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Nonlinear stability analysis of eccentrically stiffened S-FGM imperfect plates resting on elastic foundations

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Abstract. This paper presents an analytical approach to investigate effects of elastic foundation and stiffeners on the nonlinear buckling behavior of eccentrically stiffened S-FGM imperfect plates subjected to mechanical loads. By using classical plate theory, Bubnov-Galerkin's method and stress function, the close-form expressions of buckling loads and nonlinear post-buckling load-deflection curves of the S-FGM plates are determined.

Keywords: Nonlinear stability, Eccentrically stiffened S-FGM Plates, Elastic foundations, Classical plate theory.

1. Introduction

Functionally Graded Materials (FGMs) are microscopically inhomogeneous composites usually made from a mixture of metals and ceramics. By gradually varying the volume fraction of constituent materials, their effective properties exhibit a smooth and continuous change from one surface to another, thus eliminating interface problems and mitigating thermal stress concentrations. Due to essential characteristics such as high stiffness, excellent thermal resistance capacity compared with ordinary materials, FGMs have been widely used for a variety of engineering applications [1]. Cheng and Kitipornchai [2] proposed a membrane analogy to derive an exact explicit eigenvalues for compression buckling and vibration of FGM plates on a Winkler-Pasternak foundation based on the first-order shear deformation plate theory.

There are many researches on FGM plates, such as Duc et al. [3] investigated the effects of elastic foundations on nonlinear stability of FGM plates under compressive and thermal loads. However, the recent attention focuses only on P-FGM plates and shells (with metal-ceramic or ceramic-metal layers). In fact, there is a little understanding about the S-FGM with metal-ceramic-metal (M-C-M) or ceramic-metal-ceramic layers (C-M-C) plate which needs a further study. The M-C-M plate is high stiffness, can conduct electricity in both two metal sides. These have many applications in electronics and communication. Recently, Duc and Cong [4] have investigated nonlinear dynamic response of

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imperfect symmetric thin S-FGM plate with metal-ceramic-metal layers on an elastic foundation. But no one presents static stability analysis of S-FGM plate with M-C-M layers.

In order to increase the loading ability of composite structures, one usually choose stiffeners to reinforce them. Najafizadeh et al. [5] studied static buckling behavior of axially compressed stiffened cylindrical FGM shells subjected to mechanical loads. Bich et al. [6, 7] investigated the nonlinear static and dynamic responses of eccentrically stiffened FGM plates, shells and panels using an analytical approach.

As far as the authors' knowledge, the buckling behavior of eccentrically stiffened S-FGM plates (M-C-M) resting on an elastic foundation under combined loads have not been investigated. This paper studied the nonlinear buckling and post-buckling of eccentrically stiffened S-FGM imperfect plates (M-C-M) applying Galerkin's method and stress function. The effects of geometric parameters, volume fraction of constituent materials, stiffeners, elastic foundations on nonlinear buckling behavior of S-FGM plates are calculated numerically.

2. Eccentrically stiffened S-FGM (ESS-FGM) plates on elastic foundations

2.1. ESS-FGM plates on elastic foundations

We consider a rectangular ESS-FGM plate that consists of layers made of functionally graded metal-ceramic and metal materials. The outer surface layers of the plate are metal-rich, but the core of the FGM plate is pure ceramic. The plate is referred to a Cartesian coordinate system x, y, z, where xy is the midplane of the plate and z is the thickness coordinator, $-h/2 \le z \le h/2$.The length, width , and total thickness of the plate are a, b and h. Applying a Sigmoi power law distribution for S-FGM plates, the volume fractions of metal and ceramic, V_m and V_c, are assumed as [8]:

$$V_c(z) = \begin{cases} \left(\dfrac{2z+h}{h}\right)^N, & -h/2 \le z \le 0 \\ \left(\dfrac{-2z+h}{h}\right)^N, & 0 \le z \le h/2 \end{cases} \quad , \quad V_m(z) = 1 - V_c(z) \qquad (1)$$

where the volume fraction index N is a nonnegative number that defines the material distribution and can be chosen to optimize the structural response.

It is assumed that the effective properties P_{eff} of the functionally graded plate, such as the modulus of elasticity E, vary in the thickness direction z and can be determined by the linear rule of mixture as

$$P_{eff} = \Pr_m V_m(z) + \Pr_c V_c(z) \qquad (2)$$

where $\Pr$ denotes a material property, and the subscripts m and c stand for the metal and ceramic constituents, respectively.

From Eqs. (1) and (2), the effective properties of the S-FGM plate can be written as follows:

$$E(z) = E_m + E_{cm}\begin{cases}\left(\dfrac{2z+h}{h}\right)^N, & -h/2 \le z \le 0\\ \left(\dfrac{-2z+h}{h}\right)^N, & 0 \le z \le h/2\end{cases}$$

$E_{cm} = E_c - E_m$,

and the Poisson ratio is assumed to be a constant:

$$\nu(z) = \nu = const \qquad (3)$$

The load-displacement relationship of the foundation is assumed as following

$$q_0 = K_1 w - K_2 \nabla^2 w \qquad (4)$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is Laplace operator, w is the deflection of the plate, and K_1, K_2 are the Pasternak foundation stiffness.

