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Image compression using Wavelet-Vector quantization

Nguyen Vinh An*

PetroVietnam University, 173 Trung Kinh, Cau Giay, Hanoi, Vietnam

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Abstract. Wavelet decompositions is powerful tool in image processing. They decorrelate the effects on image pixels, they are also concentration of image energy in a few coefficients. Because of their multi-rate/multi-resolution and their frequency splitting, they allow for efficient coding matched to the statistics of each frequency band and to the characteristics of the human visual system. In other side, vector quantization is best way to increase the compression ratio but it has a major drawback in the amount of computations for searching codewords in codebook. This paper will present concept of image compression using both Wavelet and vector quantization. By using three different code books, the a.c energy of sub-band image can be sorted before encoding which proportional with the bit rate. The combination between Wavelet and Vector quantization for image compression will help minimizing of distortion of reproduced image.

1. Image compression overview

In the more than last ten years, there have been a lots of efforts in image compression based on wavelet transform and vector quantization. Subband and wavelet decompositions are powerful tools in image coding because of their decorrelating effects on image pixels, the concentration of energy in a few coefficients, their multi-rate/multi-resolution framework. Vector quantization (VQ) provides a means of converting the decomposed signal into bits in a manner that takes advantage of remaining inter and intraband correlation as well as of the more flexible partitions of higher dimensional vector spaces. A survey of VQ methods are presented in paper [1]. Although VQ is the best way of quantizing and compressing images, it has a major drawback in the amount of computations during the search for optimum codevector in encoding. Vector quantization codewords are designed as rectangles with ratio 1 to 2, which is more efficient in encoding different subbands with minimum error [2]. Lattice vector quantization (LVQ) reduces coding complexity and computation due to its regular structure. In the paper [3] a new multistage lattice VQ (MLVQ) is presented that concentrates on reducing the quantization error.

Image Compression is a process that creates a compact image data representation for storage and transmission purposes. Most of neighboring pixels in images are correlated and therefore contain redundant information. In general, three types of redundancy can be identified:

* Tel: 84-913508067.
E-mail: annv@pvu.edu.vn

- Spatial redundancy or correlation between neighboring pixels.

- Spectral redundancy or correlation between different color planes or spectral bands.

- Temporal redundancy or correlation between adjacent frames in a sequence of images.

The aim of image compression is reducing the number of bits needed to represent an image by removing these redundancies as much as possible. There are many transform method for image compression can be used. The most existing popular JPEG and MPEG standards are based on DCT transform.

Figure 1 illustrates the main block diagram of the compression process. The image goes through a transform, which generates a set of frequency coefficients. The transformed coefficients are then quantized to reduce the volume of the encoded data. The final step is entropy coding, where the stream of quantized data is converted to a sequence of binary symbols. The integers that occur with relatively high probability are encoded with a shorter binary symbols and versus. Huffman coding and RLC are most common encoding schemes in used.

Block transform

In the transform coder, the image pixels are converted from the space domain to the transform domain through a linear orthogonal or bi-orthogonal transform. The transform decorrelates the pixels and compacts their energy into a small number of coefficients, results in efficient coding of the transform coefficients. Since most of the energy is compacted into a few large transform coefficients, we may adopt entropy coding scheme that easily locates those coefficients and encodes them. Because the transform coefficients are decorrelated, the subsequent quantizer and entropy coder can ignore the correlation among the transform coefficients, and model them as independent random variables

In DCT transform, the input image is divided in [8x8] blocks and put into Forward DCT (FDCT). The FDCT takes a block [8x8] image as its input and decomposes it into 64 orthogonal basis signals or "DCT coefficients". The coefficient with zero frequency is called the "DC coefficient" and the remaining 63 coefficients are called "AC coefficients".

In the decoder, using the inverse DCT to form the reconstructed image.

