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Edge detection using Wavelets

Nguyễn Vĩnh An*

PetroVietnam University, 173 Trung Kinh, Cau Giay, Hanoi, Vietnam

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Abstract: Edge detection is one of very important issues in image processing. There are many traditional edge detectors nowadays such as Sobel, Roberts, Laplacian ect. but most of them had faced the problems with noisy images. In this paper, we will study the application of wavelet families in edge detection for noisy images both theoretically and experimentally. The comparison of wavelet and traditional edge detection techniques on images in noisy environment is also presented.

Keywords: Edge detection, wavelets, Sobel, Laplacian and Canny operators.

1. Introduction

Edge detection is used in computer vision applications for contours extraction of objects. Edges are large differences in value between neighboring pixels or we can say that edges are significant local changes of intensity in a image. The paper [1] studies the edge-detecting characteristics of the 2-D discrete wavelet transform. The algorithm for edge detection of noisy images is proposed in [2, 3]. A new edge detection algorithm based on wavelet transform and Canny operator is presented in [4]. In paper [5] selected methods of edge detection in magnetic resonance images are described, with the emphasis on the wavelet transform use. The classical edge detectors based on gradient detectors or Laplacian detectors usually fail to handle images with the blurred object outline or in the presence of strong noise. In this paper, we will discuss on detecting edges using wavelets and analysis of wavelet's edge detectors for noisy images.

2. Edge detection based on classical methods

The usual method is to use convolution of the image with complex filters like Sobel or Prewit. There are many ways to perform edge detection. They may be grouped into two categories: Gradient and Laplacian. The gradient method detects the edges by looking for the maximum and minimum in the first derivative of the image. The Laplacian method searches for zero crossings in the second derivative of the image to find edges. Gradient operation is an effective detector for sharp edges where the pixel gray levels change over space very rapidly. But when the gray levels change slowly from dark to bright, the gradient operation will produce a very wide edge. It is

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* Tel: 84-913508067.
E-mail: annv@pvu.edu.vn

helpful in this case to consider using the Laplace operation. The second order derivative of the wide edge will have a zero crossing in the middle of edge. It is important that the edges which occur in an image should not be missed and that there should be no spurious responses. There are some criteria that we should consider in term of edge detection. Firstly, the edge detecting methods needs to mark as many real edges in the image as possible. The second criterion is good localization, it means the distance between the actual and located position of the edge should be minimal. Thirdly, minimal response, a given edge in the image should only be marked once, and the noise in image should not create false edges. The third criterion is implemented because the first two criteria were not substantially enough to completely eliminate the possibility of multiple responses to an edge.

The gradient-based methods check the magnitude of image gradient. The gradient map is generated by 2D convolution. Edges are detected if the magnitude of image gradient greater than threshold. The Sobel operator, Prewitt operator, Robert's cross operator use masks of 3 × 3 to convolute with the image. The advantages of these are very simple, very fast. The main drawbacks of them are very susceptible to noise and not capable of detecting edges in different scales. In order to detect edges in noisy images, Canny edge detection uses Gaussian filtering to raise the image to noise ratio SNR and hysteresis threshold T_H and T_L for connectivity of edges. Canny method is easy implementation, fast speed and relatively robust and cost effect. However, the result can still be affected by strong noise. The edges in all scales do not be examined.

3. Wavelet transform

2D Discrete Wavelet Transform (*2D DWT*) decomposition of image can be described by (1)

$$C = X \cdot I \cdot Y \qquad (1)$$

where C is the final matrix of wavelet coefficients, I represent an original image, X is a matrix of row filters and Y is a matrix of column filters.

When processing image, wavelet perform separately for the horizontal and vertical directions. In the first level of decomposition of *2D DWT*, the image is separated into four parts. They are called approximation coefficients (*LowLow or LL*), horizontal (*LowHigh or LH*), vertical (*HighLow or HL*) and detail coefficients (*HighHigh or HH*) see in figure 1. Approximation coefficients obtained in the first level can be used for the next decomposition level.

Inverse 2D Discrete Wavelet Transform used in image reconstruction is defined by Eq. (2)

$$I = X^{-1} \cdot C \cdot Y^{-1} \qquad (2)$$

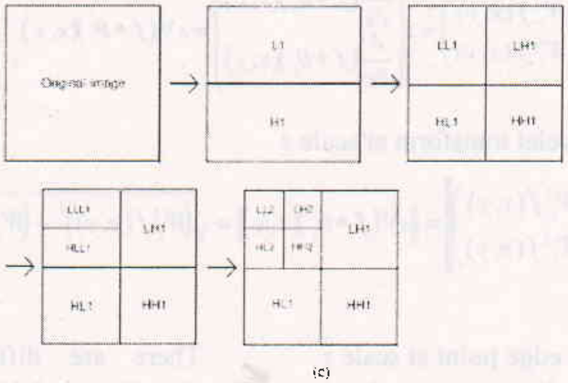

Fig.1. Two levels of 2D DWT decomposition.

