USING THE MATHEMATICAL MODELS TO STUDY THE MARINE ECOSYSTEM OF BINH THUAN-NINH THUAN SEA AREA AND TAM GIANG-CAU HAI LAGOON

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Abstract: *The marine ecosystem model has been established on the basis mathematical simulation for the material transformation through compounds of the system. The phosphor cycle is chosen as an example for eco-hydrodynamic model. In the cycle, phosphorus element is transformed through five compounds (Phytoplankton, Zooplankton, Detritus, Dissolved Organic and Inorganic phosphorus) by different bio-chemical processes. The eco-hydrodynamic model has been established on the basis turbulent diffuse model in combination with the material transformation model (as phosphor cycle model). The general model is analyzed into simple models, which are applied to the sea area of Binh Thuan-Ninh Thuan province and Tam Giang-Cau Hai lagoon of Thua Thien-Hue province. The obtained results are very important to study the biological and ecological characteristics of marine plankton communities, to simulate and predict the variation of living and non-living compounds, especially, to monitor the marine environment and ecosystem.*

1. Introduction

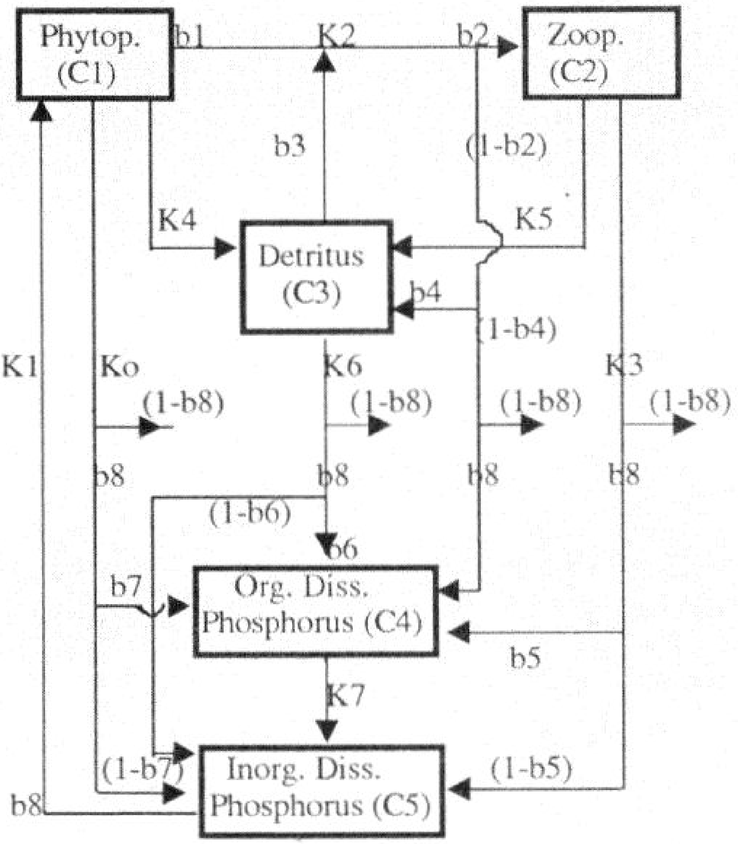
Recently, mathematical models and numerical methods have been widely used for researching the marine ecosystems. One of those is to solve the Eco-hydrodynamic problems that have been established on the basis of combination between diffusion model and the model of material transformation in the marine ecosystem. The results coming from the application of the model allow us to look totally and optically at the development tendency of the system under the influence of the marine thermo-hydrodynamic conditions. The results also allow us to determine the basic and general relationship in the marine ecosystem, to predict the variation of living and non-living compounds in the sea, especially to monitor marine environment and ecosystem.

In this paper, some kinds of marine ecological model and Eco-hydrodynamic model are introduced. The applications of these models to Binh Thuan-Ninh Thuan sea area and Tam Giang-Cau Hai lagoon in order to study characteristics and variation of compounds of marine ecosystem, especially primary productivity and in order to study the ecological effects of physical, environmental conditions show that the good results have been obtained.

2. Equations of the models

Marine Ecological Model

The marine ecological model has been established on the basis of mathematical simulation for the material transformation through the compounds of the marine ecosystem, in which the living compounds are only limited by phytoplankton and zooplankton. In this study, the phosphorus transformation cycle is chosen as an experiment for application of the Eco-hydrodynamic model. In the cycle, the phosphorus (an essential element for life) is transformed through five compounds $C_i(i = 1...5)$ by different bio-chemical processes $K_j(j = 0...7)$, fig. 1 [1]. The cycles for transformation of other elements are considered in the same way. Combining the cycles to each other, we have the general model for material transformation in the marine ecosystem.



