PAR REDUCTION IN OFDM USING TONE RESERVATION

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Abstract. Orthogonal Frequency Division Multiplexing (OFDM) systems support high data rate wireless transmission, offer excellent immunity against fading and intersymbol interference. The major drawback of OFDM is large Peak to Average power Ratio (PAR) of the transmit signals. As consequence, the power amplifier has to be highly linear. Most power amplifiers are non-linear and produce spectral distortion in the adjacent channel of the OFDM-spectrum. There are several methods for reducing the PAR in OFDM systems. This paper investigates Tone Reservation (TR) method of J.Tellado.

1. Introduction

OFDM is an attractive modulation scheme because it simplifies the equalization necessary to counteract frequency-dependent distortion [2]. OFDM is the standard modulation scheme for Asymmetric Digital Subscriber Line (ADSL), Digital Audio Broadcasting (DAB), Terrestrial Digital Video Broadcasting (DVB), Wireless Local Area Network i.e. ETSI-BRAN Hiperlan/2, IEEE 802.11a and Multimedia Mobile Access Communication Systems (MMAC)... However, OFDM has the significant drawback of having a high peak-to-average power ratio (PAR). This requires the power amplifier is very linear over a large dynamic range. Any non-linearity results in intermodulation and out-of-band power that will interfere with adjacent channels. There is a lot of PAR reduction method. This paper shows PAR reduction method using tone reservation [1] [3].

2. Background on OFDM

OFDM is a multicarrier system. This means that the available bandwidth is divided into many narrow bands. The data is transmitted as a large number of lower bit rate streams on these bands. In OFDM these band can be overlapped, but distance between them is chosen so that different bands are orthogonal. Implementation of OFDM can be archived using Fast Fourier Transform (FFT). The FFT is an efficient algorithm for calculating the discrete Fourier Transform (DFT). Figure 1 shows the block diagram of an OFDM system.

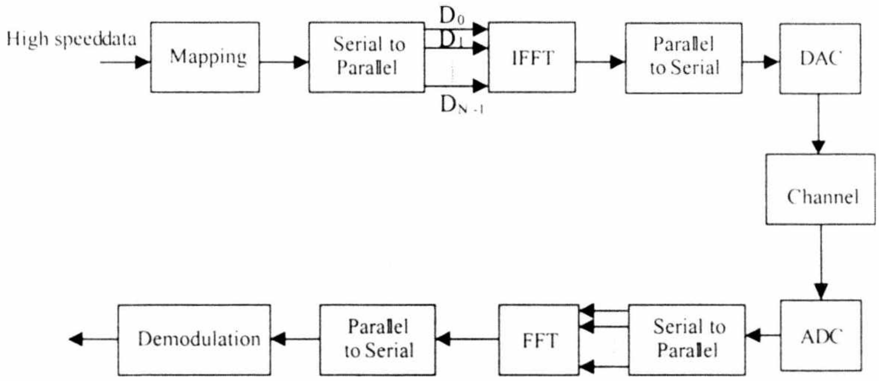

Figure 1. Block diagram of an OFDM system

The data to be transmitted is divided into lower bit rate streams and mapped onto the available subcarriers using quadrature amplitude modulation (QAM). The resulting complex data stream is divided into vectors of length N: D_0, D1, DN_{-1}. An N-point inverse DFT (IDFT) of the vector D_0, D1, DN_{-1} results in modulation and multiplexing of all of the subcarriers. These samples after IDFT must be converted to serial form, filtered converted to analogue before transmission. A symbol OFDM is:

$$d_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} D_n e^{j\frac{2\pi nk}{N}} \qquad k = 0, 1, \ldots N\text{-}1 \tag{1}$$

real parts of sequence are:

$$x_k = \sum_{n=0}^{N-1} \left[a_n \cos(2\pi f_n t_k) + b_n \sin(2\pi f_n t_k)\right] \quad k = 0, 1, \ldots N\text{-}1 \tag{2}$$

Discrete time OFDM signal can exhibit large peaks, which are caused by the addition of the several independently modulated tones. Figure 2 illustrates what the amplitudes x_k of OFDM signal could look like for a particular symbol that exhibits a large peak.

