

The impact of confined phonons on the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in compositional superlattices

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Abstract. The impact of confined phonons on the nonlinear absorption coefficient (NAC) of a strong electromagnetic wave (EMW) by confined electrons in compositional superlattices is theoretically studied by using the quantum transport equation for electrons. The dependence of the NAC on the energy (Ω), the amplitude (E_0) of external strong EMW, the temperature (T) of the system and the period (d_A) of compositional superlattices is obtained in both case of confined and unconfined phonons. Two cases for the absorption: Close to the absorption threshold $|\hbar\Omega - \hbar\omega_o| \ll \bar{\mathcal{E}}$ and far away from the absorption threshold $|\hbar\Omega - \hbar\omega_o| \gg \bar{\mathcal{E}}$ ($k = 0, \pm 1, \pm 2, \dots$; $\hbar\omega_o$ and $\bar{\mathcal{E}}$ are the energy of optical phonon and the average energy of electrons, respectively) are considered. The analytic expressions are numerically evaluated, plotted and discussed for a specific of the GaAs-Al_{0.3}Ga_{0.7}As compositional superlattices. There are more resonant peaks appearing and the values of of the NAC are much larger than they are in case of unconfined phonons.

1. Introduction

Recently, much attention has also been focused on the study of the behavior of low-dimensional system (LDS), in particular two-dimensional systems. This due to that the confinement effect in LDS considerably enhances the electron and phonon mobility and leads to unusual behaviors under external stimuli. Many papers have appeared dealing with these behaviors, for examples, electron-phonon interaction and scattering rates [1-3] and dc electrical conductivity [4, 5]. The problems of the absorption coefficient for a weak EMW in some two-dimensional systems [6-9] have also been investigated by using Kubo-Mori method. The NAC of free electrons in normal bulk semiconductors [10] and confined electrons in quantum wells [11], in doped superlattices [12] have been studied by quantum kinetic equation method. The influences of confined phonons on the NAC of

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a strong EMW in the quantum wells and the doped superlattices [13, 14], on the electron interaction with acoustic phonons in the CQW via deformation potential [15] are considered.

However, the NAC of a strong EMW, whose strong intensity and high frequency in compositional superlattices with confined phonons is opened for study. So in this paper, we study the NAC of a strong EMW by confined electrons in compositional superlattices with the influence of confined phonons. Then, we estimate numerical values for a specific case of the GaAs-Al_{0.3}Ga_{0.7}As compositional superlattices to clarify our results.

2. Nonlinear absorption coefficient in case of confined phonons

The Hamiltonian of the electron-optical phonon system in the second quantization representation can be written as:

$$H = H_0 + U \quad (1)$$

$$H_0 = \sum_{n, k_{\perp}} \varepsilon_n \left(k_{\perp} - \frac{e}{c} A(t) \right) a_{n, k_{\perp}}^{\dagger} a_{n, k_{\perp}} + \sum_{m, q_{\perp}} \omega_m b_{m, q_{\perp}}^{\dagger} b_{m, q_{\perp}} \quad (2)$$

$$U = \sum_{m, q_{\perp}} \sum_{k_{\perp}, n, n'} C_{m, q_{\perp}} I_{n, n'}^m a_{n, k_{\perp} + q_{\perp}}^{\dagger} a_{n, k_{\perp}} (b_{m, q_{\perp}} + b_{m, q_{\perp}}^{\dagger}) \quad (3)$$

where H_0 is the non-interaction Hamiltonian of the electron-phonon system, n ($n = 1, 2, 3, \dots$) denotes the quantization of the energy spectrum in the z direction, (n, k_{\perp}) and $(n', k_{\perp} + q_{\perp})$ are electron states before and after scattering, respectively $k_{\perp} (q_{\perp})$ is the in plane (x, y) wave vector of the electron (phonon), $a_{n, k_{\perp}}^{\dagger}, a_{n, k_{\perp}}$ ($b_{m, q_{\perp}}^{\dagger}, b_{m, q_{\perp}}$) are the creation and the annihilation operators of electron (phonon), respectively, $A(t)$ is the vector potential of an external EMW $A(t) = \frac{e}{\Omega} E_0 \sin(\Omega t)$ and ω_m is the energy of a free optical phonon.