2.2. Governing equations

According to the classical shell theory and geometrical nonlinearity in von Karman-Donnell sense[8], the strains at the middle surface and curvatures are related to the displacement components u, v, w in the x, y, z coordinate directions as [9]:

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{pmatrix} + z\begin{pmatrix}\chi_x\\ \chi_y\\ \chi_{xy}\end{pmatrix}, \qquad (5)$$

where

$$\begin{pmatrix}\varepsilon_{x0}\\ \varepsilon_{y0}\\ \gamma_{xy0}\end{pmatrix} = \begin{pmatrix}u_{,x} + w_{,x}^2/2\\ v_{,y} + w_{,y}^2/2\\ u_{,y} + v_{,x} + w_{,x}w_{,y}\end{pmatrix}, \quad \begin{pmatrix}\chi_x\\ \chi_y\\ \chi_{xy}\end{pmatrix} = \begin{pmatrix}-w_{,xx}\\ -w_{,yy}\\ -w_{,xy}\end{pmatrix}. \qquad (6)$$

From Eq. (5) the strains must be relative in the deformation compatibility equation

$$\frac{\partial^2 \varepsilon_x^0}{\partial y^2} + \frac{\partial^2 \varepsilon_y^0}{\partial x^2} - \frac{\partial^2 \gamma_{xy}^0}{\partial x\, \partial y} = \left(\frac{\partial^2 w}{\partial x\, \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \quad (7)$$

Hook's stress-strain relation is applied for the plate

$$\left(\sigma_x, \sigma_y\right) = \frac{E}{1-\nu^2}\left[(\varepsilon_x, \varepsilon_y) + \nu(\varepsilon_y, \varepsilon_x)\right]$$

$$\sigma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}, \quad \sigma_{xz} = \frac{E}{2(1+\nu)}\gamma_{xz}, \quad \sigma_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz} \quad (8a)$$

And for stiffeners

$$\sigma_x^{st} = E_0 \varepsilon_x$$
$$\sigma_y^{st} = E_0 \varepsilon_y \quad (8b)$$

Where E_0 is Young's modulus of stiffeners.

Taking into account the contribution of stiffeners by the smeared stiffeners technique and omitting the twist of stiffeners and integrating the stress-strain equations and their moments through the thickness of the plate, we obtain the expressions for force and moment resultant of an eccentrically stiffened S-FGM plate as [7]:

$$N_x = (A_{11} + \frac{E_0 A_1}{s_1})\varepsilon_x^0 + A_{12}\varepsilon_y^0 - (B_{11} + C_1)\chi_x - B_{12}\chi_y;$$

$$N_y = A_{12}\varepsilon_x^0 + (A_{22} + \frac{E_0 A_2}{s_2})\varepsilon_y^0 - B_{12}\chi_x - (B_{22} + C_2)\chi_y;$$

$$N_{xy} = A_{66}\gamma_{xy}^0 - 2B_{66}\chi_{xy};$$

$$M_x = (B_{11} + C_1)\varepsilon_x^0 + B_{12}\varepsilon_y^0 - (D_{11} + \frac{E_0 I_1}{s_1})\chi_x - D_{12}\chi_y; \quad (9)$$

$$M_y = B_{12}\varepsilon_x^0 + (B_{22} + C_2)\varepsilon_y^0 - D_{12}\chi_x - (D_{22} + \frac{E_0 I_2}{s_2})\chi_y;$$

$$M_{xy} = B_{66}\gamma_{xy}^0 - 2D_{66}\chi_{xy}$$

Where $A_{ij}, B_{ij}, D_{ij}\ \left(i, j = 1, 2, 6\right)$ are extensional, coupling and bending stiffeners of the plate without stiffeners.

$$
\begin{aligned}
&A_{11} = A_{22} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu^2} dz = \frac{E_1}{1-\nu^2}; A_{12} = \int_{-h/2}^{h/2} \frac{E(z)\nu}{1-\nu^2} dz = \frac{E_1\nu}{1-\nu^2};\\
&A_{66} = \int_{-h/2}^{h/2} \frac{E(z)}{2(1+\nu)} dz = \frac{E_1}{2(1+\nu)};\\
&B_{11} = B_{22} = \int_{-h/2}^{h/2} \frac{E(z)z}{1-\nu^2} dz = \frac{E_2}{1-\nu^2}; B_{12} = \int_{-h/2}^{h/2} \frac{E(z)z\nu}{1-\nu^2} dz = \frac{E_2\nu}{1-\nu^2};\\
&B_{66} = \int_{-h/2}^{h/2} \frac{zE(z)}{2(1+\nu)} dz = \frac{E_2}{2(1+\nu)};\\
&D_{11} = D_{22} = \int_{-h/2}^{h/2} \frac{z^2E(z)}{1-\nu^2} dz = \frac{E_3}{1-\nu^2}; D_{12} = \int_{-h/2}^{h/2} \frac{z^2E(z)\nu}{1-\nu^2} dz = \frac{E_3\nu}{1-\nu^2};\\
&D_{66} = \int_{-h/2}^{h/2} \frac{z^2E(z)}{2(1+\nu)} dz = \frac{E_3}{2(1+\nu)};
\end{aligned} \tag{10}
$$