Legend:

C_1, C_2, C_3, C_4, C_5 – Biomass of Phytoplankton, Zooplankton and concentration of Detritus, Dissolved Organic and Inorganic Phosphorus, respectively.

K_0, K_1, K_4 - Own speeds of respiration, photosynthesis and natural death of Phytoplankton population, respectively.

K_2, K_3, K_5 - Own speed of water filtering, respiration and natural death of Zooplankton population, respectively.

K_6, K_7 – Own speed of decomposition and mineralization of organic compounds, respectively.

b_1, b_3 - Coefficient of selecting Phytoplankton and Detritus as natural food by Zooplankton.

b_4, b_5, b_6, b_7 – Ratio of transformation into different material kinds.

b_8 – Ratio of Phosphorus in organic compounds.

Fig. 1: Schema of phosphorus transformation cycle in marine ecosystem

Mathematical model to simulate dynamic of material transformation in the cycles is following:

$$dC_i/dt = R_i; (i = 1...n) \tag{1}$$

where C_i is the biomass (or concentration) of i^{th} compound; R_i - the total speed of variation of biomass (or concentration) of i^{th} compound. For phosphorus transformation cycle, the mathematical model is following:

$$\begin{aligned}
dC_1/dt &= R_1 = (K_1 - K_0 - K_4 - b_1K_2C_2)C_1 \\
dC_2/dt &= R_2[(b_1C_1 + b_3C_3)b_2K_2 - K_3 - K_5]C_2 \\
dC_3/dt &= R_3 = K_4C_1 + K_5C_2 + (b_1C_1 + b_3C_3)(1 - b_2)b_4K_2C_2 - b_3K_2C_2C_3 - K_6C_3 \\
dC_4/dt &= R_4 = (b_1C_1 + b_3C_3)(1 - b_2)(1 - b_4)b_8K_2C_2 + b_5b_8K_3C_2 + b_6b_8K_6C_3 + \\
&\quad + b_7b_8K_0C_1 - K_7C_4 \\
dC_5/dt &= R_5 = K_7C_4 + (1 - b_5)b_8K_3C_2 + (1 - b_6)b_8K_6C_3 + (1 - b_7)b_8K_0C_1 - b_8K_1C_1
\end{aligned} \tag{2}$$

The speeds of K_j depends on environmental conditions, they can be determined by using some empirical formulae, as follows [4, 5, 6]:

$$\begin{aligned}
K_0(1/day) &= P_0Exp[Q_0(T - 20) - U_0.Ln(MP)] \\
K_1(1/day) &= K_{1\max}.\min[A, B, C, D] \\
K_2(m^3/mg.day) &= \begin{cases} 0 & if \quad S = 0 \\ K_{2\max}.S(2 - S/S_1)/S_1 & if \quad 0 < S < S_1 \\ K_{2\max} & if \quad S_1 \le S \le S_2 \\ K_{2\max}/[1 + P_2(S - S_2)] & if \quad S > S_2 \end{cases} \\
K_3(1/day) &= P_3.Exp[Q_3(T - 20) - U_3.Ln(MZ)] \\
K_{4,5}(1/day) &= P_{4,5}.exp(Q_{4,5}.T - U_{4,5}.T) \\
K_{6,7}(1/day) &= P_{6,7}Exp(Q_{6,7}T)
\end{aligned}$$

where:

$$\begin{aligned}
&K_{1\max}(1/day) = Exp(Q_1T - U_1) \\
&A = \frac{Q}{2,72.Q*}Exp\left(1 - \frac{Q}{2,72Q*}\right); B = \frac{PO_4}{P* + PO_4}; C = \frac{N}{N* + N}; D = \frac{SiO_3}{Si* + SiO_3} \\
&N = NH_4 + NO_2 + NO_3 \\
&S = C_1 + C_3
\end{aligned}$$

$$K_{2\max} = \begin{cases} X.Exp[0,08(T_{\min} - T)] & if \quad T < T_{\min} \\ X & if \quad T_{\min} \le T \le T_{\max} \\ X.Exp[0,08(T - T_{\max})] - Q_2(T - T_{\max}) & if \quad T > T_{\max} \end{cases}$$

