VNU Journal of Natural Sciences and Technology, Vol. 29, No. 3 (2013) 1-7

Analysis the Statistical Parameters of the Wavelet Coefficients for Image Denoising

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Received 14 September 2012

Revised 28 September 2012; accepted 28 June 2013

Abstract: Image denoising is aimed at the removal of noise which may corrupt an image during its acquisition or transmission. De-noising of the corrupted image by Gaussian noise using wavelet transform is very effective way because of its ability to capture the energy of a signal in few larger values. This paper proposes a threshold selection method for image de-noising based on the statistical parameters which depended on sub-band data. The threshold value is computed based on the number of coefficients in each scale *j* of wavelet decomposition and the noise variance in various sub-band. Experimental results in PSNR on several test images are compared for different de-noise techniques.

1. Introduction

Image de-noising is a common procedure in digital image processing aiming at the removal of noise which may corrupt an image during its acquisition or transmission while sustaining its quality. Noise is unwanted signal that interferes with the original signal and degrades the quality of the digital image. Different types of images inherit different types of noise and different noise models are used for different noise types.

Noise is present in image either in additive or multiplicative form [1]. Various types of noise have their own characteristics and are inherent in images in different ways. Gaussian noise is evenly distributed over the signal. Salt and pepper noise is an impulse type of noise (intensity spikes). Speckle noise is multiplicative noise which occurs in almost all coherent systems. Image de-noising is still a challenging problem for researchers as which causes blurring and introduces artifacts. De-noising method tends to be problem specific and depends upon the type of image and noise model.

De-noising based on transform domain filtering and wavelet can be subdivided into data adaptive and non-adaptive filters [2].

Image de-noising based on spatial domain filtering is classified into linear filters and non-linear filters [3, 4]. In [5, 6], the paper proposes an adaptive, data driven threshold for image denoising via wavelet soft thresholding.

A proposal of vector/matrix extension of denoising algorithm developed for grayscale images, in order to efficiently process

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multichannel is presented in [7]. In [8], authors propose several methods of noise removal from degraded images with Gaussian noise by using adaptive wavelet threshold (Bayes Shrink, Modified Bayes Shrink and Normal Shrink).

This paper is organized as follows: A brief review of DWT and wavelet filter banks are provided in session II. In session III, the wavelet based thresholding technique is explained. The methods of selection of wavelet thresholding is presented in IV. In session V the new proposed thresholding technique for denoising is presented. The experiment results of this work are compared with others in session VI and concluding remarks are given.

2. Discrete wavelet transform (DWT)

The mathematical approach of the discrete wavelet transform (DWT) is based on

$$f(t) = \sum_k a_k \psi_k(t) \qquad (1)$$

Where a_k are the analysis coefficients and $\psi_k(t)$ is the analyzing functions, which are called basic functions. If the basic functions are orthogonal, that is

$$\langle \psi_k(t), \psi_l(t) \rangle = \int \psi_k(t)\psi_l(t)dt = 0 \quad for \quad k \neq l \qquad (2)$$

The coefficients can be estimated from the following equation:

$$a_k = \langle f(t), \psi_k(t) \rangle = \int f(t)\psi_k(t)dt \qquad (3)$$

Wavelets consist of the dilations and translations of a single valued function (analyzing wavelet or basic wavelet or also known as the mother wavelet) $\psi \in L^2(R)$. The family of function $\psi_{s,\tau}$ by dilations and translations of ψ

$$\psi_{s,\tau}(t) = s^{-1/2}\psi\left(\frac{t-\tau}{s}\right) \qquad s,\tau \in R, s > 0 \qquad (4)$$

In general, a 2-D signal may be transformed by DWT as

$$f(t) = \sum_k \sum_j a_{j,k}\psi_{j,k}(t) \qquad (5)$$

Where $a_{j,k}$ and $\psi_{j,k}(t)$ are the transformed coefficients and basis functions respectively.

Another consideration of the wavelets is the sub-band coding theory or multi-resolution analysis. The signal passes successively through pairs of lowpass and high pass filters, which produce the transformed coefficients (analysis filters). By passing these coefficients successively through synthesis filters, we reproduce the original signal at the decoder. An input signal S maybe equivalently analysed as:

$$S = A_3 + D_3 + D_2 + D_1 \quad \text{Level 3} \qquad (6)$$

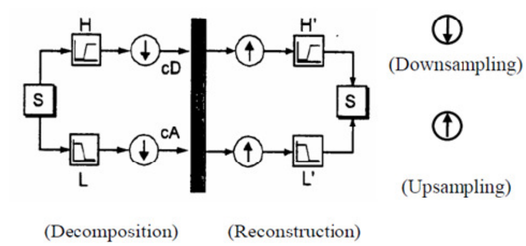
$$S = A_2 + D_2 + D_1 \quad \text{Level 2} \qquad (7)$$

$$S = A_1 + D_1 \quad \text{Level 1} \qquad (8)$$

Similarly, by using wavelet packet decomposition, the signal may be analysed as

$$S = A_1 + AAD_3 + DAD_3 + ADD_3 + DDD_3 \qquad (9)$$

The process of decomposition and reconstruction is in figure 1.