With

$$
\begin{aligned}
&E_1 = \left(E_m + \frac{E_c - E_m}{N+1}\right)h, E_2 = 0, E_3 = \left(\frac{E_m}{12} + \frac{E_c - E_m}{2(N+1)(N+2)(N+3)}\right)h^3,\\
&I_1 = \frac{b_1h_1^3}{12} + A_1z_1^2; I_2 = \frac{b_2h_2^3}{12} + A_2z_2^2;\\
&C_1 = \frac{E_0A_1z_1}{s_1}; C_2 = \frac{E_0A_2z_2}{s_2};\\
&z_1 = \frac{h_1+h}{2}; z_2 = \frac{h_2+h}{2}
\end{aligned} \tag{11}
$$

In relation (9) and (11) E_0 is Young modulus of stiffener which takes the value $E_0 = E_m$ because the stiffeners are put at the outer surface. The spacing of the longitudinal and transversal stiffeners are denoted by s_1 and s_2 respectively. The quantities A_1, A_2 are the cross-section areas of stiffeners and I_1, I_2, z_1, z_2 are the second moments of cross section areas.

The strain-force resultant relations reversely are obtained from Eq. (9)

$$
\begin{aligned}
&\varepsilon_x^0 = A_{22}^*N_x - A_{12}^*N_y + B_{11}^*\chi_x + B_{12}^*\chi_y;\\
&\varepsilon_y^0 = A_{11}^*N_y - A_{12}^*N_x + B_{21}^*\chi_x + B_{22}^*\chi_y;\\
&\gamma_{xy}^0 = A_{66}^*N_{xy}
\end{aligned} \tag{12}
$$

Where

$$A_{11}^{*}=\frac{1}{\Delta}\left(A_{11}+\frac{E_0A_1}{s_1}\right);A_{22}^{*}=\frac{1}{\Delta}\left(A_{22}+\frac{E_0A_2}{s_2}\right);$$
$$A_{12}^{*}=\frac{A_{12}}{\Delta};A_{66}^{*}=\frac{1}{A_{66}};$$
$$\Delta=\left(A_{11}+\frac{E_0A_1}{s_1}\right)\left(A_{22}+\frac{E_0A_2}{s_2}\right)-A_{12}{}^{2}; \qquad (13)$$
$$B_{11}^{*}=A_{22}^{*}C_1;B_{22}^{*}=A_{11}^{*}C_2;B_{12}^{*}=-A_{12}^{*}C_2;B_{21}^{*}=-A_{12}^{*}C_1;$$

Substituting Eq. (12) into Eq. (9) yields:

$$M_x=B_{11}^{*}N_x+B_{21}^{*}N_y-D_{11}^{*}\chi_x-D_{12}^{*}\chi_y;$$
$$M_y=B_{12}^{*}N_x+B_{22}^{*}N_y-D_{21}^{*}\chi_x-D_{22}^{*}\chi_y; \qquad (14)$$
$$M_{xy}=-2D_{66}^{*}\chi_{xy}.$$

Where

$$D_{11}^{*}=D_{11}+\frac{E_0I_1}{s_1}-C_1B_{11}^{*};$$
$$D_{22}^{*}=D_{22}+\frac{E_0I_2}{s_2}-C_2B_{22}^{*};$$
$$D_{12}^{*}=D_{12}-C_1B_{12}^{*}; \qquad (15)$$
$$D_{21}^{*}=D_{12}-C_2B_{21}^{*};$$
$$D_{66}^{*}=D_{66}.$$

Based on the classical plate theory we have:

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0;$$
$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0; \qquad (16)$$
$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}+N_y\frac{\partial^2 w}{\partial y^2}+K_1w-K_2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=0$$

The first two of Eq. (16) are satisfied automatically by choosing a stress function as: φ

$$N_x = \frac{\partial^2 \varphi}{\partial y^2}; N_y = \frac{\partial^2 \varphi}{\partial x^2}; N_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y} \tag{17}$$