It is well known that in the low-dimensional structures, the energy levels of the electron become discrete in the confined direction, which are different between different dimensionalities. In this paper, we assume that the quantization direction is the z direction and only consider intersubband transitions ($n \neq n'$) and intrasubband transitions ($n = n'$). In this system, the electron-optical phonon interaction constants $C_{m, q_{\perp}}$, the electron energy $\varepsilon_{n, k_{\perp}}$ and the electron form factor $I_{n, n'}^m$ can be written as [16]:

$$|C_{m, q_{\perp}}|^2 = \frac{2\pi e^2 \omega_m}{\varepsilon_0 V} \left(\frac{1}{\chi_{\infty}} - \frac{1}{\chi_0} \right) \frac{1}{q_{\perp}^2 + q_z^2}; \quad q_z = \frac{m\pi}{L}; \quad m = 1, 2, 3, \dots \quad (4)$$

$$I_{n, n'}^m = \int_0^{S^d} \psi_n^*(z) \psi_{n'}(z) e^{iq_z z} dz \quad (5)$$

$$\varepsilon_{n,k_{\perp}} = \varepsilon_n + \frac{\hbar^2 k_{\perp}^2}{2m^*} - \Delta_n \cos k_{\parallel} d \quad (6)$$

Here, V and ε_o are the normalization volume and the electron constant, χ_o and χ_{∞} are the static and the high frequency dielectric constants, m^* and e are the effective mass and the charge of the electron, respectively. $\psi_n(z)$ is the wave function of the n -th state in one of the one-dimensional potential wells which compose the superlattices potential, d is the superlattices period, S_o is the number of superlattices period, ε_n and Δ_n are the energy levels of an individual well and the width of the n -th miniband, which is determined by the superlattices parameters.

In order to establish the quantum kinetic equations for the electrons in compositional superlattices with case of confined phonons, we use general quantum equation for electrons distribution function $n_{n,k_{\perp}} = \langle a_{n,k_{\perp}}^{\dagger} a_{n,k_{\perp}} \rangle_t$ [6,10]:

$$i \frac{\partial n_{n,k_{\perp}}}{\partial t} = \langle a_{n,k_{\perp}}^{\dagger} a_{n,k_{\perp}} ; H \rangle_t \quad (7)$$

Where $\langle \psi \rangle_t$ denotes a statistical average value at the moment t and $\langle \psi \rangle_t = \text{Tr}(\hat{W} \hat{\psi})$ (\hat{W} being the density matrix operator).

The carrier current density formula in compositional superlattices is taken the form:

$$j(t) = \frac{e}{m_e} \sum_{n,k_{\perp}} \left(k_{\perp} - \frac{e}{c} A(t) \right) n_{n,k_{\perp}} \quad (8)$$

Because the motion of electrons is confined along z direction in superlattices, we only consider the in plane (x, y) current density vector of electrons, $\mathbf{j}_{\perp}(t)$. Starting from Hamiltonian (1, 2, 3) and realizing operator algebraic calculations, we obtain the expression of $n_{n,k_{\perp}}(t)$ by solving the quantum kinetic equations. Substituting $n_{n,k_{\perp}}(t)$ into Eq.(8), then using the electron-optical phonon interaction potential $C_{m,q_{\perp}}$ in Eq.(4) and the relation between the NAC of a strong EMW with the carrier current density $\mathbf{j}_{\perp}(t)$, we obtain the NAC in compositional superlattices:

$$\alpha = \frac{16\pi^3 e^2 \Omega k_B T}{\varepsilon_o c \sqrt{\chi_{\infty}} E_o^2} \left(\frac{1}{\chi_{\infty}} - \frac{1}{\chi_o} \right) \sum_{\substack{m,q_{\perp} \\ n,n',k_{\perp}}} \sum_{k=1}^{\infty} \frac{k J_k^2}{q_{\perp}^2 + \left(\frac{m\pi}{L} \right)^2} |I_{n,n'}^m|^2 \quad (10)$$

$$\times (\bar{n}_{n,k_{\perp}} - \bar{n}_{n',k_{\perp}+q_{\perp}}) d (\varepsilon_{n',k_{\perp}+q_{\perp}} - \varepsilon_{n,k_{\perp}} - \Delta_n (\cos k_{\parallel}^n d - \cos k_{\parallel}^n d) + \omega_o - k \Omega)$$

Eq. (10) is the general expression for the nonlinear absorption of a strong EMW in compositional superlattices. In this paper, we will consider two limiting cases for the absorption, close to the absorption threshold and far away from absorption threshold, to find out the explicit formula for the absorption coefficient α .