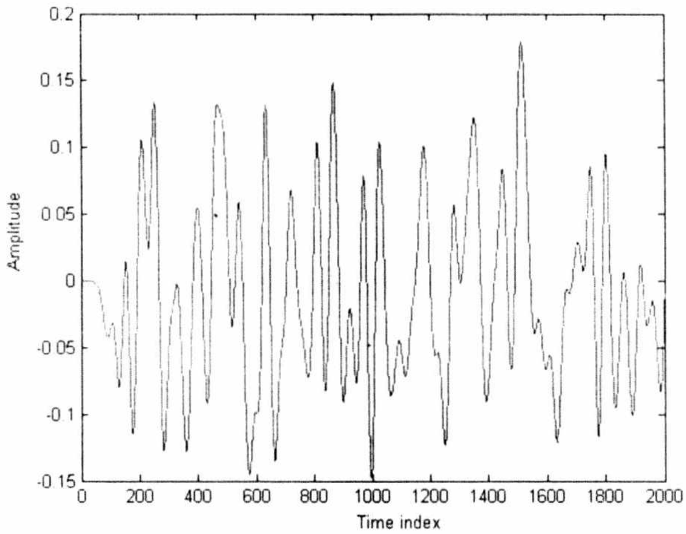

Figure 2. Amplitude of OFDM signal

3. PAR in OFDM symbols

3.1. Definition

The peak-to-average power ratio (PAR) of an OFDM symbol is defined as

$$\mathrm{PAR}(x) = \frac{\max\limits_{n \in 0,\dots,N-1} |x_n|^2}{E\left[|x_n|^2\right]} \tag{3}$$

where E(.) denotes taking expected value.

PAR of OFDM symbols is large, which are caused by the addition of the several independently modulated tones. This high PAR could cause problems when the signal is applied to a non-linear device such as a power amplifier, since it results in in-band distortion and spectral spreading. To counteract these effects, the amplifier needs to be highly linear or operated with a large back-off. Both approaches result in a severe power efficiency penalty and are expensive. That is why solutions have been proposed over the years, to counteract the PAR problem.

3.2. Statistical Properties

Assuming that the OFDM symbol size is large and that the input data are uncorrelated, the central limit theorem applies and the time domain transmitted flow is approximately distributed as complex Gaussian with zero mean and variance σ_x^2.

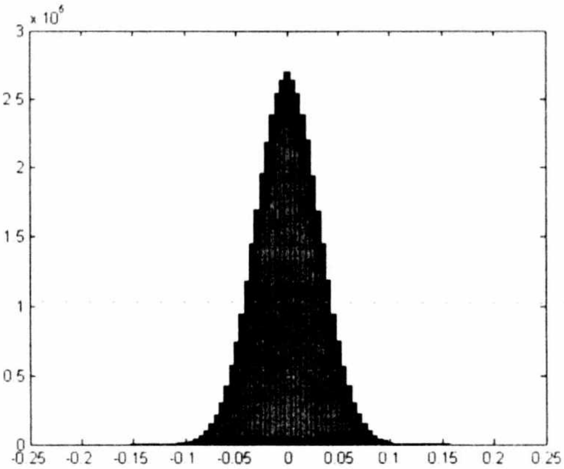

Figure 3. Histogram of OFDM signal

Because output of IFDT is Gaussian distributed, its modulus $u_n = |x_n|$ is Rayleigh distributed with probability density function

$$p(u) = \frac{2u}{\sigma_x^2} e^{-\frac{u^2}{\sigma_x^2}} \tag{4}$$

Probability that the amplitude of a sample with OFDM symbol exceeds a threshold $x_{th}>0$ is:

$$\Pr(|x| \geq x_{th}) = \int_{x_{th}}^{+\infty} p(u)du = \int_{x_{th}}^{+\infty} \frac{2u}{\sigma_x^2} e^{-\frac{u^2}{\sigma_x^2}} du = e^{-\frac{x_{th}^2}{\sigma_x^2}} \tag{5}$$