The substitution of Eq. (12) into the compatibility Eq. (7) and Eq. (17) into the third of Eq. (16) we have a system of equations:

$$A_{11}^* \frac{\partial^4 \varphi}{\partial x^4} + (A_{66}^* - 2A_{12}^*) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + A_{22}^* \frac{\partial^4 \varphi}{\partial y^4} + B_{21}^* \frac{\partial^4 w}{\partial x^4} + B_{12}^* \frac{\partial^4 w}{\partial y^4} + (B_{11}^* + B_{22}^*) \frac{\partial^4 w}{\partial x_1^2 \partial y^2} = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} . \frac{\partial^2 w}{\partial y^2} \tag{18}$$

$$D_{11}^* \frac{\partial^4 w}{\partial x^4} + (D_{12}^* + D_{21}^* + 4D_{66}^*) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}^* \frac{\partial^4 w}{\partial y^4} - B_{21}^* \frac{\partial^4 \varphi}{\partial x^4} - (B_{11}^* + B_{22}^*) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} - B_{12}^* \frac{\partial^4 \varphi}{\partial y^4} - \frac{\partial^2 \varphi}{\partial y^2} . \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 \varphi}{\partial x \partial y} . \frac{\partial^2 w}{\partial x \partial y}$$
$$- \frac{\partial^2 \varphi}{\partial x^2} . \frac{\partial^2 w}{\partial y^2} + K_1 w - K_2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0 \tag{19}$$

For an S-FGM imperfect plate, following [9], Eqs. (18) and (19) are modified into form as

$$A_{11} \frac{\partial^4 \varphi}{\partial x^4} + (A_{66}^* - 2A_{12}^*) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + A_{22}^* \frac{\partial^4 \varphi}{\partial y^4} + B_{21}^* \frac{\partial^4 w}{\partial x^4} + (B_{11}^* + B_{22}^*) \frac{\partial^4 w}{\partial x^2 \partial y^2} + B_{12}^* \frac{\partial^4 w}{\partial y^4} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{\partial^2 w}{\partial x^2} . \frac{\partial^2 w}{\partial y^2}$$
$$-2 \frac{\partial^2 w}{\partial x \partial y} . \frac{\partial^2 w_*}{\partial x \partial y} + \frac{\partial^2 w}{\partial x^2} . \frac{\partial^2 w_*}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} . \frac{\partial^2 w_*}{\partial x^2} = 0, \tag{20}$$

$$D_{11}^* \frac{\partial^4 w}{\partial x^4} + (D_{12}^* + D_{21}^* + 4D_{66}^*) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}^* \frac{\partial^4 w}{\partial y^4} - B_{21}^* \frac{\partial^4 \varphi}{\partial x^4} - (B_{11}^* + B_{22}^*) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} - B_{12}^* \frac{\partial^4 \varphi}{\partial y^4} -$$
$$\frac{\partial^2 \varphi}{\partial y^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_*}{\partial x^2} \right) + 2 \frac{\partial^2 \varphi}{\partial x \partial y} \left(\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w_*}{\partial x \partial y} \right) - \frac{\partial^2 \varphi}{\partial x^2} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_*}{\partial y^2} \right) + K_1 w - K_2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0, \tag{21}$$

in which $w_* = w_*(x, y)$ is a known function representing initial small imperfection of the plate and w is additional deflection of plate.

The couple of nonlinear Eqs. (18) and (19) or Eqs.(20) and (21) in terms of two dependent unknown w and φ are used to investigate the stability of in-plane compressed ESS-FGM plates on elastic foundation.

3. Solution of basic equations

Consider an ESS-FGM imperfect plate with simply supported and subject to in-plane compressive loads of intensities p_1 and p_2 respectively. In this case the boundary conditions are

$$w=0,\ M_x=0,\ N_x=N_{x0}=-p_1h,\ N_{xy}=0 \text{ at } x=0,\ a,$$
$$w=0,\ M_y=0,\ N_y=N_{y0}=-p_yh,\ N_{xy}=0 \text{ at } y=0,\ b. \tag{22}$$

The approximate solutions of Eq. (20) and (21) satisfying the mentioned conditions (22) are chosen in the form as

$$w=f\sin\lambda_m x\sin\mu_n y,$$
$$w_*=\xi h\sin\lambda_m x\sin\mu_n y, \tag{23}$$

where $\lambda_m=\dfrac{m\pi}{a}$, $\mu_n=\dfrac{n\pi}{b}$, m and n are the half-wave numbers along the x-axis and the y-axis, respectively, and ξ is a imperfection size of the ESS-FGM plates.