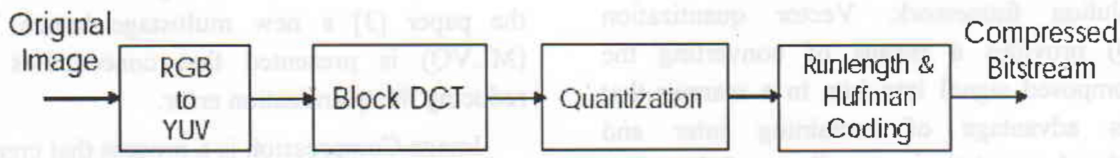

Figure 1. Block scheme of DCT Image Compression.

Quantization: After output from the FDCT, each of the 64 DCT coefficients is quantized in conjunction with a 64 element quantization table. The purpose of quantization is to achieve further compression by representing DCT coefficients with less precision, because of that the quality of reproduced image is not as good as original. The quantization is defined as division of each DCT coefficient by its corresponding quantizer's step size and then rounding to the nearest integer as in (1).

$$F^Q(u,v) = Integer \quad Round\left(\frac{F(u,v)}{Q(u,v)}\right) \quad (1)$$

The quantizer's step size will determine the quality of the image reproduction as illustratted.in figure 2.

Figure 2. The effect of the change of quantization step.

Dequantization is the inverse function as shown in (2)

$$F^{Q'}(u,v) = F^Q(u,v) * Q(u,v) \quad (2)$$

All of the quantized coefficients are order into the "zig-zag" sequence, which help to place low - frequency coefficients (big amplitude) before high - frequency coefficients (small amplitude).

Entropy Coding: The final step is entropy coding. This step achieves additional compression by encoding the quantized DCT coefficients more compactly based on their statistical characteristics. There are some common entropy coding methods include Huffman coding, Arithmetic coding.

Measurement of the difference between the original and the reconstructed image

It is natural to raise the question of how much an image can be compressed and still preserve sufficient information for a given clinical application. This section discusses some parameters used to measure the trade-off between image quality and compression ratio.

Compression ratio is defined as the nominal bit depth of the original image in bits per pixel (bpp) divided by the bpp necessary to store the compressed image. For each compressed and reconstructed image, an error image was calculated. From the error data, maximum absolute error (MAE), mean square error (MSE), root mean square error (RMSE), signal to noise ratio (SNR) and peak signal to noise ratio (PSNR) were calculated.

The MAE is calculated as (3)

$$MAE = \max\left| f(x,y) - f^*(x,y) \right| \quad (3)$$

Where $f(x, y)$ is the original image data and $f^*(x, y)$ is the compressed image value.

The formulae for calculated image matrices are

$$MSE = \frac{1}{NM}\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\left[f(x,y) - f^*(x,y) \right] \quad (4)$$

$$RMSE = \sqrt{MSE}$$

Where M, N are the matrix dimensions in x and y.

$$SNR = 10\log\left|\frac{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1} f(x,y)^2}{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\left[f(x,y)-f^{'}(x,y)\right]}\right| \quad (5)$$

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right)$$

2. Wavelet transform

The main disadvantage of DCT transform is when the coded bit rate is lower than a certain value (0.25 bits/pixel), there are blocking effects in the decoded image. Also, if the image is directly transmitted over noisy channels, blocks are lost because Huffman coding is a variable length code, which is usually the case for wideband mobile networks. The noisier the channel is, the more blocks are lost.

Recently, the Wavelet decomposition has been proved to be a better tool for image compression. Similar to sinusoids used in DCT, wavelets are used as a type of basis function to represent arbitrary functions. There are many different sorts of wavelets and the type is used can vary with the application. Individual wavelet functions are characterized by definition over a finite interval and a mean value of zero. Wavelet transform are linear and square integrable transforms with a mother waveform and daughter wavelets. The daughter wavelets are formed by scaling and dilating of mother waveform. These wavelets form a complete orthogonal set. From a mathematical perspective, wavelets have general form:

$$\psi^{(a,b)}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \quad (6)$$

The variables a is used to scale the wavelet by powers of two and b is used to translate the wavelet in integer amounts.