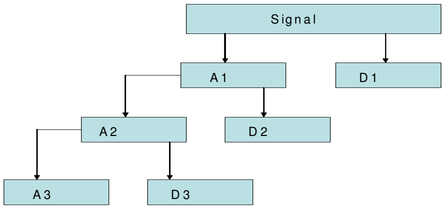


Fig. 1. Wavelet decomposition and reconstruction.

3. Wavelet thresholding

Let $f = \{f_{ij}, i, j = 1, 2, ...M\}$ (10)

denote the $M \times M$ matrix of the original image to be recovered and M is some integer power of 2. Assume the signal function f is corrupted by independent and identically distributed (*i.i.d*) zero mean, white Gaussian noise n_{ij} with standard deviation σ i.e, n_{ij} ~ N(0, σ^2), so that the noisy image is obtained.

$$g_{ij} = f_{ij} + \sigma n_{ij} \tag{11}$$

The goal is to estimate an $\hat{f}_{ij}$ from noisy g_{ij} (*M, N* are width and height of image) such that Mean Squared Error (MSE) is calculated in (12)

$$MSE = \frac{1}{MN}\sum_{j=1}^{M}\sum_{i=1}^{N}\left(\hat{f}_{ij} - f_{ij}\right)^2 \tag{12}$$

The observation model is expressed as follows:

$$Y = X + V \tag{13}$$

Here Y is wavelet transform of the noisy degraded image, X is wavelet transform of the original image and V denotes the wavelet transform of the noise components in Gaussian distribution $N(0, \sigma_v^2)$. Since X and V are mutually independent, we have

$$\sigma_y^2 = \sigma_x^2 + \sigma_v^2 \tag{14}$$

It has been shown that the noise standard deviation σ_v^2 can be estimated from the first decomposition level diagonal subband HH_1 by the robust and accurate median estimator [5].

$$\sigma_v^2 = \left[\frac{median\left(\left|HH_1\right|\right)}{0.6745}\right]^2 \tag{15}$$

The variance of the sub-band of noisy image can be estimated as (A_m are wavelet coefficients of subband under consideration. M is the total number of wavelet coefficient in that sub-band)

$$\sigma_y^2 = \frac{1}{M}\sum_{m=1}^{M} A_m^2 \tag{16}$$

In figure 1 shown wavelet decomposition in 3 levels. The su-bands HH^k, HL^k, LH^k are called the details (*k* is level ranging from 1 to the largest number *J*). The LL^J is the low resolution residue. The size of the subband at scale *k* is $\frac{M}{2^k} \times \frac{M}{2^k}$.

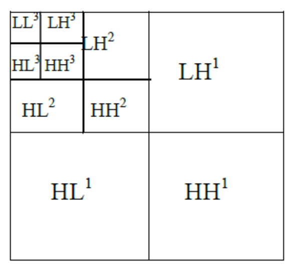


Fig.2. Sub-bands of the 2-D orthogonal wavelet transform with 3 decomposition levels (H- High frequency bands and L-Low frequency bands).

The wavelet threshold denoising method filters each coefficient from the detail subbands with a threshold function to obtain modified coefficients. Threshold plays an important role in the denoising process. There are two thresholding methods in used. The hard thresholding operator is defined as

$D(U,\lambda) = U$ for all $|U| > \lambda$ and $D(U,\lambda) = 0$ *otherwise* (17)

The soft thresholding operator on the other hand is defined as

$$D(U,\lambda) = \mathrm{sgn}(U) * \max(0, |U| - \lambda) \quad (18)$$

Hard thresholding is "keep or kill" procedure and it introduces artifacts in the recover images. Soft thresholding is more efficient and it is used to achieved near minmax rate and to yield visually more pleasing images. The soft-threshold function (shrinkage function) and the hard threshold as depicted in figure 3.

