A MODEL FOR WAVE PROPAGATION IN THE NEAR-SHORE AREA

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Abstract: *A numerical model based on the Mild-Slope Equation for simulating the wave of wave propagation in shallow water and wave energy dissipation due to wave breaking was developed. Some computational experiments were carried out for the verification of the model in the case of theoretical condition as well as of experimental condition. The good agreements during verification stage had been obtained. An example of computation for 2-D case also was given.*
Keywords: *Mild Slope Equation, Energy Dissipation, Model Verification.*

1. Introduction

In the near-shore area, the actions of wave and currents are the main causes driving the transportation of sediment and the erosion, accretion of the seashore. So an accurate prediction of waves, currents and their interaction in this area is very important not only for the requirements of design and construction but also for protection of shore and coastal structures.

The mild slope equation derived by Berkhoff (1972) has been widely used in the numerical computation of diffraction and refraction of regular waves. In the past, many solutions of the elliptic problem for open coastal zone have been obtained by using a parabolic approximation, which treats the forward-propagating portion of the wave field only. With the approximation, the reflection parts of waves are no longer considered. Thus, the applicability of the parabolic approximations is limited to the regions without complicated structural boundaries. In addition, the sea waves are random and the randomness of sea waves has a significant effect on the wave transformation especially due to refraction and diffraction. In 1992, James T. Kirby derived a Time-Dependent Mild Slope Equation applying for unsteady wave trains; however, the energy dissipation due to wave breaking was not included in the equation.

The purpose of this study is to develop a numerical model for calculating the time evolution of random waves in the near-shore area based on the combination of the time-dependent Mild Slope Equation [4] and the wave breaking model derived by Isobe (1987, 1994). Some computational experiments in theoretical condition and experimental condition were carried out for verification of the model. Comparisons between the computed results and experimental data showed that the good agreement was reached. An example of computation in a 2-D domain with a breakwater was also realized. Discussion in detail will be shown in the followings.

2. Model Formulation

Governing equations of the model

The Time-Dependent Mild Slope Equations derived by Kirby (1992) are

$$\frac{\partial \eta}{\partial t} = -\nabla_h \left(\frac{CC_g}{g} \nabla_h \tilde{\Phi} \right) + \frac{\omega^2 - k^2 CC_g}{g} \tilde{\Phi} \tag{1}$$

$$\frac{\partial \tilde{\Phi}}{\partial t} = -g\eta, \tag{2}$$

where, η is the water surface displacement; $\tilde{\Phi}$ is the velocity potential at the surface; C and C_g are the phase velocity and group velocity respectively; k is the wave number; g is the gravitational acceleration; ω is the wave frequency; t is the time and ∇_h is the horizontal gradient operator.

The equation (1) and (2) can be combined to become

$$\frac{\partial^2 \tilde{\Phi}}{\partial t^2} = \nabla_h (CC_g \nabla_h \tilde{\Phi}) - (\omega^2 - k^2 CC_g) \tilde{\Phi}.$$

This equation can exactly be reduced to the mild slope equation derived by Berkhoff [1] by taking the time derivations equal to zero.

Because of wave breaking, a part of wave energy is dissipated when the waves propagating over the surfzone. In order to account for this energy dissipation, the equation (1) need adding a dissipation term, then we have

$$\frac{\partial \eta}{\partial t} = -\nabla_h \left(\frac{CC_g}{g} \nabla_h \tilde{\Phi} \right) + \frac{\omega^2 - k^2 CC_g}{g} \tilde{\Phi} - f_d \eta, \tag{3}$$

where f_d is the energy dissipation coefficient, which can be determined according to Isobe's wave breaking model [3].

According to Isobe, the energy dissipation due to wave breaking is modeled as follows: there are critical values γ_b and $\gamma = |\tilde{\eta}|/d$ that if γ is greater than γ_b, the individual wave is judged to be breaking. After breaking, if γ become smaller than $\gamma_r = 0.135$, the individual wave is judged to have recovered. Where $|\tilde{\eta}|$ is the amplitude at the wave crest; d is the water depth; γ_b is expressed as equation (4)

$$\gamma_b = 0.8 \left[0.53 - 0.3 \exp(-3\sqrt{d/L_0}) + 5(\tan\beta)^{1.5} \exp[-45(\sqrt{d/L_0} - 0.1)^2] \right], \tag{4}$$

where L_0 is the representative wave length in deep water; $\tan\beta$ is the bottom slope.