To analyze data at different resolutions, a scaling function $\phi(x)$ is used in conjunction with the based (mother) wavelet

$$\phi(x) = \sum_{k=0}^{M-1} C_k\phi(2x-k) \quad (7)$$

Where M is the number of non zero coefficients.

In equation (7), C_k are the wavelet coefficients which satisfy the constraints

$$\sum_{k=0}^{N-1} C_k = 2 \quad (8)$$

This allows for the defining coefficients to be varied according to the wavelet system used. Equation (7) is orthogonal to its translations and its dilations or scales i.e

$$\int \phi(x)\phi(x-k)dx = 0 \quad (9)$$

$$\int \psi(x)\psi(2x-k)dx = 0 \quad (10)$$

where $\psi(x) = \sum_k (-1)^k c_{1-k}\phi(2x-k)$ (11)

The DCT is calculated on block of pixels independently, therefore, coding error causes discontinuity between the blocks which leads to annoying blocking artifact. On the contrary, the wavelet transform operates on the entire image, (or a tile of a component in the case of large color image). It offers better energy compaction than the DCT, and no blocking artifact after coding.

A sample one scale 2D-separate wavelet transform is shown in figure 3.

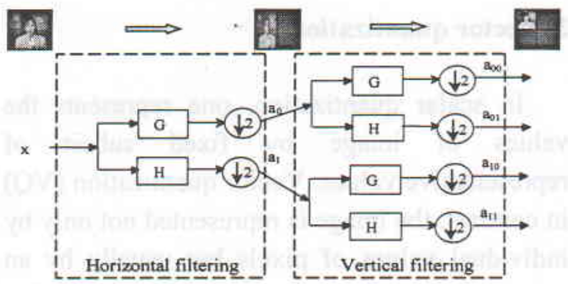

Figure 3. 2D wavelet transform.

The 2D data array of the image is first filtered in the horizontal direction, which results in two sub-bands, - a horizontal low and a horizontal high pass subband. Each sub-band then passes through a vertical wavelet filter. The image is thus decomposed into four sub-bands, - sub-band LL (low pass horizontal and vertical filter), LH (low pass vertical and high pass horizontal filter), HL (high pass vertical and low pass horizontal filter) and HH (high pass horizontal and vertical filter). Because the wavelet transform is a linear transform, we may switch the order of the horizontal and vertical wavelet filter, and still reach the same effect. A multi-scale dyadic wavelet pyramid shows in figure 4 can be obtained by further decomposing the sub-band LL with another 2D wavelet transform.

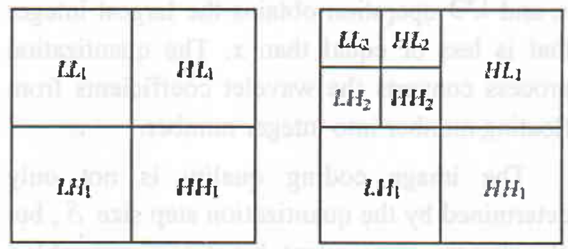

Figure 4. 2D DWT applied one (a) and twice (b).

The wavelet transform decomposes the image into an 4-level dyadic wavelet pyramid, as shown in figure 5.

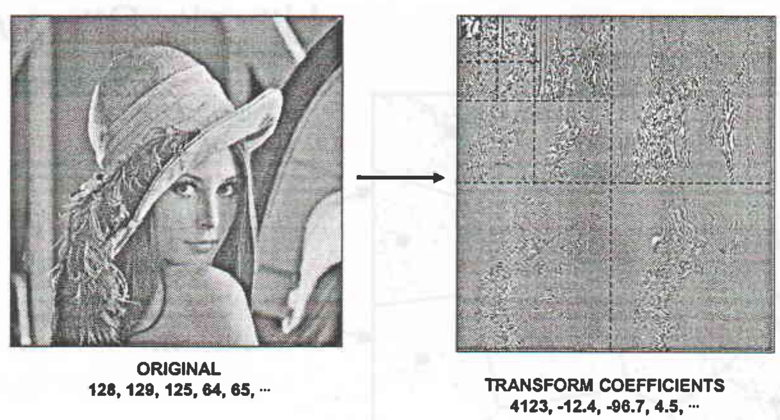

Figure 5. 2D Wavelet transform of image "Lena".