$$X = 0,000682Exp[-U_2.Ln(MZ)]$$

In the empirical formulae: $T(^oC)$ is the environmental temperature; $T_{\min}, T_{\max}(^oC)$ - the minimal and maximal temperature in the optimal temperature interval for growth of zooplankton population, respectively; $Q, Q*$ (cal/cm^2/min) - energy of photosynthetically active radiation and half-saturation coefficient of light intensity, respectively; e -

base of natural logarithm; $PO_4, NH_4, NO_2, NO_3, SiO_3 (mgP, N, Si/m^3)$ - concentration of phosphate, ammonium, nitrite, nitrate and silicon, respectively (here, PO_4 is compound $C5$); $P*, N*, Si*$ $(mgP, N, Si/m^3)$- half-saturation coefficient for the salts of phosphor, nitrogen and silicon, respectively; S_1, S_2 (mg) - optimal interval of food concentration of zooplankton; M_P, M_Z - the mean size of phytoplankton cell and zooplankton individual, respectively; $P_K, Q_K, U_K (k = 0...7)$ - experimental coefficients.

Eco-hydrodynamic model

By adding the functions R_i of system (2) into the equations of diffusion model, the general model to simulate for time and space variation of compounds under the influence of thermo-hydrodynamic processes can be obtained, as follows:

$$\begin{aligned} &\frac{\partial C_i}{\partial t} + U\frac{\partial C_i}{\partial x} + V\frac{\partial C_i}{\partial y} + (W + W_i)\frac{\partial C_i}{\partial z} = \\ &= \frac{\partial}{\partial x}\left(A_{Li}\frac{\partial C_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_{Li}\frac{\partial C_i}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_{Zi}\frac{\partial C_i}{\partial z}\right) + R_i \end{aligned} \tag{3}$$

where U, V, W are the components of velocity of current in the x, y, z directions, respectively; W_i - the sedimentation velocity of i^{th} compound ($W_i = 0$ for dissolved compounds); A_{Li}, A_{Zi} - the horizontal and vertical turbulent diffusion coefficients of i^{th} compound. The boundary and initial conditions are chosen suitable with the normal diffusion model.

The input values are velocity and direction of current, diffusion coefficients, water temperature, intensity of photosynthetically active radiation at the water surface, concentration of $PO_4, NH_4, NO_2, NO_3, SiO_3$... depending on the objective of study and the source data, the model can be simplified for some certain cases of study.

3. Some study results

Dynamic model for material transformation in the phosphorus cycle

The model (2) with initial conditions $C_i = C_i^0$ at $t = t_0$ is solved by Runge-Kutta method. The solutions are the time variations of biomass (or concentration) of the compounds in the phosphorus cycle. The biological productivity of plankton community and the ecological effects of autotrophic and first heterotrophic hierarchy are also calculated from the model.

The mentioned model is named PMOD and has been applied to the sea area of Binh Thuan-Ninh Thuan province in order to study the annual variations of compounds of phosphorus cycle [1]. The results are depicted in figure 2.