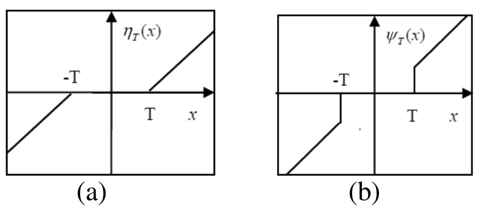


Fig. 3. Thresholding function (a) Soft threshold (b) Hard threshold.

4. Methods of threshold selection for image denoising

4.1. Universal threshold

Universal threshold can be defined as

$$T = \sigma\sqrt{2\log(N)} \quad (19)$$

N being the signal length i.e the size of the image, σ is noise variance.

This is easy to implement but provide a threshold level much depend on the size *N* of image resulting in smoother reconstructed image. This threshold estimation does not care of the content of the data and provide the value larger than other.

4.2. Visu Shrink

Visu Shrink was introduce by Donoho [6]. It uses a threshold value that is proportional to the standard deviation of the noise. The estimation of σ was defined by

$$\sigma = \frac{median\left(\left|g_{j-1,k}\right| : k = 0,1,...2^{j-1} - 1\right)}{0.6745} \quad (20)$$

Where $g_{j-1,k}$ corresponds to the details coefficients in the DWT. Visu Shrink does not deal with minimizing the mean squared error and can not remove speckle noise. It can only deal with an additive noise and follow the global threshold scheme. Visu shrink has a limitation of not dealing with minimizing the mean squared error, i.e it removes overly smoothed.

4.3. Sure Shrink

In Sure Shrink, a threshold is choosen based on Stein's Unbiased Risk Estimator(SURE) by Donoho and Johnstone. It is a combination of the universal threshold and SURE threshold [7] so to be smoothness adaptive. This method specifies a threshold value t_j for each resolution level *j* in the DWT. The goal of SURE is to minimize the MSE, the threshold *T* is defined as

$$T = \min\left(t, \sigma\sqrt{2\log N}\right) \quad (21)$$

Where *t* denotes the value that minimizes SURE, σ is the noise variance and *N* is the size of the image. This method threshold the empirical wavelet coefficients in groups rather than individually, making simultaneous decisions to retain or to discard all the coefficients within non-overlapping blocks.

4.4. Bayes Shrink (BS)

Bayes Shrink was proposed by Chang, Yu and Vetterli. The Bayes threshold T_B is defined as

$$T_{BS} = \sigma_v^2 / \sigma_x \quad (22)$$

Where $$\sigma_x = \sqrt{\max\left(\sigma_y^2 - \sigma_v^2\right)} \quad (23)$$

σ_v^2 is the noise variance which is estimated from the sub-band *HH* and σ_y is the variance of the original image. Note that in the case where $\sigma_v^2 \geq \sigma_y^2, \sigma_x^2$ is taken to be zero. In practice, we can choose $T_{BS} = \max\{|A_m|\}$ and all coefficients are set to zero.

Noise is not being sufficiently removed in an image using Bayes Shrink method. So the paper [8] referred to Modified Bayes Shrink (MBS). It performs the threshold values that are different for coefficients in each sub-band. The threshold *T* can be determined as follows:

$$T_{MBS} = \beta\sigma_v^2 / \sigma_x \quad (24)$$

where

$$\beta = \sqrt{\frac{\log N}{2 \times j}} \quad (25)$$

N is the total of coefficients of wavelet, j is the wavelet decomposition level present in the sub-band under scrutiny.

4.5. Normal Shrink

The threshold value which is adaptive to different sub-band characteristics

$$T_N = \beta\sigma_v^2 / \sigma_y \quad (26)$$

Where the scale parameter β has computed once for each scale using the following (27):

$$\beta = \sqrt{\log\left(\frac{L_K}{J}\right)} \quad (27)$$

L_k means the length of the sub-band at k^{th} scale. J is the total number of decomposition. Where σ_v^2 is the noise variance which is estimated from the equation (15) and σ_y is the variance of the noisy image which is calculated by equation (16).

5. The new proposal method

In Modified Bayes Shrink, the value of β in equation (25) only count for *N* is the total of coefficients of wavelet. So that the value of β is something "globally", which does not count for the length of the sub-band at k^{th} scale. We present a new proposal function for threshold T_N MBS in equation (24) $T_{MBS} = \beta\sigma_v^2 / \sigma_x$

In our proposed method, the value of β is substituted by

$$\beta_N = \sqrt{\frac{\log\left(\frac{N}{2^k}\right)}{2 \times k}} \quad (28)$$

Here $N/2^k$ is the length of the sub-band at scale *k*.