Substituting Eq. (23) into Eq. (20) and solving obtained equation for unknown φ, lead to

$$\varphi=\varphi_1\cos 2\lambda_m x+\varphi_2\cos 2\mu_n y+\varphi_3\sin\lambda_m x\sin\mu_n y+\frac{1}{2}N_{x0}y^2+\frac{1}{2}N_{y0}x^2 \tag{24}$$

Where denote

$$\varphi_1=\frac{\mu_n^2 f(f+2\xi h)}{32A_{11}^*\lambda_m^2},$$
$$\varphi_2=\frac{\lambda_m^2 f(f+2\xi h)}{32A_{22}^*\mu_n^2}, \tag{25}$$
$$\varphi_3=\frac{-f\left[B_{21}^*\lambda_m^4+(B_{11}^*+B_{22}^*)\lambda_m^2\mu_n^2+B_{12}^*\mu_n^4\right]}{A_{11}^*\lambda_m^4+(A_{66}^*-2A_{12}^*)\lambda_m^2\mu_n^2+A_{22}^*\mu_n^4}$$

Substituting the expressions Eqs. (24) and (25) into Eq. (21) and using Bubnov-Galerkin method for the resulting equation yield

$$\frac{ab}{4}\left(D^*+\frac{B^{*2}}{A^*}+D_k\right)f-\frac{2mn\pi^2\delta_m\delta_n}{3ab}.\frac{B^*}{A^*}.f(f+\xi h)+\frac{mn\pi^2\delta_m\delta_n}{6ab}.H^*f(f+2\xi h)$$
$$+\frac{m^2n^2\pi^4}{64ab}L^*f(f+\xi h)(f+2\xi h)+\frac{ab}{4}(N_{x0}\lambda_m^2+N_{y0}\mu_n^2)(f+\xi h)=0, \tag{26}$$

Where:

$$D^* = D_{11}^*\lambda_m^4 + (D_{12}^* + D_{21}^* + 4D_{66}^*)\lambda_m^2\mu_n^2 + D_{22}^*\mu_n^4,$$
$$D_k = K_1 + K_2(\lambda_m^2 + \mu_n^2),$$
$$B^* = B_{21}^*\lambda_m^4 + (B_{11}^* + B_{22}^*)\lambda_m^2\mu_n^2 + B_{12}^*\mu_n^4,$$
$$A^* = A_{11}^*\lambda_m^4 + (A_{66}^* - 2A_{12}^*)\lambda_m^2\mu_n^2 + A_{22}^*\mu_n^4, \quad (27)$$
$$H^* = \frac{B_{21}^*}{A_{11}^*} + \frac{B_{12}^*}{A_{22}^*}, L^* = \frac{\mu_n^2}{A_{11}^*\lambda_m^2} + \frac{\lambda_m^2}{A_{22}^*\mu_n^2},$$
$$\delta_m = 1-(-1)^m, \delta_n = 1-(-1)^n, m,n = 1,2,...$$

By introducing

$$D^* = \frac{\overline{D}}{h}, B^* = \frac{\overline{B}}{h^3}, A^* = \frac{\overline{A}}{h^5}, H^* = \overline{H}h^2, L^* = \overline{L}.h, f = \overline{f}.h, \lambda = a/b$$

Eq. (26) can be rewritten as

$$p_1m^2 + p_2\lambda^2n^2 == \frac{1}{\pi^2}\left(\overline{D} + \frac{\overline{B}^2}{\overline{A}} + D_kh\right)\left(\frac{a}{h}\right)^2.\frac{\overline{f}}{\overline{f}+\xi} - \frac{8}{3}mn\delta_m\delta_n\frac{\overline{B}}{\overline{A}}\left(\frac{h}{b}\right)^2.\overline{f} \quad (28)$$
$$+\frac{2mn\delta_m\delta_n}{3}\left(\frac{h}{b}\right)^2.\overline{H}.\frac{\overline{f}(\overline{f}+2\xi)}{\overline{f}+\xi} + \frac{\pi^2m^2n^2}{16}\left(\frac{h}{b}\right)^2\overline{L}.\overline{f}(\overline{f}+2\xi).$$

Eq. (28) is used to analyze the buckling and post-buckling of ESS-FGM imperfect plates resting on elastic foundations and under the in-plane compressive loads.

With only axial compressive load p_1, Eq. (28) is reduced to

$$p_1 = \frac{1}{\pi^2m^2}\left(\overline{D} + \frac{\overline{B}^2}{\overline{A}} + D_kh\right)\left(\frac{a}{h}\right)^2.\frac{\overline{f}}{\overline{f}+\xi} - \frac{8n\delta_m\delta_n}{3m}.\frac{\overline{B}}{\overline{A}}.\left(\frac{h}{b}\right)^2.\overline{f} + \frac{2n\delta_m\delta_n}{3m}\left(\frac{h}{b}\right)^2.\overline{H}.\frac{\overline{f}(\overline{f}+2\xi)}{\overline{f}+\xi} + \frac{\pi^2n^2}{16}\left(\frac{h}{b}\right)^2\overline{L}\overline{f}(\overline{f}+2\xi).$$

(29)

For perfect plates, $\xi = 0$, Eq. (29) representing the load-deflection curve of plate, leads to

$$p_1 = \frac{1}{\pi^2m^2}\left(\frac{a}{h}\right)^2\left(\overline{D} + \frac{\overline{B}^2}{\overline{A}} + D_kh\right) - \left(\frac{2n\delta_m\delta_n}{3m}\right)\left(\frac{h}{b}\right)^2\left[\frac{4\overline{B}}{\overline{A}} - \overline{H}\right].\overline{f} + \frac{\pi^2n^2}{16}\left(\frac{h}{b}\right)^2.\overline{L}.\overline{f}^2. \quad (30)$$