To evaluate the spatial distribution of the energy dissipation coefficient f_d, we first determine $f_{d\max}$ at each crest of breaking waves by using equation (5), then obtain the energy dissipation coefficient f_d by interpolating $f_{d\max}$ linearly [5].

$$f_{d\max} = 2.5\tan\beta\sqrt{\frac{1}{k_0 d}}\sqrt{\frac{\gamma - \gamma_s}{\gamma_s - \gamma_r}}, \tag{5}$$

where $\gamma_s = 0.4(0.57 + 5.3\tan\beta)$

Boundary condition

Solid boundary: At this boundary, two kinds of boundaries are employed.

The fully reflective boundary condition:

$$\frac{\partial\tilde{\Phi}}{\partial n} = 0. \tag{6}$$

The arbitrary reflective boundary with the reflection coefficient K_r:

$$\frac{\partial\eta}{\partial n} = \frac{1 - K_r}{1 + K_r}\frac{k}{\omega}\frac{\partial\eta}{\partial t}. \tag{7}$$

Open boundary: In order to allow the reflected waves from the computation domain to go freely through the boundary, the radiation boundary condition is applied for outgoing waves:

$$\frac{\partial\eta_{out}}{\partial t} + C\frac{\partial\eta_{out}}{\partial n} = 0, \tag{8}$$

where n is in the direction normal to the boundary; C is phase velocity.

Incident- wave boundary: Incident waves arriving at this boundary can be expressed in two forms: For a harmonic wave:

$$\eta = 0.5H\cos(k.x - \omega t). \tag{9}$$

For random waves:

$$\eta(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\cos(k_m x\cos\alpha_n + k_m y\sin\alpha_n - 2\pi f_m t + \xi_{mn}), \tag{10}$$

where a_{mn} and ξ_{mn} respectively are the amplitude and phase of a representative component wave for the range of frequency $[f_m, f_m + \Delta f_m]$ and for the range of direction $[\alpha_n, \alpha_n + \Delta\alpha_n]$. ξ_{mn} is random; a_{mn} can be determined by using the frequency specteral density function proposed by Bretshneider (1968) and Mitsuyasu (1990) (for more detail, see Horikawa, 1988).

Initial condition

At the initial time, the water are assumed to be still so all of the values of η and $\tilde{\Phi}$ in side the computation domain are set to be equal to zero, except the values of those at the offshore boundary are taken not equal to zero but determined by using the equation of boundary condition.

3. Application and Verification of the model

In order to verify the model, three experiments of computation have been done:

The first experiment: Assume a harmonic wave arriving normal to the open boundary at one-end of a wave flume, a vertical wall closes the other end of the wave flume. We compute time evolution and distribution of wave heights in the wave flume. If the model well simulates the wave propagation and the boundary conditions applied are good, we will obtain a distribution of standing waves in the wave flume and a stability of wave amplitude at each point on the water surface of the wave flume. Figure 1. depicts the wave flume and shows the computation conditions.

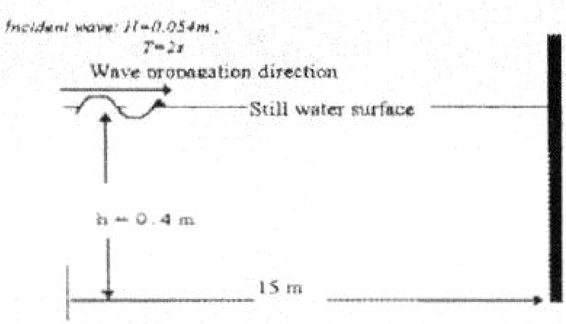

Figure 1.

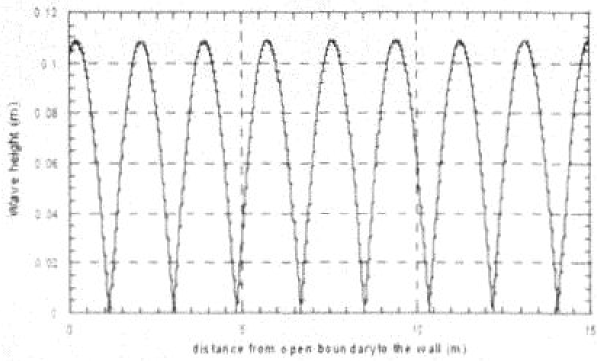

Figure 2. Distribution of computed wave height

On the Fig. 2, it is clear that the distribution of wave heights in the wave flume is a standing wave distribution with a system of Nodes and Anti-nodes, which is resulted from the combination between incident waves and reflected waves from the wall.