2.1. The absorption far away from threshold

In this case, for the absorption of a strong EMW in compositional superlattices the condition $|k\Omega - \omega_o| \gg \bar{\varepsilon}$ must be satisfied. Here, $\bar{\varepsilon}$ is the average energy of an electron in compositional superlattices. Finally, we have the explicit formula for the NACof a strong EMW in compositional superlattices for the case of the absorption far away from its threshold, which is written:

$$\alpha = \frac{2\pi^2 e^4 n_o (k_B T)^2}{\varepsilon_o c \sqrt{\chi_\infty} m^* \Omega^3} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \sum_{m,n,n'} |I_{n,n'}^m|^2 \times \left\{ 1 + \frac{3e^2 E_o^2}{16m^* \Omega^4} B \right\} \times \left\{ 1 - \exp\left[-\frac{\xi}{k_B T} (\Omega - \omega_o)\right] \right\} \times \frac{2m^* B^{3/2}}{2m^* B + \left(\frac{m\pi \Omega}{L}\right)^2} \tag{11a}$$

Here $\xi = \omega_k (n' - n) + \omega_o - \Omega;$

$$B = \frac{\pi^2 \Omega^2}{2m^* L^2} (n'^2 - n^2) - \Delta_n (\cos p_{||}^{n'} d - \cos p_{||}^n d) + \omega_o - \Omega$$

When quantum number m characterizing confined phonons reaches to zero, the expression of the NAC for the case of absorption far away from its threshold in compositional superlattices without influences of confined phonons can be written as:

$$\alpha = \frac{4\pi^2 e^4 n_o k_B T}{\varepsilon_o c \sqrt{\chi_\infty} m^* \Omega^3} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \sum_{m,n,n'} |I_{n,n'}|^2 \times \left(\frac{\pi^2 (n'^2 - n^2)}{L^2} + \frac{2m^* (\Omega - \omega_o)}{\Omega} - \frac{2m^*}{\Omega^2} \Delta_n (\cos p_{||}^{n'} d - \cos p_{||}^n d) \right)^{1/2} \times \left(1 + \frac{3}{32} \frac{e^2 E_o^2}{m^{*2} \Omega^4} \left[\frac{\pi^2 (n'^2 - n^2)}{L^2} + \frac{2m^* (\Omega - \omega_o)}{\Omega} - \frac{2m^*}{\Omega^2} \Delta_n (\cos p_{||}^{n'} d - \cos p_{||}^n d) \right] \right) \times \left(1 - \exp\left[-\frac{\xi}{k_B T} \left(\frac{\pi^2 n^2}{2m^* L^2} + \Omega - \omega_o - \Delta_n \cos p_{||}^n d \right) \right] \right) \tag{11b}$$

Here, $I_{n,n'}$ the electron form factor in case of unconfined phonons.

2.2. The absorption close to the threshold

In this case, the condition $|k\Omega - \omega_o| \ll \bar{\varepsilon}$ is needed. Therefore, we can't ignore the presence of the vector \vec{k}_\perp in the formula of δ function. This also means that the calculation depends on the electron distribution function n_{n,k_\perp} . Finally, the expression of the NAC for the case of absorption close to its threshold is obtained:

$$\begin{aligned}
\alpha = & \frac{\pi e^4 n_o (k_B T)^2}{2c \sqrt{\chi_\infty} \Omega^3 \square^4} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \sum_{m,n',n} |I_{n,n'}^m| \times \left(\exp\left[\frac{1}{k_B T} (\Omega - \omega_o) \square - 1 \right] \right. \\
& \times \exp\left[-\frac{1}{k_B T} \left(\frac{\pi^2 \square^2 n^2}{2m^* L^2} - \Delta_n \cos p_{||}^{n'} d \right) \right] \times \exp\left[-\frac{1}{k_B T} (B + |B|) \right] \\
& \left. \times \left[1 + \frac{3e^2 E_o^2 k_B T}{8m^* \square^2 \Omega^4} \left(1 + \frac{1}{2k_B T} B \right) \right] \right) \quad (12a)
\end{aligned}$$