The resultant wavelet coefficients can be easily scaled in resolution: by not using the wavelet coefficients at the finest M-levels, we may reconstruct an image that is 2^M times smaller than the original one. The multi-resolution capability of the wavelet transform lends it ideally to scalable image coding.

After the wavelet transform, all wavelet coefficients are uniformly quantized by the following:

$$w_{m,n} = sign(s_{m,n})\left[\frac{\lfloor s_{m,n} \rfloor}{\delta}\right] \quad (9)$$

where $s_{m,n}$ is the transform coefficients, $w_{m,n}$ is the quantization result, δ is the quantization step size, *sign(x)* returns the sign of coefficient *x*, and $\lfloor x \rfloor$ operation obtains the largest integer that is less or equal than *x*. The quantization process converts the wavelet coefficients from floating number into integer number.

The image coding quality is not only determined by the quantization step size δ, but also by the subsequent bit-stream assembler. The entropy coder measures both the coding rate and distortion during the encoding process. The coding rate is derived directly through the length of the coding bit-stream at certain instances, e.g., at the end of each sub-bitplane. The coding distortion is obtained by measuring the distortion between the original coefficient and the reconstructed coefficient at the same instance

3. Vector quantization

In scalar quantization, one represents the values of image by fixed subset of representative values. Vector quantization (VQ) in contrast, the image is represented not only by individual values of pixels but usually by an array of them. VQ gives a good compression without the blocking effects usually found in other coding techniques.

A vector quantizer maps *k-dimensional* vectors in the space $\Re^k$ into a finite set of vectors Y = {y_i ; i=1,2,…,N}. Each vector y_i is called a *codeword* and the set of all the codewords is called a *codebook*. Associated with each codeword, y_i, is a nearest neighbor region called *Voronoi* region, it is defined by

$$V_i = \{x \in R^k : |x - y_i| \le |x - y_j|\} \ for\ all\ j \ne j \quad (10)$$

The set of Voronoi regions partition the entire space $\Re^k$ such that:

$$\bigcup_{i=1}^{N} V_i = \Re^k \qquad \bigcap_{j=1}^{N} V_j = \phi \ for\ all\ j \ne j \quad (11)$$

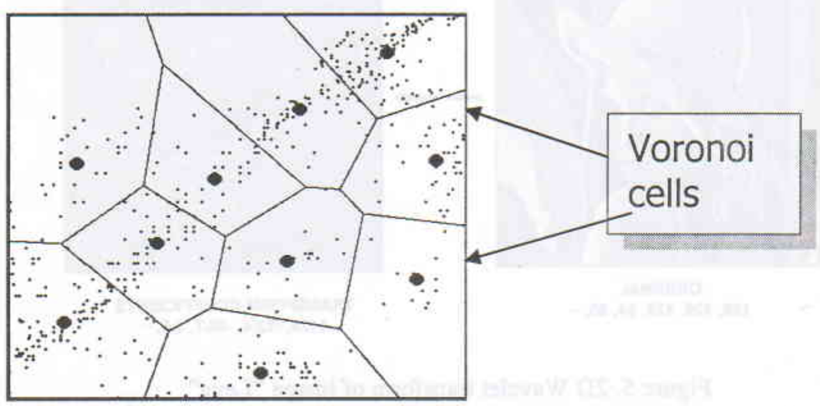

Figure 6. Vector quantization.