If the samples of the OFDM symbol are independent, we get

$$\Pr\left(\max_{n \in 0,\ldots,N-1} |x_n| > x_{th}\right) = 1 - \Pr\left(\max_{n \in 0,\ldots,N-1} |x_n| \leq x_{th}\right) \tag{6}$$

where

$$\begin{gathered} \Pr\left(\max|x_n| \leq x_{th}\right) = \prod_{n=0}^{N-1} \Pr\left(|x_n| \leq x_{th}\right) = \\ \Pr(x_0 \leq x_{th})\Pr(x_1 \leq x_{th}).....\Pr(x_{N-1} \leq x_{th}) = \left(1 - e^{\frac{x_{th}^2}{\sigma_x^2}}\right)^N \qquad n \in 0,\ldots,N-1 \end{gathered} \tag{7}$$

then, we obtain the corresponding probability for the PAR

$$\Pr(PAR(x) \geq PAR_0) = 1 - \left(1 - e^{-PAR_0}\right)^N \tag{8}$$

where $PAR_0 = \frac{x_{th}^2}{\sigma_x^2}$

This probability, corresponding to the Complementary Cumulative Distribution Function (CCDF) of the PAR. Which is useful to quantify the improvements of method for reducing the PAR. It represents the probability that an OFDM symbol has peak that exceed a given threshold.

4. PAR reduction method

There are a lot of PAR reduction technique e.g. block coding, windowing filter, selected mapping, partial transmit sequences, tone injection, tone reservation... Tone reservation method was developed by J.Tellado a former Stanford Ph.D. student in Professor John Cioffi's research group. This method involves setting aside preselected tones for PAR reduction. The information transmitted in these tones is used to subtract from the signal envelope, thus reducing the PAR.

By not transmitting data on certain tones, one can use those tones for the purpose of PAR reduction. If $X_j = 0$ for $j \in \{i_1,\ldots,i_L\}$, then the transmitter can add any vector $\overline{C}$ that satisfies $C_j = 0$ for $j \notin \{i_1,...,i_L\}$ to the data vector and remove it at the receiver. Denoting the IFFT matrix by Q, the new time sequence is:

$$x + c = IFFT(X+C) = Q(X + C) \tag{9}$$

Calling $\hat{C} = \left[C_{i_1},....C_{i_L}\right]^T$ the vector of nonzero values of $\overline{C}$ and $\hat{Q} = \left[q_{i_1},....,q_{i_L}\right]$ the matrix constructed by selecting column $(i_1,...,i_L)$ of Q, minimizing the PAR of the vector x + c is equivalent to solving:

$$\min_{c}\|x + c\|_{\infty} = \min_{\hat{C}}\left\|x + \hat{Q}.\hat{C}\right\|_{\infty} \tag{10}$$

or

$$\min_{c}\ \max\|x + c\| = \min_{\hat{C}}\ \max\left\|x + \hat{Q}.\hat{C}\right\| \tag{11}$$

where $c = Q.C = \hat{Q}.\hat{C}$ and $\|x\|_{\infty}$ denotes ∞-norm

This can recast as a linear programming (LP)

$$\min_{\hat{C}}\ t\ \text{ subject to } \begin{array}{l}\hat{Q}\hat{C} - t.1_N \underset{N}{\leq} -x \\ \hat{Q}\hat{C} + t.1_N \underset{N}{\geq} -x\end{array} \tag{12}$$

This LP can be solved with complexity O(NlogN). However, we can get good approximate solutions to Eq. (11) with complexity O(N). After some calculations, one arrives at the following iterative algorithm:

$$c^{(k+1)} = c^{(k)} - \mu\alpha_k p[((n - n_k))_N] \tag{13}$$

$$n_k = \arg\max_{n}\left|x_n + c_n^{(k)}\right| \tag{14}$$

where α_k is scale factor depending on the maximum peak found at iteration k, p is called Peak Reduction Kernel is a function of the tone locations $(i_1,...,i_L)$. The notation $p[((n - n_k))_N]$ means that the kernel has been circularly shifted in time by a value of n_k.