When quantum number m characterizing confined phonons reaches to zero, the expression of the NAC for the case of absorption close from its threshold in compositional superlattices without influences of confined phonons can be written as:

$$\begin{aligned}
\alpha = & \frac{\pi^2 e^4 n_o (k_B T)^2}{\epsilon_o c \sqrt{\chi_\infty} \square^4 \Omega^3} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \sum_{m,n',n} |I_{n,n'}^m|^2 \\
& \times \exp\left\{ -\frac{1}{k_B T} \left(\frac{\pi^2 \square^2 (n^2 - n'^2)}{2m^* L^2} - \Delta_n (\cos p_{||}^{n'} d - \cos p_{||}^{n''} d) \right) \right\} \\
& \times \left\{ \exp\left[\frac{\square}{k_B T} (\Omega - \omega_o) - 1 \right] \right\} \times \exp\left[-\frac{\square}{k_B T} \left(\frac{\pi^2 \square^2 n^2}{2m^* L^2} - \Delta_n \cos p_{||}^{n''} d \right) \right] \\
& \times \left(1 + \frac{3e^2 E_o^2 k_B T}{8m^* \square^2 \Omega^4} \left[1 + \frac{1}{k_B T} \left(\frac{\pi^2 (n^2 - n'^2)}{L^2} + \square (\Omega - \omega_o) - \Delta_n (\cos p_{||}^{n'} d - \cos p_{||}^{n''} d) \right) \right] \right) \quad (12b)
\end{aligned}$$

3. Numerical results and discussion

In order to clarify the mechanism for the NAC of a strong EMW in compositional superlattices with case of confined phonons, in this section, we will evaluate, plot and discuss the expression of the NAC for the case of the GaAs-Al_{0.3}Ga_{0.7}As compositional superlattices. We use some results to make the comparison with case of unconfined phonons. The parameters used in the calculations are as follows [4,7,8,16]: $\chi_o = 12.9$, $\chi_\infty = 10.9$, $n_o = 10^{23}$, $\Delta_n = 0.85 \text{ meV}$; $L = 118 \text{ \AA}$; $m = 0.067 m_o$, m_o being the mass of a free electron, $\square \omega_o = 36.25 \text{ meV}$ and $\Omega = 2.10^{14} \text{ s}^{-1}$, $d_A = 134.10^{-10} \text{ m}$, $d_B = 16.10^{-10} \text{ m}$.

3.1. The absorption far away from its threshold

Figures (1a-1b) shows the NAC of a strong EMW in a compositional superlattice as function of E_o for the case of the absorption far away from its threshold in both case of confined and unconfined phonons. The curve increases following E_o rather fastly. The value of the NAC is higher and higher when m increases.

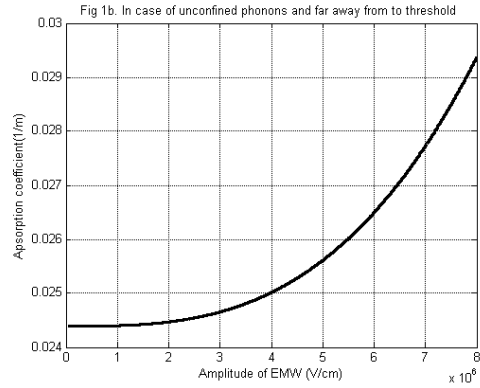
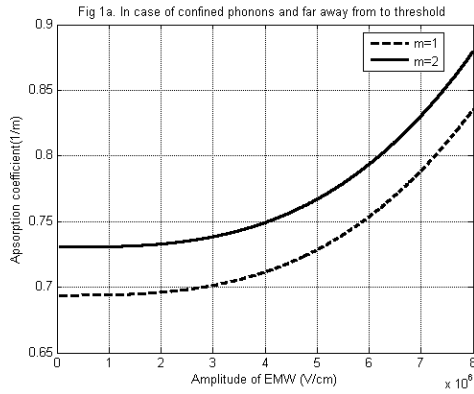


Fig 1a & 1b. The dependence of α on the amplitude E_0 in compositional superlattices in case confined phonon (1a) and in case unconfined phonons (1b).