The representative codeword is determined to be the closest in Euclidean distance from the input vector. The Euclidean distance is defined by

$$d(x, y_i) = \sqrt{\sum_{j=1}^{k} (x_j - y_{ij})^2} \quad (12)$$

where x_j is the j th component of the input vector and y_{ij} is the j th component of the codeword y_i.

A vector quantizer is composed of two parts: encoder and decoder as in figure 7. An encoder will compare each input vector with all codevector in the codebook and generate index based on the minimum Euclidean distance between them. A decoder takesthe indexes to match the codevector in the codebook and generate the output vectors. The most popular technique in codebook design is the LBG algorithm in which the whole image is partitioned into subblocks and all these subblocks are used to train this codebook.

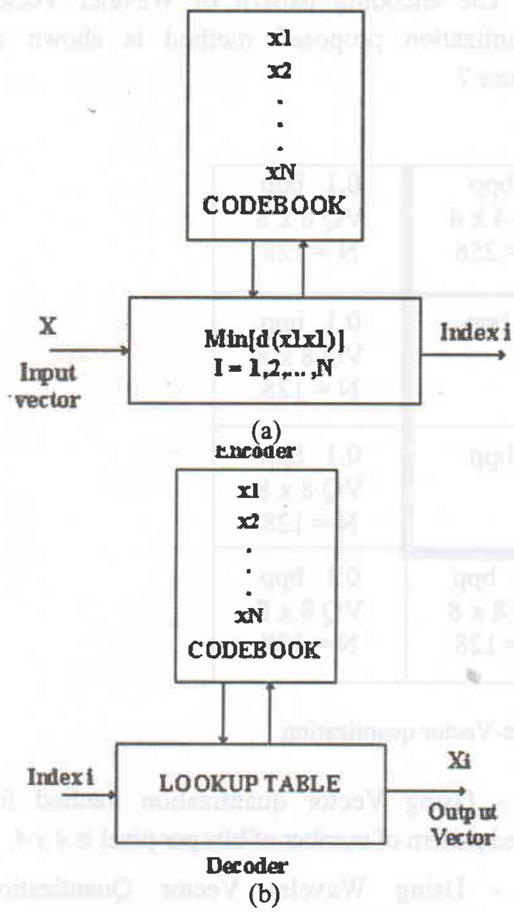

Figure 7. Vector quantizer (a) Encoder (b) Decoder.

Using vector quantization to compress image can achieve high compression ratio compare with another compression methods. However, VQ is not widely implemented because two things. Firstly, when image is divided into $N \times N$ blocks, assuming that L is quantization level, R is bit rate (bpp) then we have $R = \frac{1}{N}\log_2 L$ and total number of calculations is $N2^{NR}$. That means the complexity and number of calculations are increased exponentially with N. Secondly, it takes time to generate the codebook and the speed up of the search. In full search, an input vector is compared with every codeword in the codebook. If there were M input vectors, N codewords and each vector is in k dimensions, then the number of comparisons becomes $MN(k-1)$. This is an expensive method.

4. Wavelet - Vector quantization

Wavelet transform is a powerful tool in image analysis. For one stage (level) of wavelet transform, the image will be divided into LL, LH HL and HH sub-bands. Usually, the sub-band of high frequency image HH will be omitted in the reconstruction process. However, the high frequency sub-band image may be contained of significant a.c energy, so that the edges of the reconstructed image can not be detected and the mean square error can be introduced. In order to improve the mean square error, all subband must be calculated and classified with a proportion of bit rate. Therefore, the subband image with high a.c energy will be encoded with high bit rate while subband with low a.c energy will be encoded with low bit rate or may be discarded.