Fig. 3 shows the time evolution of water surface elevation at a point 7.5m far from the open boundary. After about 18 seconds from the beginning, the amplitude of water surface elevation changed to be nearly equal to 2 times of the incident wave amplitude. That means reflected waves from the wall reached the point and combined with the incident waves. It is clear that after the change, the amplitude of water surface elevation nearly remained the new value for all time. This proved that the reflected waves were not be reflected at the open boundary but going through freely. This also means that applying the open boundary condition is reasonable.

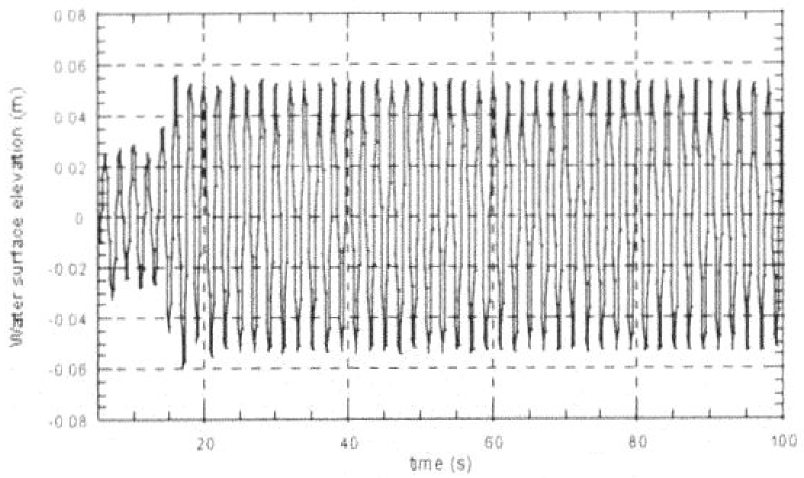

Figure 3. Time variation of water surface elevation at 7.5m far from the open boundary

Second experiment: In order to verify the model against experiment, we compute the propagation of random waves in a wave flume. The computational conditions depicted on Fig. 4 are the same as those of the experiment done by Watanabe et al [7], in which, the peak frequency f_p and significant wave height $H_{1/3}$ of incident wave train are 0.5 Hz and 5.4 cm, respectively.

Fig. 5 shows the comparison between the results of this computation and the experimental data. It is clear that computed significant wave height distribution agrees satisfactorily with experimental data. A small difference between computed and observed data may be due to the nonlinear nature of wave propagation on shallow water.

