosen to be 0.3.

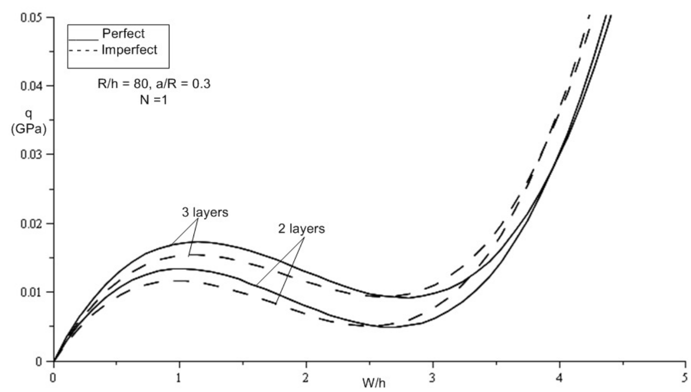

Fig.3. Comparison of the stability of 2 layers (P-FGM) and 3 layers (S-FGM) thin spherical shells.

The Fig.3. shows that the loading capability of 3-layers S-FGM spherical shell is better than 2-layers P-FGM when they have the same of shell thickness. Moreover, at the postbuckling period, the imperfect S-FGM spherical shell (μ = 0.1) has loading capability better than the perfect S-FGM spherical shell (μ = 0).

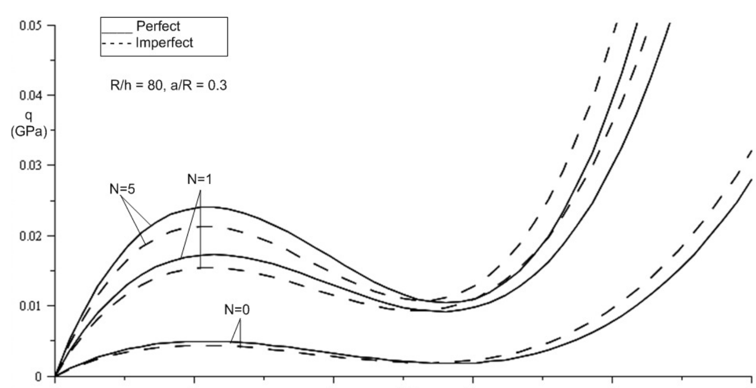

Fig. 4. Effects of volume fraction index N on the nonlinear response of the S-FGM shallow spherical shell.

Fig.4. shows the effects of volume fraction index N (0, 1 and 5) on the nonlinear response of the S-FGM spherical shell subjected to external pressure. As can be seen, the load–deflection curves become lower when N increases. However, the increase in the extremum-type buckling load and load carrying capacity of the ceramic-rich spherical shells is paid by a more severe snap-through behavior, i.e. a bigger difference between upper and lower buckling loads.

Fig. 5 depicts the effects of curvature radius-to-thickness ratio *R/h* (60, 70 and 80) on the nonlinear behavior of the externally pressurized S-FGM spherical shell. As can be observed, the load bearing capability of the S-FGM spherical shell is considerably enhanced as *R/h* ratio decreases. Furthermore, the increase in *R/h* ratio is accompanied by a drop of nonlinear equilibrium paths and a more severe snap-through response.

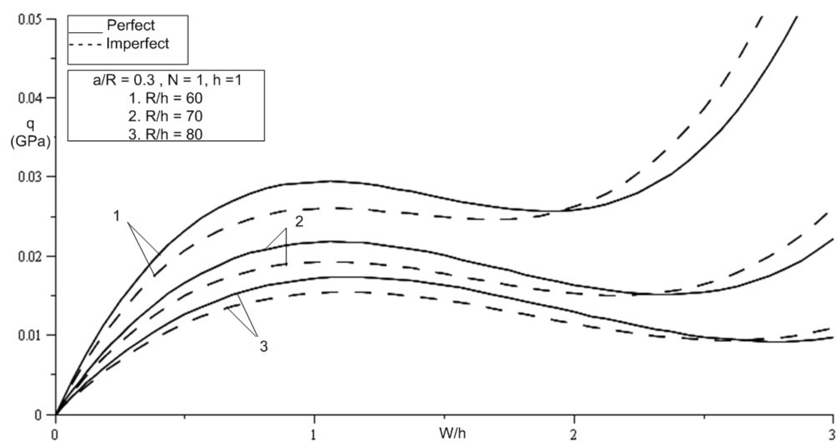

Fig. 5. Effects of curvature radius-to-thickness ratio on the nonlinear response of S-FGM shallow spherical shells