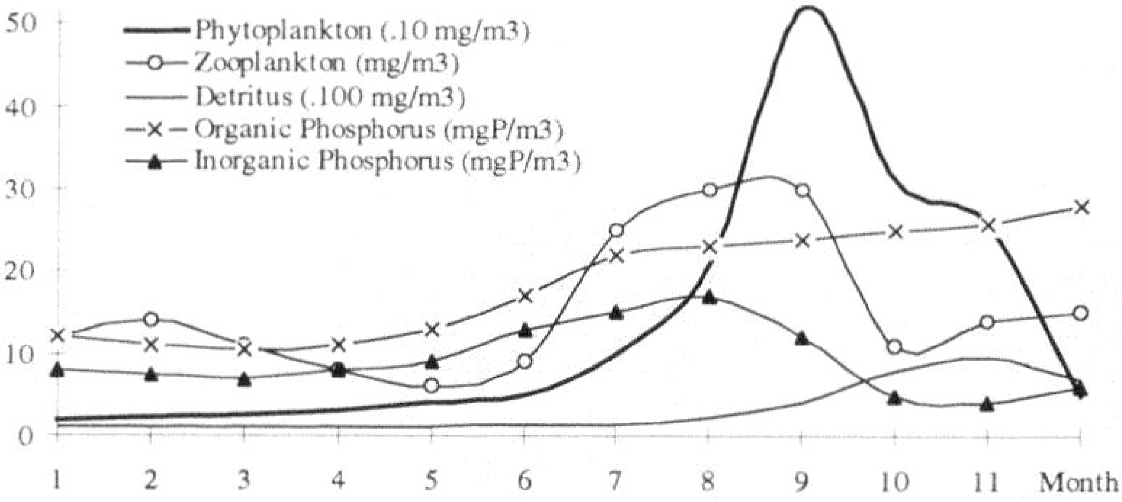


Fig. 2: The annual variation of compounds of phosphorus cycle at the sea area of Binh Thuan-Ninh Thuan province (results estimated by PMOD)

The Model of competition in marine plankton communities Separating first two equations of the system (2), we have model of competition in marine plankton communities (the model is called PLAMOD). The model is the same as the model of "prey-predator" relationship of Voltera. Where, zooplankton is considered as "predator" and phytoplankton as "prey". In the model, the concentration of detritus (C_3) is determined through the value of BOD or COD. The PLAMOD has been applied to Tam Giang-Cau Hai lagoon, Thua Thien-Hue province in order to study the daily variations of biomass and biological productivity of plankton community [3]. The input values of the PLAMOD are the environmental data at 24-hours continuous stations. The results are depicted in figure 3.

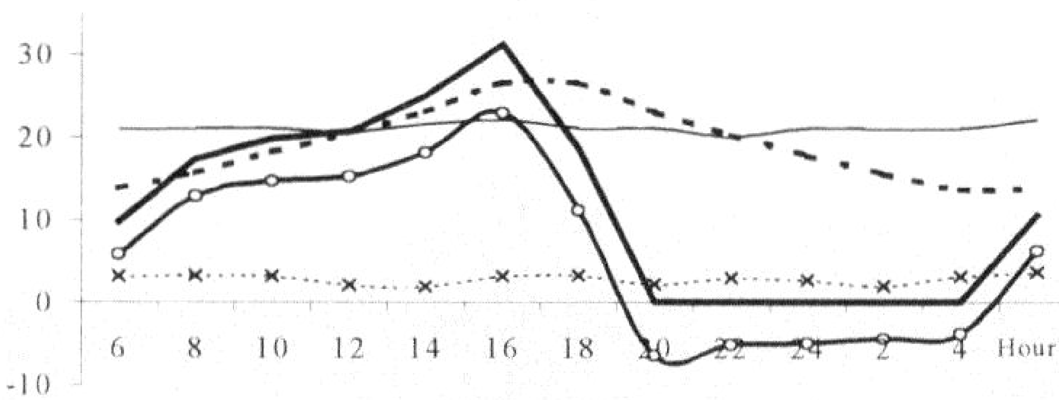


Fig. 3: Daily variation of biomass and primary productivity of phytoplankton at Thuan An Stations (Tam Giang-Cau Hai lagoon) in October 1995 (results estimated by PLAMOD)

Two-dimensions model for distribution of the compounds in phosphorus cycle

Integrate equation (3) in the vertical direction from free surface to depth D and consider the steady state, diffusion coefficient is constant, to have the two-dimension steady model. This model is named P2DMOD, as follows:

$$U\frac{\partial C_i}{\partial x} + V\frac{\partial C_i}{\partial y} = A_{Li}\left(\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2}\right) + \frac{(W + W_i)C_i}{D} + R_i \tag{4}$$

The boundary conditions used in this model are: $aC_i + b(\partial C_i/\partial L) = aC_i^*$, with $a = 1$ and $b = 0$ if at the solid boundary, $a = 0$ and $b = 1$ if at the open boundary, L is the unit vector normal to the open boundary, C_i^* is the given value of C_i at the solid boundary. In the case of having no values C_i^* we can use the condition $\partial C_i/\partial n = 0$ (n is the unit vector normal to the solid boundary).

The P2DMOD can be reduced to solve the competition problem in the plankton community. In this case, system (4) includes 2 equations, one for phytoplankton, another for zooplankton, R_1 and R_2 are first two equation from system (2). The model has also been applied for Binh Thuan-Ninh Thuan sea area in order to estimate ecological role of upwelling in the primary production process [2]. The input data got from investigation in upwelling sea area in 8/1992. Figure 4 shows the results.