With un-stiffened S-FGM plates($A_1 = A_2 = 0$ and $I_1 = I_2 = 0$) Eq. (28) become

$$p_1m^2 + p_2\lambda^2n^2 = \frac{1}{\pi^2}\left[D_{11}(\lambda_m^2 + \mu_n^2)^2h + D_kh\right]\left(\frac{a}{h}\right)^2\frac{\overline{f}}{\overline{f}+\xi} + \frac{\pi^2m^2n^2}{16}\left(\frac{h}{b}\right)^2\frac{E_1}{h}\left(\frac{\mu_n^2}{\lambda_m^2} + \frac{\lambda_m^2}{\mu_n^2}\right)\overline{f}\left(\overline{f}+2\xi\right).$$

(31)

4. Numerical results and discussions

In this paper, some examples will be presented for perfect and imperfect simply supported–symmetric S-FGM plate. We use the above formulations to investigate the effect of material parameters on the buckling and post-buckling behavior of the eccentrically stiffened S-FGM plate. We consider a square metal-ceramic-metal plate which consists of aluminum and alumina with geometrical parameters:

$$h = 0.005(m); m = 1; n = 1;\ \nu = 0.3;\ \ Em = 70\ GPa;\ Ec = 380\ GPa; b/h = 40\ ;\ a/b = 1;$$

This ESS-FGM plate with parameters of stiffeners:

$$s_1 = 0.2(m); s_2 = 0.2(m); h_1 = 0.003(m); h_2 = 0.003(m); b_1 = 0.004(m);\ b_2 = 0.004(m);$$

and elastic modulus of stiffeners are taken by

$$E_0 = E_m = 70(GPa)$$

In Fig.1, f/h denotes the dimension-less maximum deflection of plate, the plate act on axially compression and foundations interaction are ignored.

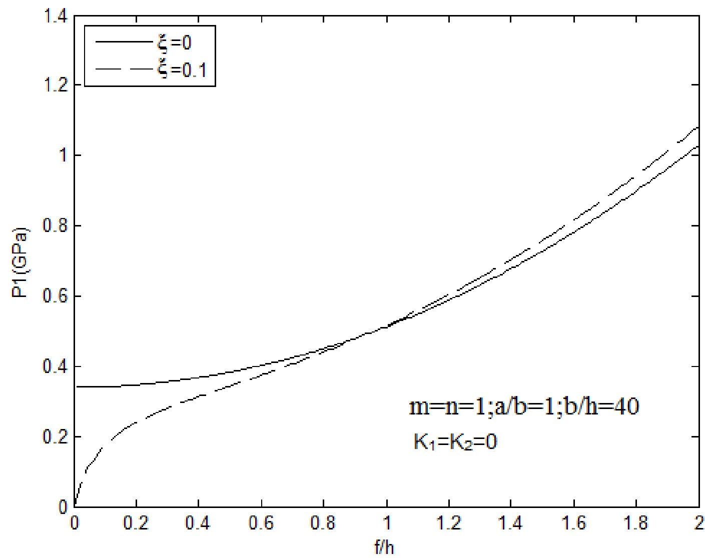

Fig.1. Post-buckling curves for ESS-FGM perfect and imperfect plates under axial compression without elastic foundations

To illustrate the effects of foundations on the post-buckling behavior of ESS-FGM plate, we consider three aspects: axially compressed plate resting on the Winkler foundation $(K_1 \neq 0,\ K_2 = 0)$, on the Pasternak foundation $(K_1 = 0,\ K_2 \neq 0)$, and on both foundations with $(K_1 \neq 0,\ K_2 \neq 0)$.