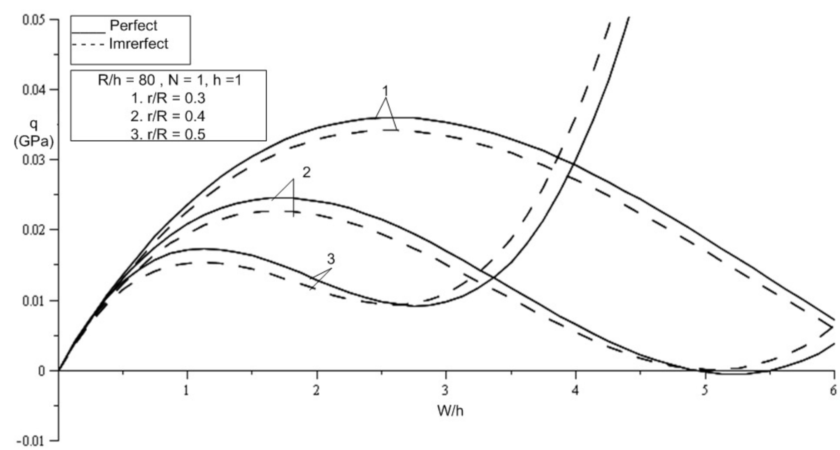

Fig. 6. Effects of radius of base-to-curvature radius ratio *r/R* on the nonlinear response of the S-FGM shallow spherical shells.

Fig.6. analyzes the effects of radius of base-to-curvature radius ratio *r/R* (0.3, 0.4 và 0.5) on the nonlinear response of S-FGM spherical shell subjected to uniform external pressure. It is shown that the nonlinear response of S-FGM spherical shells is very sensitive with change of *r/R* ratio characterizing the shallowness of spherical shell. Specifically, the enhancement of the upper buckling loads and the load carrying capacity in small range of deflection as *r/R* increases is followed by a very severe snap- through behavior. In other words, in spite of possessing higher limit buckling loads, deeper spherical shells exhibit a very unstable response from the postbuckling point of view.

The effects of environment temperature on the thermomechanical behavior of the S-FGM shallow spherical shell with immovable clamped edge are analyzed in Fig.7. In this case, the shell is exposed to temperature field prior to applying external pressure. It is evident that the spherical shell exhibits a bifurcation-type buckling behavior due to the presence of temperature field. Specifically, no deflection occurring until external pressure reaches a bifurcation value represented by intersection of curves with q axis. The bifurcation point external pressure depending on the temperature difference may be predicted by the last term in Eq. (29). This behavior of the S-FGM spherical shells can be explained as follows: The temperature field makes the shell to deflect outwards (negative deflection) prior to application of mechanical load. With the action of uniform external pressure, the outward deflection is reduced and when external pressure exceeds bifurcation point load an inward deflection occurs. The enhancement of temperature difference is accompanied by the increase in bifurcation points, load bearing capability in the small region of deflection and the intensity of snap-through behavior of the spherical shells. In addition, it is interesting to note that there exists an intersection point where all q(W) curves with various values of temperature difference pass.

Fig. 8 analyzes the effects of through the thickness temperature gradient on the nonlinear response of the clamped immovable S-FGM shallow spherical shell. In this figure, the curves are plotted for three various values of temperature at ceramic top surface T_{c1}(27^{0}C, 400^{0}C and 800^{0}C), whereas temperature at ceramic bottom surface is retained at T_{c2} = 27^{0}C (room temperature). It seems that bifurcation points are lower and the intensity of snap-through is weaker under temperature gradient in comparison with their uniform temperature counterparts.