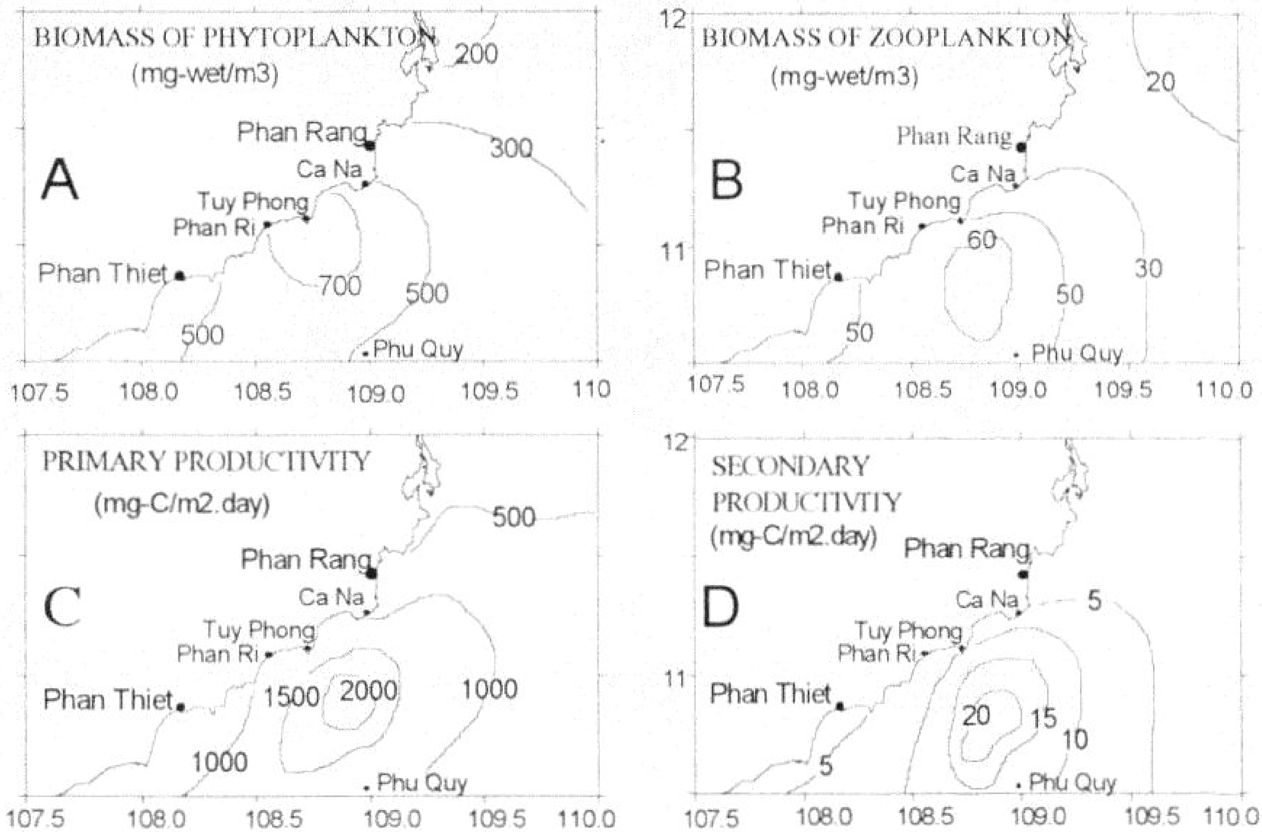


Fig. 4: Distribution of biomass of phytoplankton (A), of zooplankton (B) in the surface layer and total of primary productivity (C), of secondary productivity of zooplankton (D) in $1m^2$ water column of the photosynthetically layer in Binh Thuan-Ninh Thuan sea area in Aug. 1992 (results estimated by P2DMOD)

One-dimension model for vertical distribution of the compounds in cycle

The governing equations:

$$(W + W_i)\frac{\partial C_i}{\partial z} = \frac{\partial}{\partial z}\left(A_{Zi}\frac{\partial C_i}{\partial z}\right) + R_i \tag{5}$$

Boundary condition:

- At the free surface $(z = 0), C_i = C_{i0}$ (C_{i0} is the given values). If there are no given values C_{i0}, the balance condition is applied: $A_{zi}(\partial C_i/\partial z) - W_i C_i = 0$.