Basic form of continuous wavelet transform (*CWT*)

$$W_{\psi} f(a,b) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-a}{b}\right) dt \qquad (3)$$

In which $\psi(t)$ is the mother wavelet. a is the dimension of translation and b is the dimension of dilation.

The function f belongs to $L^2(R)$, that is $\int_{-\infty}^{\infty} |f(t)|^2 \, dt < \infty$ (finite energy). The functions generated by mother wavelet should be a basis of the $L^2(R)$ space.

Let *f*(*x*) be a function in $L^2(R)$, $\theta(x)$ be a smoothing function. (impulse response of a low-pass filter) then

$$\theta_s(x) = \frac{1}{s}\theta\left(\frac{x}{s}\right) \qquad (4)$$

be the stretched version of $\theta(x)$

$$\psi^1(x) = \frac{d\theta(x)}{dx} \quad \text{and} \quad \psi^2(x) = \frac{d^2\theta(x)}{dx^2} \qquad (5)$$

$$W_s^1 f(x) = f * \left(s\frac{d\theta_s}{dx}\right)(x) = s\frac{d}{dx}(f * \theta_s)(x) \qquad (6)$$

$$W_s^2 f(x) = f * \left(s^2\frac{d^2\theta_s}{dx^2}\right)(x) = s^2\frac{d^2}{dx^2}(f * \theta_s)(x) \qquad (7)$$

We can easily generalize this to 2D signals:

$$\psi^1(x,y) = \frac{\partial\theta(x,y)}{\partial x} \quad \text{and} \quad \psi^2(x,y) = \frac{\partial\theta(x,y)}{\partial y} \qquad (8)$$

Given

$$\psi_s^1(x,y) = (1/s)^2 \psi^1(x/s, y/s)$$
$$\psi_s^2(x,y) = (1/s)^2 \psi^2(x/s, y/s) \qquad (9)$$

which are the s-dilation of $\psi^1(x,y)$ and $\psi^2(x,y)$ respectively $(s = 2^{j}, j \in Z, j \in (-\infty, \infty))$. Then, the wavelet transform defined with respect to $\psi^1(x,y)$ and $\psi^2(x,y)$ has two components:

$$W_s^1 f(x,y) = f * \psi_s^1(x,y)$$
$$\text{and} \quad W_s^2 f(x,y) = f * \psi_s^2(x,y) \qquad (10)$$

We can easily prove for equation (11) as

$$\begin{pmatrix} W_s^1 f(x,y) \\ W_s^2 f(x,y) \end{pmatrix} = s \begin{pmatrix} \frac{\partial}{\partial x}(f*\theta_s)(x,y) \\ \frac{\partial}{\partial y}(f*\theta_s)(x,y) \end{pmatrix} = s\nabla(f*\theta_s)(x,y) \qquad (11)$$

The modulus of the wavelet transform at scale s

$$M_s f(x,y) = \left\| \begin{pmatrix} W_s^1 f(x,y) \\ W_s^2 f(x,y) \end{pmatrix} \right\| = \left\| s\nabla(f*\theta_s)(x,y) \right\| = \sqrt{\left(W_s^1 f(x,y)\right)^2 + \left(W_s^2 f(x,y)\right)^2}\,, \qquad (12)$$

A point is a multi-scale edge point at scale s if the magnitude of the gradient attains a local maximum.

There are different types of wavelet families. Some of them can be listed in figure 2 (Haar wavelet), figure 3 (Daubechies wavelets), figure 4 (biorthogonal) and figure 5 (coiflets).

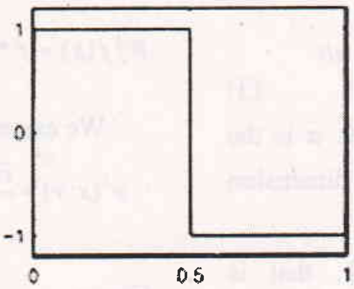

Fig. 2. Haar wavelet.