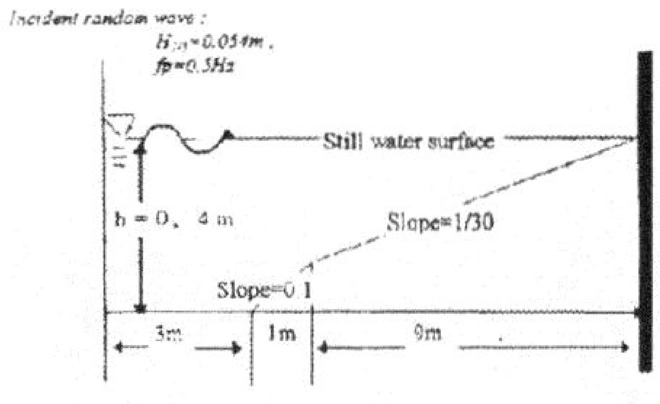

Figure 4. Sketch of the experiment for random waves

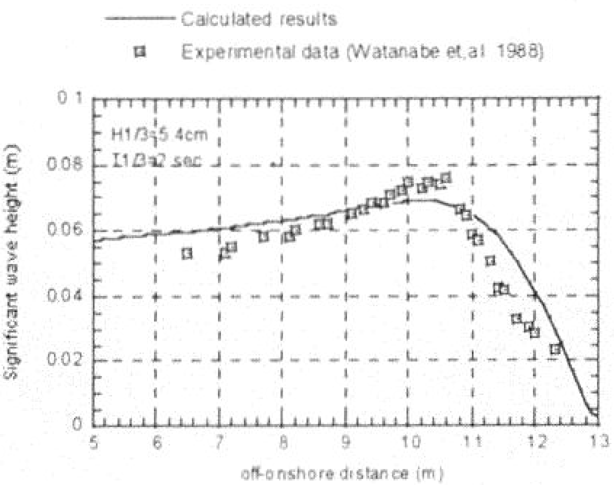

Figure 5. Comparison between computed results and experimental data

Third experiment: This experiment is an example of computation for random waves propagating on a shallow area, which has a breakwater inside. Bottom slope tan β is uniform and equal to 0.02. The incident wave train coming in the direction normal to shoreline has a significant wave height $H_{1/3} = 1.0m$ and a significant period $T_{1/3} = 6$

sec. Figure 6. shows the distribution of computed significant wave heights around the breakwater.

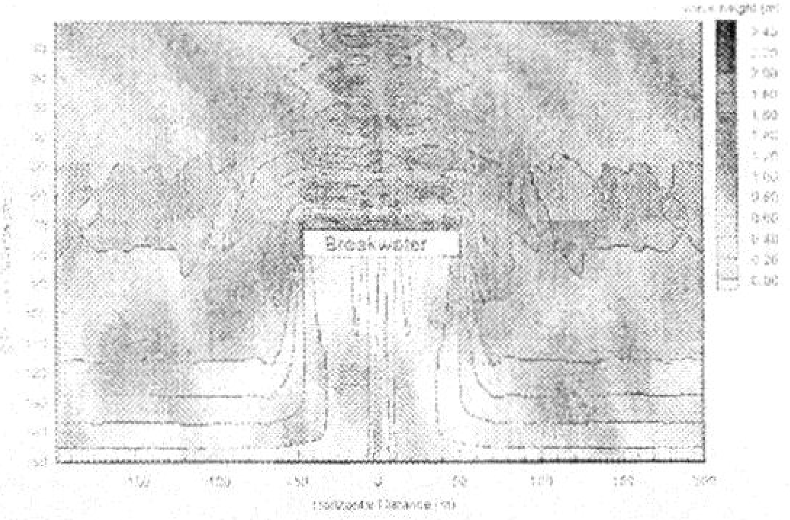

Figure 6. Distribution of computed significant wave heights

4. Conclusion and recommendation

A numerical model for wave propagation in the near-shore area including the simulation of energy dissipation due to wave breaking has been built. The preliminary verifications of the model with theoretical and experimental conditions showed that the model has well simulated the propagation of waves in the near-shore area. Because of the lack of measured data in the field of two-dimension, the verification of the model against measured data could not be held here. The model should be developed for practicalities.

5. References

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MÔ HÌNH TRUYỀN SÓNG TRONG VÙNG VEN BỜ

Phùng Đăng Hiếu
Khoa Khí tượng Thuỷ văn & Hải dương học
Đại học Khoa học Tự nhiên - ĐHQG Hà Nội

Trong vùng ven bờ, sự tác động của sóng và dòng chảy là nguyên nhân chủ yếu chế ngự quá trình vận chuyển trầm tích, nó điều khiển quá trình bồi, xói của vùng bờ. Vì vậy, tính toán chính xác trường sóng và dòng chảy phân bố trong vùng ven bờ là một vấn đề hết sức quan trọng phục vụ cho thiết kế, xây dựng cũng như bảo vệ bờ biển và đảm bảo an toàn giao thông hàng hải. Trong bài viết này, chúng tôi đã phát triển một mô hình toán mô phỏng truyền sóng trong vùng nước nông có tính đến tiêu tán năng lượng do sóng đổ gây ra và giải nó trên máy tính bằng phương pháp sai phân hữu hạn. Việc áp dụng mô hình tính toán theo các điều kiện lý thuyết và thực nghiệm đã được thực hiện nhằm kiểm chứng mô hình. So sánh kết quảtính toán số liệu thí nghiệm với kết quả lý thuyết đã cho thấy có sự phù hợp rất tốt; mô hình đã mô phỏng được quá trình truyền sóng trong vùng biển nông. Mô hình cũng được áp dụng tính toán cho trường hợp phân bố trường sóng trong vùng biển ven bờ xung quanh một đê chắn sóng. Một số kết luận và kiến nghị cho việc hoàn thiện mô hình cũng được trình bày trong bài này.