- At the level $z = D$ (D is under boundary of photosynthesis layer): if there are the given values C_{iD}, the equation will be taken as $C_i = C_{iD}$; if there are not, we assume that the vertical gradients of compounds are zero $(\partial C_i/\partial z = 0)$. If D is bottom in the deep-water area, where the light can not reach, the biomass can be taken to be zero $(C_1 = C_2 = 0)$.

This model is called P1DMOD. The results of application of this model for Binh Thuan-Ninh Thuan sea are shown in figure 5.

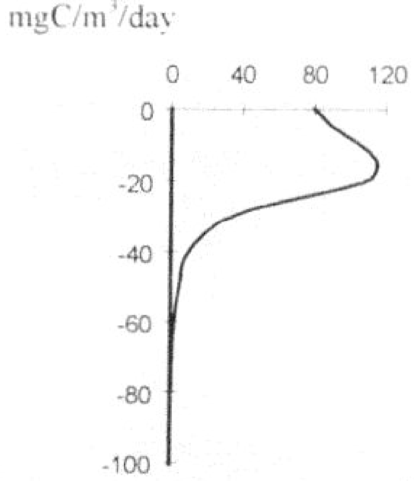


Fig. 5: Vertical profile of primary productivity of phytoplankton at Binh Thuan-Ninh Thuan sea area in Aug. 1992 results estimated by P1DMOD)

4. Conclusion

The Eco-hydrodynamic model has been established on the basis of turbulent diffusion model in combination with the phosphorus transformation model. Four simplified forms of the model were also given, as follows: the model for material transformation in the phosphorus cycle (PMOD), the model of competition in marine plankton communities (PLAMOD), the two-dimensions model (P2DMOD) and one-dimension model (P1DMOD) for distribution of the compounds in phosphorus cycle.

The models were applied to the sea area of Binh Thuan-Ninh Thuan province and Tam Giang-Cau Hai lagoon of Thua Thien-Hue province. The received results are very important for studying the biological and ecological characteristics of marine plankton communities and also for simulating and predicting the variation of living and non-living compounds, especially, for monitoring the marine environment and ecosystem.

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TẠP CHÍ KHOA HỌC ĐHQGHN, KHTN, t.XVII, $n^0$2 - 2001

SỬ DỤNG MÔ HÌNH TOÁN TRONG NGHIÊN CỨU HỆ SINH THÁI VÙNG BIỂN BÌNH THUẬN-NINH THUẬN VÀ ĐẦM PHÁ TAM GIANG-CẦU HAI

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Mô hình sinh thái biển được xây dựng trên cơ sở mô phỏng toán học quá trình chuyển hoá vật chất qua các hợp phần của hệ. Ở đây, chu trình chuyển hoá phosphor đã được xây dựng làm thí nghiệm cho việc triển khai bài toán sinh thái-thuỷ động lực. Trong chu trình, nguyên tố phosphor được chuyển hoá qua 5 hợp phần (thực vật nổi,

động vật nổi, chất vẩn, phosphor hữu cơ và vô cơ hoà tan) nhờ các quá trình sinh hoá khác nhau. Mô hình sinh thái thuỷ động lực được xây dựng trên cơ sở kết hợp mô hình chu trình chuyển hoá vật chất (ở đây là chu trình phosphor) với mô hình khuếch tán rối, nó cho phép đánh giá ảnh hưởng của các điều kiện vật lý, môi trường tới chiều hướng phát triển của hệ sinh thái vùng biển nghiên cứu.

Mô hình chung được phân tích thành các mô hình giản đơn và các mô hình giản đơn này đã được triển khai tại vùng biển Bình Thuận-Ninh Thuận và vùng đầm phá Tam Giang-Cầu Hai (mô hình động lực sự chuyển hoá vật chất trong chu trình phosphor, mô hình cạnh tranh trong quần xã plankton biển, mô hình 2 chiều ngang, 1 chiều thẳng đứng phân bố các hợp phần của chu trình phosphor và năng suất sinh học plankton). Các kết quả nhận được rất có ý nghĩa trong việc nghiên cứu định lượng các đặc trưng sinh học, sinh thái học quần xã plankton biển, tính toán và dự báo biến động các hợp phần vô sinh, hữu sinh, đặc biệt trong việc kiểm soát môi trường và hệ sinh thái biển. Những kết quả này cũng mở ra triển vọng sử dụng các mô hình toán và các phương pháp số trong nghiên cứu hệ sinh thái vùng biển nhiệt đới Việt Nam.