5. Proposal and Results

By using vector quantization to the sub-band images they obtained from wavelet transform, this paper will combine the advantages of Wavelet and VQ to get better ratio of compression and image quality. For a given bit rate, each sub-band image has been assigned the number of bit per pixel which is proportioned with the sub-band a.c energy. We introduce a group of 3 codebooks for encoding and decoding. The first one has 256 codewords for *2 x 2* pixels block size which provide the bit rate of 4 bpp. The second one has 256 codewords but for *4 x 4* pixel block size which provide the bit rate of 2 bpp. The third one has 128 codewords for *8 x 8* pixels block size which provide the bit rate of 0.1 bpp. This group of codebooks will be assigned to any sub-band image in order to maintain the given bit rate. The first group is applied to the sub-band image with high a.c energy (the volume of Wavelet coefficients is over 50%), the second one is applied for sub-band with average a.c energy (from 25% to 50 %) and the third one is used for sub-band image with low a.c energy (3% to 25%). For a given bit rate of compressed image, the subband image with the highest amount of the total a.c energy (LL_3) will be encoded in the scalar 8 bpp. If the remainder bit rate is great enough, one of the next subbands image (LL_2, LL_1, HL_3, LH_3) may be encoded in the first group or second group, the rest can be encoded in third group and so on.

The encoding pattern of Wavelet Vector quantization proposed method is shown in figure 7

8 bpp	4 bpp VQ 2 x 2, N = 256	2 bpp VQ 4 x 4 N = 256	0.1 bpp VQ 8 x 8 N = 128
4 bpp VQ 2 x 2 N = 256	2 bpp VQ 4 x 4 N = 256	0 bpp	0.1 bpp VQ 8 x 8 N = 128
2 bpp VQ 4 x 4 N = 256	0 bpp	0 bpp	0.1 bpp VQ 8 x 8 N = 128
0.1 bpp VQ 8 x 8 N = 128	0.1 bpp VQ 8 x 8 N = 128	0.1 bpp VQ 8 x 8 N = 128	0.1 bpp VQ 8 x 8 N = 128

Figure 7. Coding pattern of Wavelet-Vector quantization.

The results of Mean Square Error is shown in table 1 for testing of image "Lena" has size 512 x 512 pixels for the following cases:

- Using Vector quantization method for fixed pattern of number of bits per pixel is *8 x 8*.
- Using Vector quantization method for fixed pattern of number of bits per pixel is *4 x 4*.
- Using Wavelet Vector Quantization method for 3 codebooks as we are proposing.

Table 1. Mean Square Error

Image	1 VQ 8 x8	1 VQ 4 x 4	3 VQ proposed
Lena (512 x 512)	30.2715	26.1326	15.74129

6. Conclusion

The paper has proposed a solution of improvement of the image quality based on combining of Wavelet and Vector Quantization. The reproduced image can preserve the quality and edges of image in different directions. By using three different code books, the a.c energy of sub-band image can be sorted before encoding which proportional with the bit rate. The combination between Wavelet and Vector quantization for image compression will help minimizing of distortion of reproduced image.

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Nén ảnh dùng phương pháp Wavelet-lượng tử hóa vector

Nguyễn Vĩnh An

Trường Đại học Dầu Khí Việt Nam, 173 Trung Kính, Cầu Giấy, Hà Nội, Việt Nam

Phân tích Wavelet là một công cụ rất mạnh dùng trong xử lý ảnh. Sử dụng wavelets sẽ làm đơn giản hóa sự ảnh hưởng lẫn nhau của các điểm ảnh, làm cho năng lượng của ảnh tập trung vào một số ít các hệ số. Do tính chất đa phân giải và đa tốc độ, chúng cho phép xử lý một cách hiệu quả các dải băng con. Mặt khác, lượng tử hóa vector được sử dụng rộng rãi để làm tăng tỷ số nén. Nhược điểm cơ bản của nó là làm tăng thời gian tính toán do phải tìm kiếm các từ mã. Trong bài báo này, tác giả trình bày phương pháp nén ảnh dùng kết hợp wavelet và lượng tử hóa vector với các bộ từ mã khác nhau tương thích với tốc độ truyền để làm tăng hiệu quả nén và giảm sai số.