In contrast with the Figures.(1a&1b), it is seen that the values of the NAC decrease following $\hbar\Omega$ in figures(2a&2b). But when the temperature T of the system increases, its absorption coefficient increases very slowly. This dependence is similar to the figures.(3a&3b) which show that the NAC changes very slowly following the period d_A of compositional superlattice.

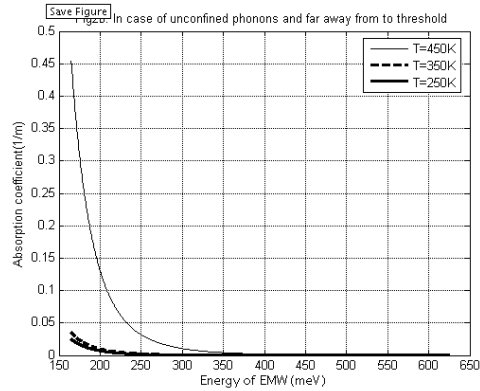
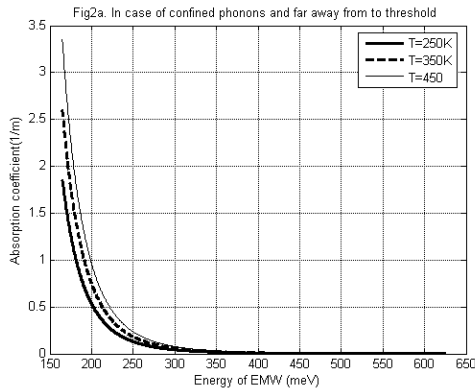


Fig 2. The dependence of α on $\hbar\Omega$ in compositional superlattices in case confined phonon (2a) and in case unconfined phonons (2b).

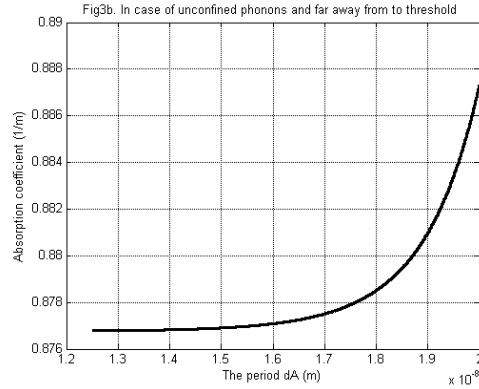
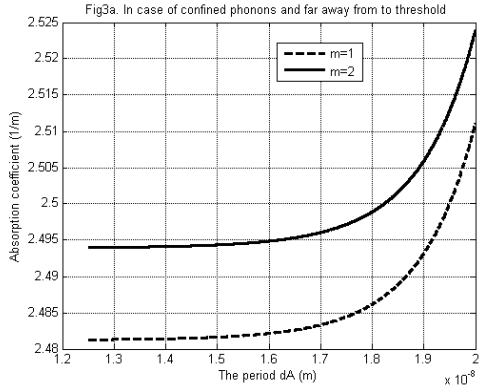


Fig 3a & 3b. The dependence of α on d_A in compositional superlattices in case confined phonon (3a) and in case unconfined phonons (3b).

All that figures show that the values of the NAC are much higher than these in case of unconfined phonons, and its depends very strong on m . The results are quite similar to those in [13, 14].

3.2. The absorption close to the threshold

In this case, the dependence of the NAC α on other parameters are quite similar with case of the absorption far away from its threshold. But, the values of the NAC are much greater than above case. The first difference in figures (4a&ab), it is seen that the values of the NAC increase following the T faster than above case.