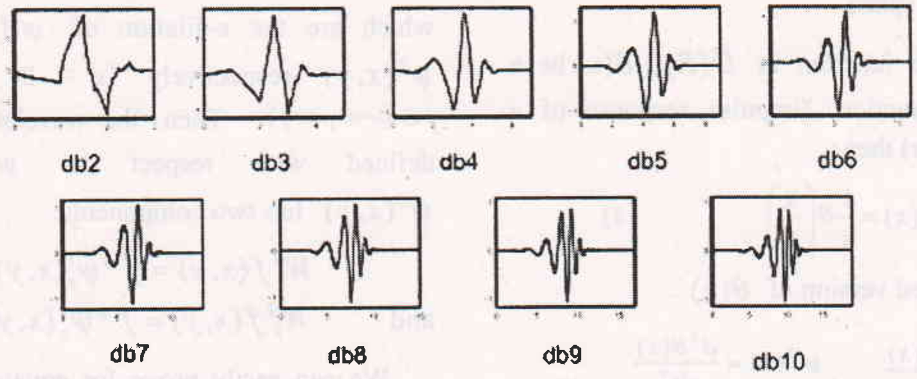

Fig. 3. Daubechies.

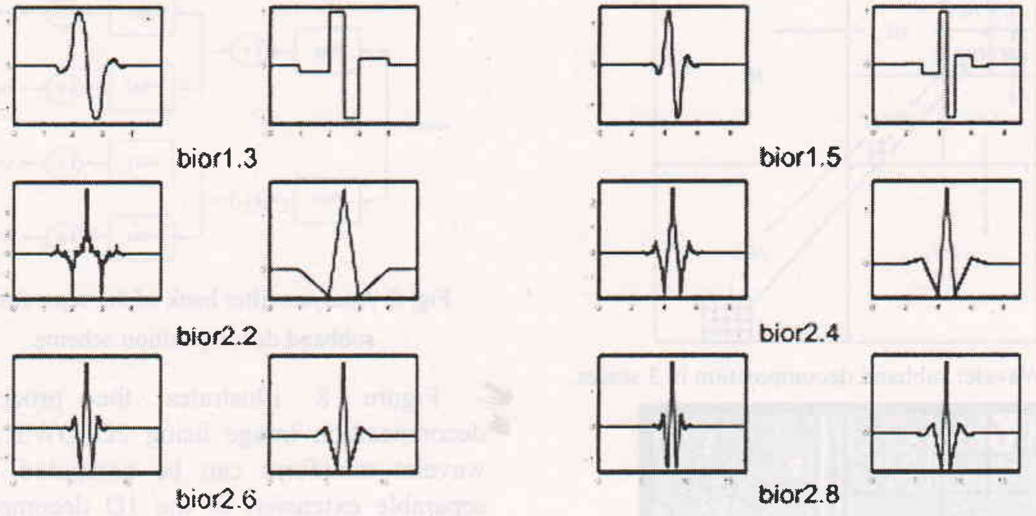

Fig. 4. Biorthogonal.

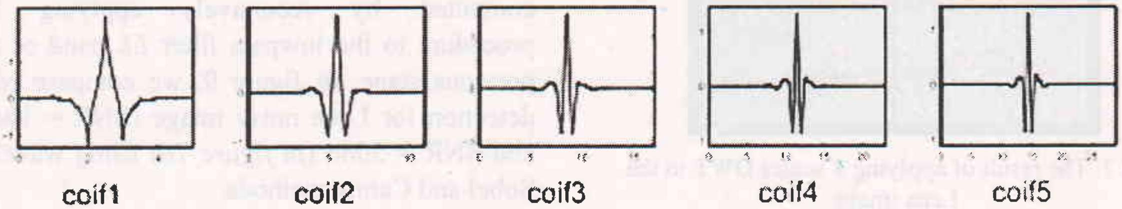

Fig. 5. Coiflets.

4. Edge detection using wavelet transform

Wavelet transform has widely applied in image processing. Wavelets are used for edge detection to eliminate the difficulties such as the inability of handling large contrast between images and the inability to handle large translations of features. By using multiple levels of wavelet's decomposition, edge detection using wavelet can be worked well with the noisy image and images of large size.

The image is used here for the wavelet transform. The wavelet decomposition is applied on image which creates different sub-bands like *LL, LH, HH* and *HL* (*figure 6, 7*). The wavelet transform basically is a convolution operation, which is equivalent to passing an image through low-pass and high-pass filters (*fig. 8*). Let the original image be *I(w, h)*, then the *LH* sub-band represents the vertical edges, *HL* sub-band represents the horizontal edges and *HH* sub-band represents the diagonal edges of *I(w, h)*. We can use these properties of the *LH, HH* and *HL* sub-bands to construct an edge image.