The image denoising algorithms that use the wavelet transform consist of the following steps:

1- Calculate the multiscale decomposition wavelet transform of the noisy image.
2- Estimate the noise variance σ_v^2 from the sub-band HH^k and σ_x is variance of the original image.
3- For each level k, compute length N of the data.
4- Compute threshold based on equation (24) and (28)

5- Apply soft threshold to the noisy coefficients.
6-Meger low frequency coefficients with denoise high frequency coefficients in step 5.
7- Invert the wavelet transform to reconstruct the denoised image.
8- Difference of noisy image and original image is calculated using **imsubract** *command.*
9- Size of the matrix obtains in step 8 is calculated
10- Each of the pixels in the matrix obtained in the steps 8 is squared and calculate sum of all the pixels.
11- MSE is obtained by taking the ratio of value obtained in step 10 to the value obtained in the step 9 as in equation (12).
12- PSNRis calculated by dividing 255 with MSE, taking log base 10 as in (29)

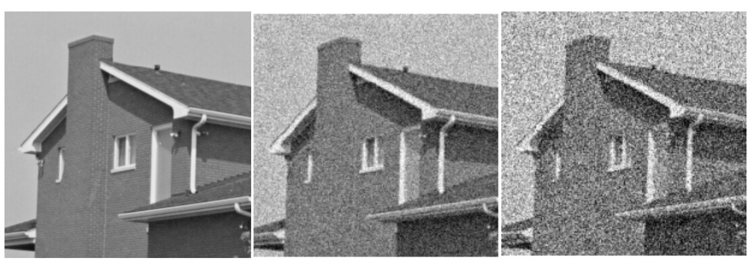
The performance of noise reduction algorithm is measure using Peak Signal to Noise Ratio (PSNR) which is defined as

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \quad dB \qquad (29)$$

6. Experimental results and discussions

We try to compare above algorithm on several test gray image like image of Lena and image of House at Gaussian noise level with noise standard deviation σ = 0.01 and σ = 0.04 using Daubechies wavelet with 3 level decomposition.

Original Lena (Left) and noisy Lena with σ = 0.01(Middle) and with σ = 0.04 (Right)



Original House (Left) and noisy house with σ = 0.01 (Middle) and σ = 0.04 (Right)

Fig. 4. Images of Lena and House using for testing of denoising methods.

The original image and noised images of Lena and House is in figure 4. Performance of noise reduction is measured using Peak Signal to Noise Ratio (PSNR) as in table 1.

From table 1, by using equation (24) and (28) we calculated the values of PSNR for Lena image and House image. The results by our proposal method is significantly improved than by using other method in term of denoising images those are corrupted by Gaussian noise during transmission which is normally random in nature.

Tabel 1. Comparision of PSNR of different wavelet thresholding selection for images corrupted by Gaussian noise

Image	Noise level	Universal threshold	Visu shrink	Bayes shrink	Modified Bayes shrink	Normal shrink	Proposed method
Lena	0.001	69.06	73.21	74.11	75.87	75.34	76.24
	0.004	56.23	59.12	61.67	62.07	61.55	62.77
House	0.001	69.02	73.56	74.38	75.89	75.23	76.04
	0.004	55.27	59.67	61.22	62.13	61.78	62.45

The proposed threshold estimation is based on the adaptation of the statistical parameters of the sub-band coefficients. Since the value of proposed threshold is calculated dependent on decomposition level with sub-band variance estimation, the method yields significantly superior quality and better PSNR.

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Phân tích các tham số thống kê của các hệ số wavelet dùng cho tách nhiễu ảnh

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Tóm tắt: Tách nhiễu cho ảnh nhằm mục đích khôi phục lại ảnh bị giảm chất lượng khi thu nhận và trong quá trình truyền. Dùng biến đổi wavelet để thực hiện việc tách nhiễu Gaussian là rất hiệu quả do hầu hết năng lượng của tín hiệu được dồn tập trung vào một số ít các hệ số. Trong bài báo này, tác giả sẽ đề xuất một phương pháp lựa chọn mức ngưỡng trong quá trình tách nhiễu cho ảnh dựa vào các tham số thống kê dữ liệu trong các dải băng con. Giá trị ngưỡng được tính toán căn cứ vào số các hệ số trong mỗi mức phân tích j của phép phân tích wavelet và phương sai của nhiễu trong các dải băng con khác nhau. Cuối cùng tác giả sẽ sẽ tiến hành so sánh hiệu quả của các phương pháp bằng thực nghiệm dựa vào tỷ số tín hiệu trên nhiễu PSNR của một số bức ảnh có nội dung khác nhau để đánh giá hiệu quả tách nhiễu.