5. Simulation Results

When the number of subcarrier increases, the PAR increases accordingly. The PAR under various numbers of subcarriers are shown in Figure 4

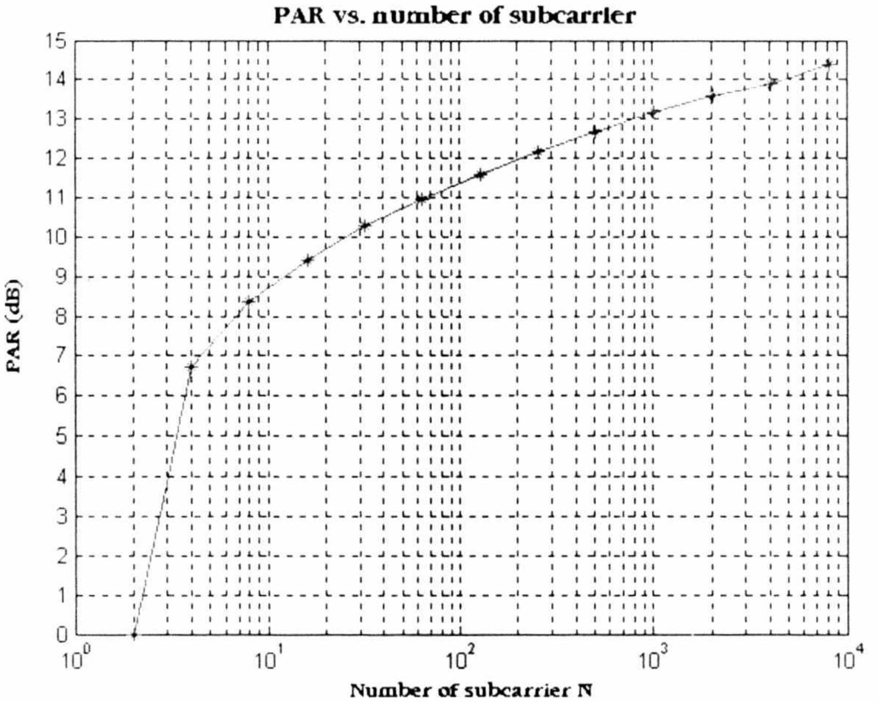

Figure 4. PAR vs. the number of subcarriers

In our simulation, we use OFDM scheme using QPSK modulation, number carriers are 64. We archive CCDF of PAR with different ratio of reserved to data tones (L/N).

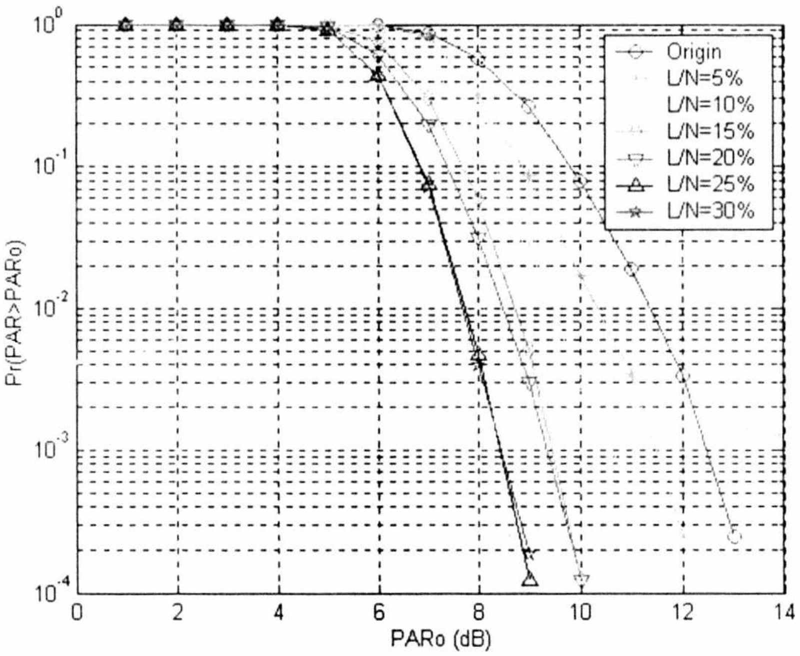

Figure 5. CCDF of PAR before and after using TR

In figure 5, PAR decreases when unused tones L increase. However, PAR reduction technique has the limitation. L/N ratio is conventionally chosen about 20%.