One essential issue in the edge detection is how to threshold to filter out the noises. If we choose the value of threshold too large, weak edges will be removed. If the value of threshold is too small, noises can not be filtered out. The choice of the wavelet thresholding function and wavelet threshold can be seen in [6].

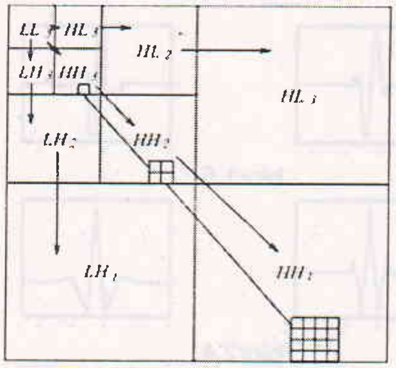

Fig. 6. Wavelet subband decomposition in 3 scales.

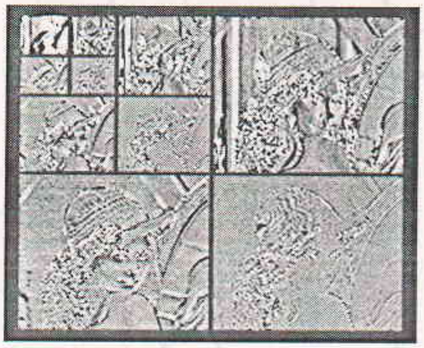

Fig. 7. The result of applying 3 scales DWT to the Lena image

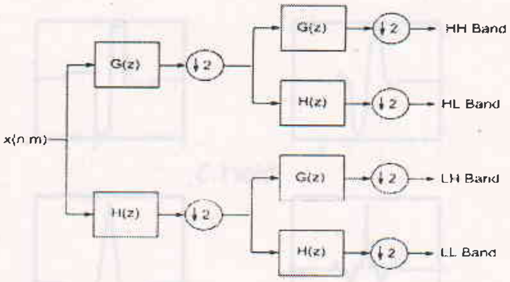

Fig. 8. Analysis filter bank of the separable 2D subband decomposition scheme.

Figure 8 illustrates the process of decomposition image using 2D DWT. A 2D wavelet transform can be computed with a separable extension of the 1D decomposition algorithm [7] as shown in figure *8*. Further stages of the 2D wavelet decomposition can be computed by recursively applying the procedure to the lowpass filter *LL* band of the previous stage. In figure 9, we compare edge detection for Lena noisy image (SNR = 10db) and SNR = 30db (*in figure 10*) using wavelet, Sobel and Canny methods.

Fig. 9. Edge detection for a Lena image with noise (a) Lena image with SNR=10db (b) Edge detection by Sobel (c) Edge detection by Canny (d) Edge detection using wavelets.

2D DWT decomposition seperates an image into four parts, each of them contains different information of the original image. Detail coefficients represent edges in the image, approximation coefficients are low frequencies and noise. The easiest way to detect edges is modification of approximation coefficients properly.

The simplest method of edge detection is replacing all approximation coefficients less than threshold by zeros. By doing this, low frequencies and the noise will be removed from image. The image is reconstructed by using the remaining wavelet coefficients.

As we mention above, edge detection also be performed by modification of approximation coefficients using Canny, Sobel, Prewitt detector.

In figure 10, we compare for different methods of edge detection. The simple detector is applied to the approximation coefficients obtained in the first level of decomposition. From the remaining coefficients and the modified coefficients the image is reconstructed. This is the simple method that provides sufficient results, especially with Canny detector use.

Fig. 10. Edge detection for a Lena image with noise (a) Lena image with SNR=30db (b) Edge detection by Sobel (c) Edge detection by Canny (d) Edge detection using wavelets.

5. Results and conclusions

In this paper, we have presented the edge detection for noisy image using wavelets. From figure 9 and figure 10 we can see that, edge detection for strong noisy image using wavelet given better result than using traditional methods. A comparison of different wavelet's detectors for noisy image can be seen in figure 11. Quantitative analyses of wavelet's edge detectors are on the graphs in figure 12 and figure 13.

