THE GAP EQUATION IN THE NUCLEAR SIGMA - OMEGA MODEL

Phan Hong Lien
Technical Military University

Abstract. *The CJT effective action and quantum potential for the nuclear sigma-omega model is obtained when we consider the interaction of the spinor and pion fields and electromagnetic field in nuclear matter. The gap equation is derived directly. It is shown that the symmetry breaking part for inverse of the propagator is proportional to* τ_3

1. INTRODUCTION

In recent years, a systematic method for deriving the Schwinger - Dyson equations nd ethe-Salpeter equations has been proposed by many authors [1], [5]. It starts from the ffective action Γ (and effective potential), in particular from the CJT* effective action or the composite fields [6], [8], which is obtained by the double Legendre transformation.

The purpose of the present paper is to apply the same method for deriving the ap equation, when the interaction of the spinor field and the electromagnetic field are onsidered.

The equations for the nucleon propagator and the boson propagators are obtained. 'his is the nonlinear integral equation, when the equation is simplified by linearizing the ropagator, it returns the ladder Bethe - Salpeter equation.

Our work is organized as follows:

In Sec. II, by introducing the electromagnetic field in the Lagrangian QHD- I** or Sigma - Omega model, we derive the formula for effective action (and potential). The ap equation is derived directly based on this approach in Sec.III. Sec. IV is devoted to everal discussions and conclusions.

2. THE LAGRANGIAN AND THE EFFECTIVE ACTION

. The Lagrangian

First of all, we will dealt with the Lagrangian of Sigma - Omega model:

$$
\begin{aligned}
L = {} & \overline{\Psi}\left[i\gamma^{\mu}\partial_{\mu} - g\left(\pi\vec{\tau}\gamma^{5}\right) - g_{\omega}\gamma^{\mu}V_{\mu}\right]\Psi \\
& + \frac{1}{2}\left[\left(\partial_{\mu}\sigma\right)^{2} + \left(\partial_{\mu}\vec{\pi}\right)^{2}\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\mu^{2}}{2}\left(\sigma^{2}+\vec{\pi}^{2}\right) \\
& - \frac{\lambda^{2}}{4}\left(\sigma^{2}+\vec{\pi}^{2}\right)^{2} + \frac{m_{\omega}^{2}}{2}V^{\mu}V_{\mu} + c\sigma,
\end{aligned}
\tag{2.1}
$$

---

* J. Cornwall, R.Jackiw and E. Tomboulis

** The full field theory with baryons, neutral scalar meson and neutral vector meson is called QHD-I [9] where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$; $\overline{\Psi}, \Psi, \sigma, \vec{\pi}$ and V_μ represent the field operators of nucleon, sigma meson, pion and omega meson, respectively. The model has three coupling constant g, g_ω and λ. The last term leads to the explicit breaking of symmetry.

Let us consider the interaction of spinor field and electromagnetic field, i.e., replace $\hat{p}_\mu$ with $\hat{p}_\mu - eA_\mu$, where A_μ is the electromagnetic field vector.

In the case of nucleon field, we can write:

$$\partial_\mu \to \partial_\mu - ieQA_\mu, \tag{2.2}$$

where the charge Q is

$$Q = T_3 + \frac{S+B}{2} = T_3 + \frac{Y}{2} = \begin{cases} 1 \text{ for proton} \\ 0 \text{ for notron} \end{cases}$$

$$\begin{aligned} L =& \overline{\Psi}\left[i\gamma^\mu(\partial_\mu - ieQA_\mu) - g(\sigma + i\vec{\pi}\vec{\tau}\gamma^5) - g_\omega\gamma^\mu V_\mu\right]\Psi \\ &+ \frac{1}{2}\left[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \\ &- \frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2)^2 + \frac{m_\omega^2}{2}V_\mu V^\mu + c\sigma, \end{aligned} \tag{2.3}$$

where $G_{\mu\nu} = \partial_\mu A_v - \partial_\nu A_\mu$ is the electromagnetic field tensos and $\begin{pmatrix} \Psi_p \\ \Psi_n \end{pmatrix}$ is a two - component field in the "isospin" space and τ is the usual Pauli matrix.