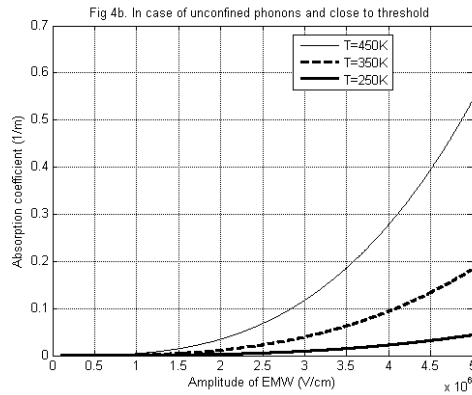
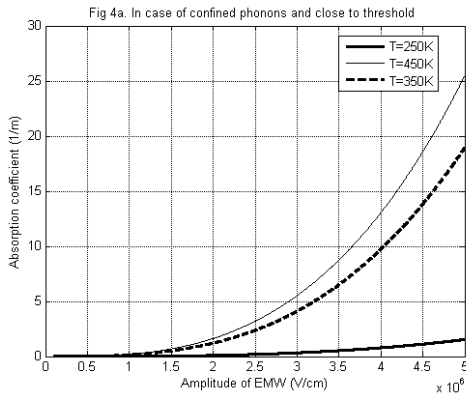


Fig4a & 4b. The dependence of α on the amplitude E_0 in compositional superlattices in case confined phonon (4a) and in case unconfined phonons (4b)

Also, it is seen that the NAC depends much strongly on the energy of EMW $\hbar\Omega$ that there are appearing resonant peaks. Especially, fig.5a show that there is appearing more resonant peak than case

of unconfined phonons in fig.5b. The position of the first resonant peak is similar to its in case unconfined phonon (fig.5b) but its value is much higher. The second ones which appears when $\Omega > \omega_o$ is higher than the first ones.

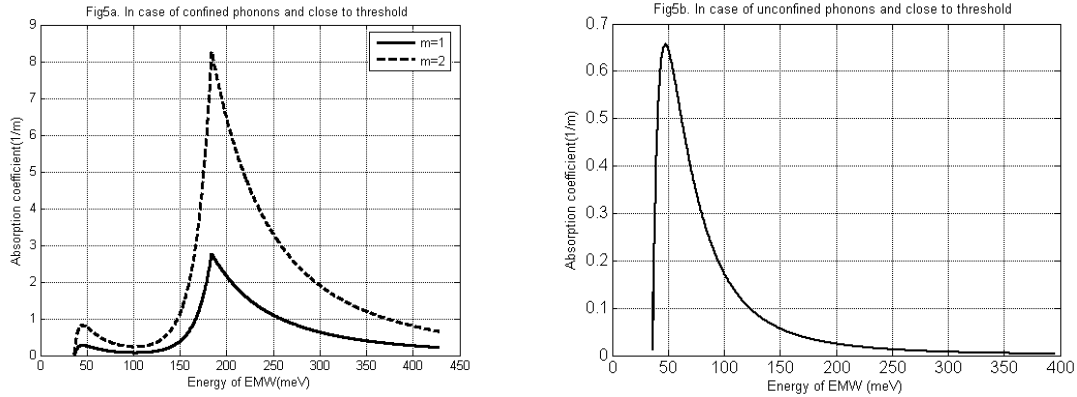


Fig 5a & 5b. The dependence of α on $\hbar\Omega$ in compositional superlattices
in case confined phonon (5a) and in case unconfined phonons (5b)

In short, all figures show that the NAC depends strongly on quantum number m characterizing confined phonons, it increases following m . The values of NAC in case of confined phonons much higher than those in case unconfined phonons. The great impact of confined phonons on NAC is expressed by the above results.

4. Conclusion

In this paper, we have theoretically studied the nonlinear absorption of a strong EMW by confined electrons in compositional superlattices under the influences of confined phonons. We have obtained a quantum kinetic equation for electrons in compositional superlattices. By using the tautology approximation methods, we can solve this equation to find out the expression of electrons distribution function. So that, we received the formulae of the NAC for two limited cases, which are far away from the absorption threshold, Eq. (11a&11b) and close to the absorption threshold, Eq. (12a&12b). We numerically calculated and graphed the NAC for compensated GaAs-Al_{0.3}Ga_{0.7}As compositional superlattices to clarify the theoretical results. Numerical results present clearly the dependence of the NAC on the amplitude (E_o), energy ($\hbar\Omega$) of the external strong NAC, the temperature (T) of the system, the period (d_A). There are more resonant peaks of the absorption coefficient appearing and the values of the NAC are larger than they are in case of unconfined phonons. The NAC depends strongly on the quantum number m characterizing confined phonons. In short, the confinement of phonons in compositional superlattices makes the nonlinear absorption of a strong NAC by confined electrons stronger.

Acknowledgments

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