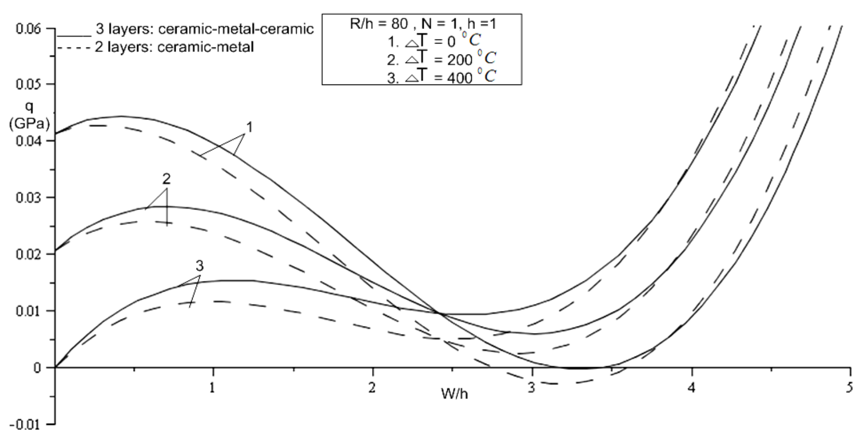

Fig. 7. Effects of temperature field on the nonlinear response of the S-FGM shallow spherical shell under uniform external pressure.

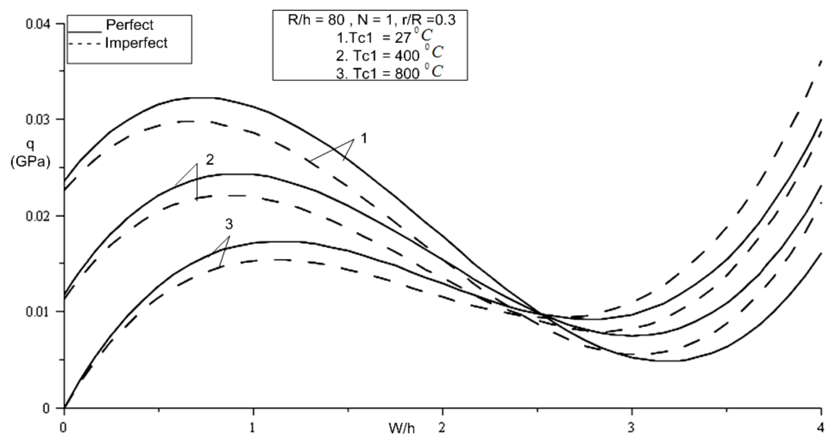

Fig. 8. Effects of temperature gradient on the non-linear response of the S-FGM shallow spherical shell with ceramic-metal-ceramic layers under external pressure.

6. Conclusions

This paper presents an analytical approach to investigate the nonlinear axisymmetric response of thin FGM shallow spherical shells with ceramic – metal – ceramic layers under uniform external pressure and temperature. The effective properties of functionally graded material are expressed as power functions of thickness variable whereas the properties of constituents are assumed to be temperature-independent. Formulation for axisymmetrically deformed thin S-FGM spherical shell is based on the classical shell theory with both geometrical nonlinearity and initial geometrical imperfection is incorporated. One-term deflection mode is approximately assumed and explicit expressions of buckling loads and load–deflection curves for a clamped spherical shell under mentioned loads are determined by applying Galerkin procedure. From these explicit expressions, the nonlinear axisymmetric response of the shell is analyzed and the results are illustrated in graphic form. The results show that the nonlinear response of the FGM spherical shell is complex and greatly influenced by the material and geometric parameters and in-plane restraint. The study also reveals important role of pre-existent temperature conditions and weak effect of initial imperfection on the nonlinear response of a thin FGM shallow spherical shell under uniform external pressure.

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