The continuous time transmit signal x(t) is obtained by supplying the discrete time samples to the D/A converter and filtering the outputs using a pulse shaping function. This time signal x(t) also exhibits these large peaks. PAR of two cases, continuous and discrete, is shown in Figure 6.

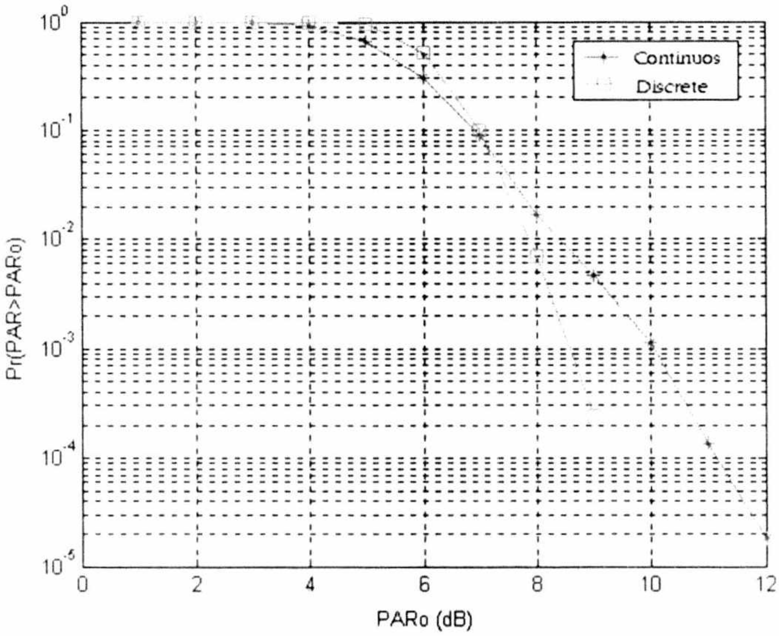

Figure 6. CCDF of PAR of continuous and discrete signal

The continuous PAR is larger than discrete PAR. The reason for this may be stated as follows. The maximum of discrete samples is not corresponding to the maximum peak of continuous signals, even though the obtained sample may be close to the maximum peak. As a result, the discrete PAR underestimates the continuous.

6. Conclusion

In this paper, we have considered a method for PAR reduction that is call tone reservation. Tone reservation is a preventative measure for high PAR that works by adding optimized PAR reduction vectors to the transmit vectors as to subtract from the signal envelope. This method has advantage of very low complexity at the transmitter, but that has the limitation.

Finally, the difference of PAR between discrete and continuous signal is shown.

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TẠP CHÍ KHOA HỌC ĐHQGHN, KHTN & CN, T.XX, Số 3, 2004

GIẢM TỶ SỐ PAR TRONG HỆ OFDM BẰNG PHƯƠNG PHÁP DÀNH RIÊNG TẦN

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Hệ thống ghép kênh tần số trực giao (OFDM) đảm bảo việc truyền dẫn vô tuyến với tốc độ dữ liệu cao, kháng nhiễu tốt trên kênh pha-đinh đa đường và chống được nhiễu xuyên ký hiệu. Tuy nhiên, hệ OFDM lại có một nhược điểm lớn là tỷ số công suất đỉnh trên công suất trung bình (PAR) cao. Điều này dẫn tới bộ khuếch đại công suất phải có độ tuyến tính cao, nếu không sẽ dẫn tới méo phi tuyến gây nhiễu kênh kề trong hệ OFDM. Bài báo này nghiên cứu các thuộc tính của tỷ số PAR và thực hiện giảm nó bằng phương pháp dành riêng tần của J.Tellado