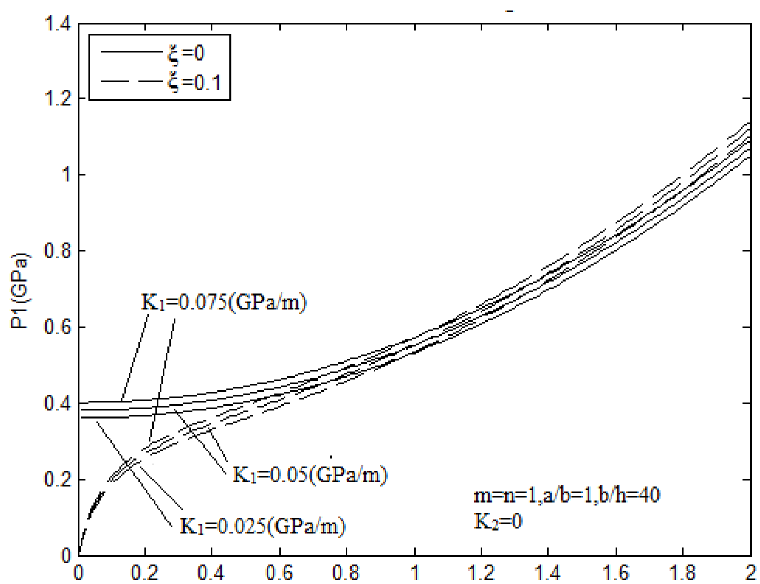

Fig.2. Effects of Winkler foundation on nonlinear buckling for the ESS-FGM plate

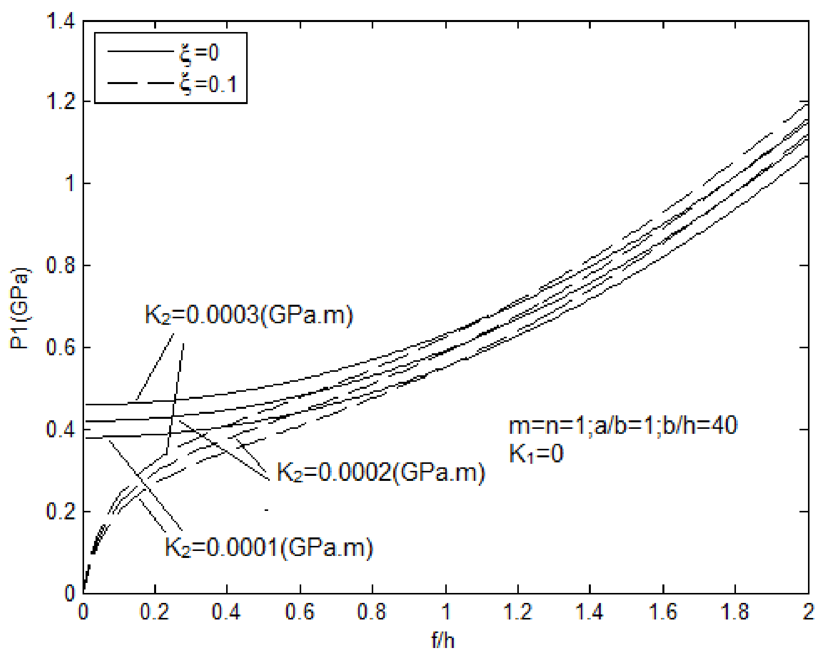

Fig.3. Effects of Pasternak foundation on nonlinear buckling for the ESS-FGM plate

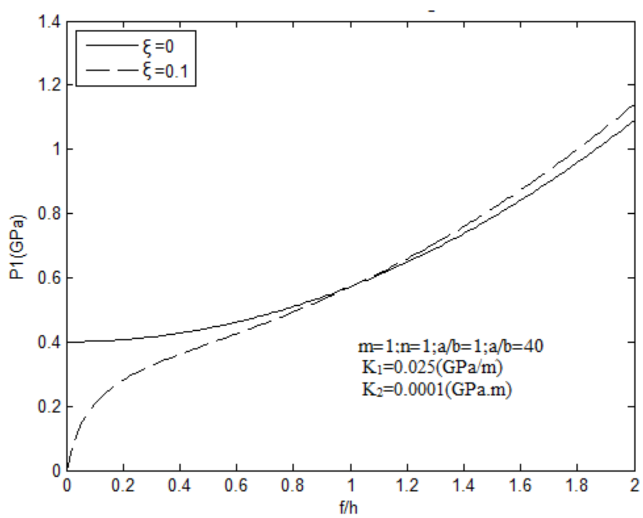

Fig. 4. Effects of both elastic foundations on nonlinear buckling for the ESS-FGM plate

Figs. 2, 3 and 4 show that a ESS-FGM plate resting on Winkler and Pasternak foundations have better behavior than a single Pasternak foundation or Winkler foundation, and the Pasternak foundation has stronger effect than Winkler foundation.

Fig. 5 shows influence of volume fraction index on nonlinear postbuckling for ESS-FGM plates with the ratio $b/h = 40$ under compression loads on one side with three volume fraction indexes $N = (0,1,2)$. As we can see, the smaller the volume fraction coefficient N is, the stronger buckling and postbuckling capacity loads are. Indeed, the similar feature has been found in [4] for S-FGM plates with ceramic-metal-ceramic layers without stiffeners.