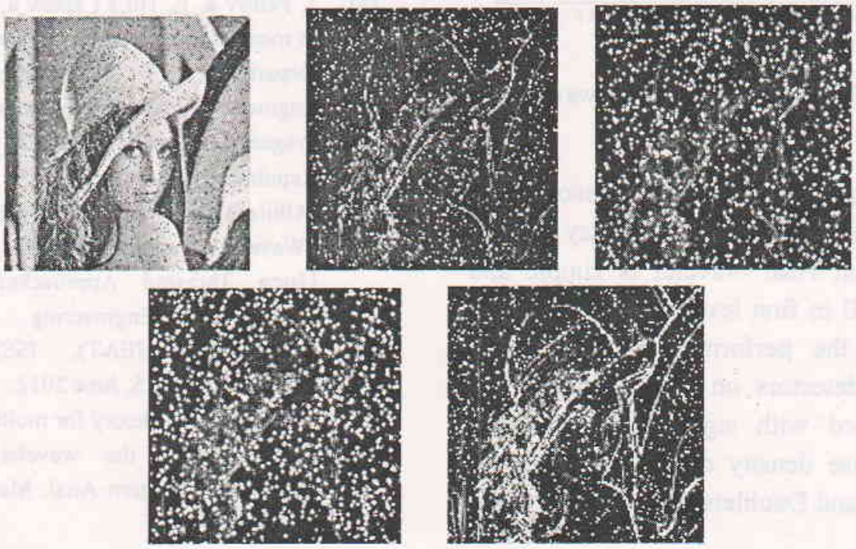

Fig. 11. Edge detection of noisy lenna image using 1st level DWT (a) Original lenna image (b) Edge detection using Haar (c) Edge detection using Db2 (d) Edge detection using coif1 (e) Edge detection using Bior1.3.

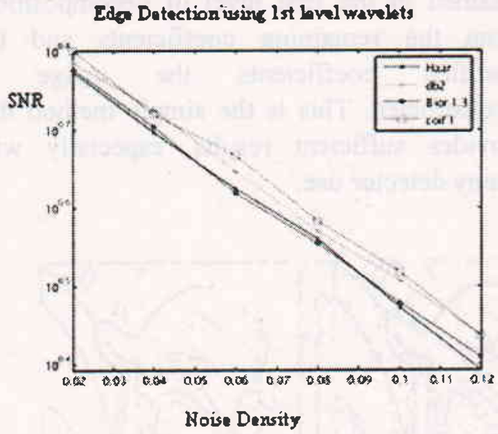

Fig. 12. Edge detection using 1st level wavelets.

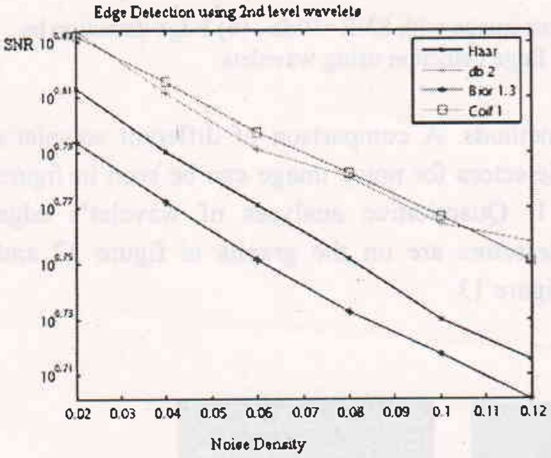

Fig.13. Edge detection using 2nd level wavelets.

From the graph of the performance of 1st level wavelet edge detectors on noisy images we can see that Haar wavelet is simple and works very well in first level. Figure 13 shows the graph of the performance of 2nd level wavelet edge detectors on noisy images. The graph is plotted with signal to noise ratio against the noise density of the salt &pepper noise. Coiflets and Daublets perform well at the 2nd level. They suppress a lot of noise and have a higher SNR compare to Haar and Bior 1.3.

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Phát hiện biên dùng Wavelets

Nguyễn Vĩnh An

Trường Đại học Dầu Khí Việt Nam, 173 Trung Kính, Cầu Giấy, Hà Nội, Việt Nam

Tóm tắt: Phát hiện biên là một trong những nội dung quan trọng của xử lý tín hiệu ảnh. Hiện nay có rất nhiều các toán tử phát hiện biên đang được sử dụng rất thông dụng như Sobel, Roberts, Laplacian... nhưng hầu hết các toán tử này đều làm việc không hiệu quả đối với các ảnh bị nhiễu mạnh. Trong bài báo này, chúng ta sẽ xem xét kỹ thuật phát hiện biên sử dụng biến đổi wavelets cho các ảnh bị nhiễu cả về góc độ lý thuyết và thực tế. Ngoài ra, bài báo cũng tiến hành so sánh hiệu quả của một số kỹ thuật phát hiện biên truyền thống và kỹ thuật phát hiện biên dùng biến đổi wavelets đối với ảnh bị tác động của nhiễu mạnh.