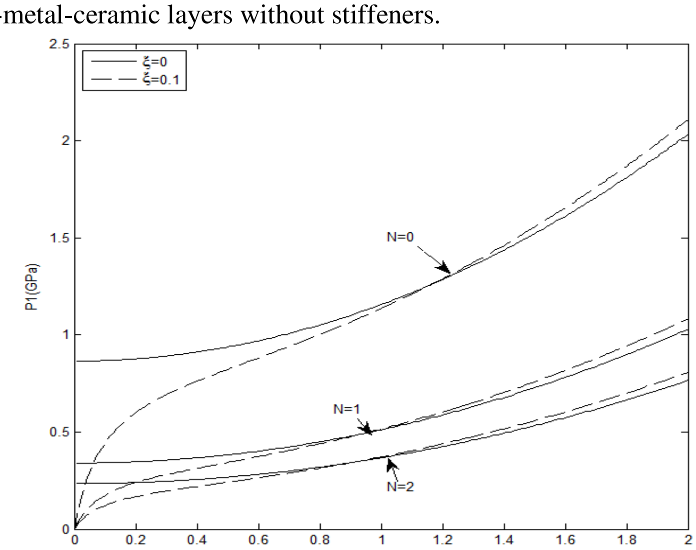

Fig.5. Effects of volume fraction index on postbuckling of the ESS-FGM plate.

Finally, we compare behaviors of the ESS-FGM plate and the un-stiffened S-FGM plates resting on elastic foundations ($K_1, K_2 \neq 0$). To illustrate these effects we consider ESS-FGM plates with geometrical parameters:

$$h = 0.012(m); s_1 = 0.2(m); s_2 = 0.2(m);\ h_1 = 0.01(m); h_2 = 0.01(m); b_1 = 0.016(m); b_2 = 0.016(m);$$

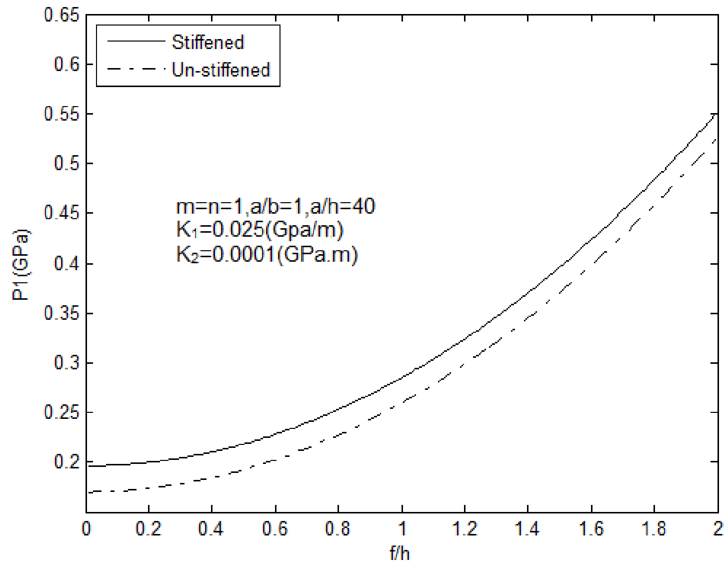

Fig. 6. Effect of stiffeners on buckling of perfect ESS-FGM plate

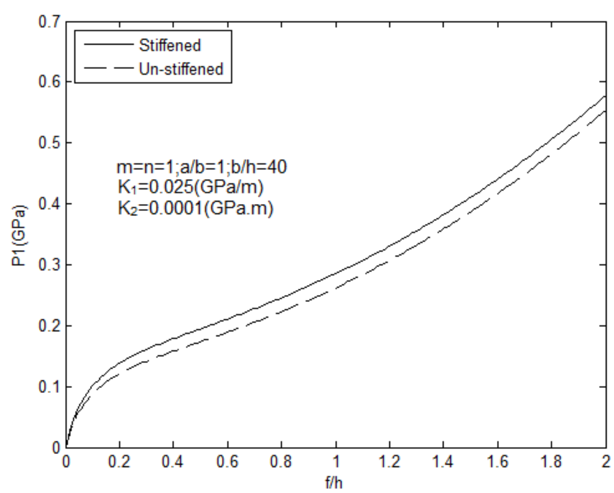

Fig. 7. Effect of stiffeners on buckling of imperfect ESS-FGM plate

The influence of perfection or imperfection on the stability of ESS-FGM and un-stiffened S-FGM plates under compression and resting on elastic foundation are shown in figs. 6 and 7. The loading ability of the ESS-FGM plates are better than S-FGM plates, in both cases of perfect and imperfect plates.

5.Conclusions

Analytical approach has been developed in this paper for studying nonlinear buckling and postbuckling behaviors of eccentrically stiffened S- FGM imperfect rectangular thin plates resting on elastic foundations under axial compression loads.

The governing equations of eccentrically stiffened S-FGM are based upon the classical plate theory and smeared stiffeners technique. The modulus of elasticity E changes with the thickness in z directions and constant Poisson's ratio. The basic equations to investigate the stability of eccentrically stiffened S-FGM plates and some expression of nonlinear buckling and post-buckling behavior of the S-FGM plates are determined in this paper.

The influences of Winkler and Pasternak foundations, stiffeners and imperfection sensitivity on nonlinear stability of plates are discussed in details. ESS-FGM plate have better loading capacity than the un-stiffened S-FGM plate under axial compression loads.

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