Making the shift of the fields in nuclear matter :

$$\begin{aligned} \sigma &= \sigma_0 + s. \\ V_\mu &= \delta_{0\mu}\omega + w_\mu. \end{aligned} \tag{2.4}$$

In nuclear matter (the density is finite) the expectation values of fields tend to their ground state mean values, where

$$\begin{aligned} \overline{\Psi} &= \langle F|\overline{\Psi}|F\rangle = \langle F|\Psi|F\rangle = \Psi = 0, \\ \overline{\sigma} &= \langle F|\sigma|F\rangle = \sigma_0 + s \quad \langle F|s|F\rangle = 0, \\ \overline{V}_\mu &= \langle F|V_\mu|F\rangle = \delta_{0\mu}\omega \quad \langle F|w_\mu|F\rangle = 0, \\ \overline{\pi}_i &= \langle F|\pi_i|F\rangle = 0, \end{aligned}$$

where F is the ground state, so the Langrangian takes the following formation:

$$
\begin{aligned}
L = & \overline{\Psi}_p(i\gamma^\mu\partial_\mu - M_N + e\gamma^\mu A_\mu - gs)\Psi_n \\
& + \overline{\Psi}_n(i\gamma^\mu\partial_\mu - M_N - gs)\Psi_n - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \\
& - ig(\overline{\Psi}_p\vec{\tau}\Psi_n + \overline{\Psi}_n\vec{\tau}\gamma^5\Psi_p)\vec{\pi} + \frac{1}{2}[(\partial_\mu\vec{\pi})^2 - m_\pi^2\vec{\pi}^2] \\
& \frac{1}{2}[(\partial_\mu s)^2 - m_\sigma^2 s^2] - \lambda^2\overline{\sigma}s(s^2+\vec{\pi}^2) - \frac{\lambda^2}{4}(s^2+\vec{\pi}^2)^2 \\
& - g_\omega\overline{\Psi}_p\gamma^\mu\delta_{0\mu}\omega\Psi_p - g_\omega\overline{\Psi}_p\gamma^\mu w_\mu\Psi_p - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& - g_\omega\overline{\Psi}_n\gamma^\mu\delta_{0\mu}\omega\Psi_n - g_\omega\overline{\Psi}_n\gamma^\mu w_\mu\Psi_n + \frac{m_\omega^2}{2}(\omega^2 + w_\mu^2) + m_\omega^2\delta_{0\mu}\omega w_\mu \\
& + c(\overline{\sigma} - \sigma_0) - \frac{\lambda^2}{4}(\overline{\sigma^2} - \sigma_0^2) - \frac{m_\pi^{(0)2}}{2}(\overline{\sigma^2} - \sigma_0^2), \qquad (2.5)
\end{aligned}
$$

where

$$
\begin{aligned}
M_N &= g\overline{\sigma}, \\
m_\sigma^2 &= \mu^2 + 3\lambda^2\overline{\sigma}^2, \\
m_\pi^2 &= \mu^2 + \lambda^2\overline{\sigma}^2
\end{aligned}
$$

are in-medium mass of nucleon, sigma meson and pion are the mass of pion in the vacuum.

. The Effective Action

The generating functional for connected Green's functions in the *presence* of the currents J and the sources K coupled to the corresponding field is:

$$
\begin{aligned}
Z[J,K] &= expiW[J,K] \\
&= \frac{1}{Z[0,0]}\int[D\overline{\Psi}][D\Psi][D\sigma][D\vec{\pi}][DV_\mu][DA_\mu]expi\{S+ \\
&+ \int d^4x[\ \overline{\Psi}(x)\eta(x) + \overline{\eta}(x)\Psi(x) + \sigma(x)J_\sigma(x) + \vec{\pi}_i(x)J_i(x) + V_\mu(x)J_\mu^\nu(x) + A_\mu(x)J_\mu^A(x)] \qquad (2.6) \\
&+ \frac{1}{2}\iint d^4xd^4y[\ \overline{\Psi}(x)K_{\overline{\psi}\psi}(x,y)\Psi(y) + \sigma(x)K_\sigma(x,y)\sigma(y) + \vec{\pi}_i(x)K^{ik}_{(x,y)}\vec{\pi}_k(y) \\
&\qquad + V_\mu(x)K^\nu_{\mu\nu}(x,y)V_\nu(y) + A_\mu(x)K^A_{\mu\nu}(x,y)A_\nu(y)]
\end{aligned}
$$

The mean values of field operators $\overline{\Psi}, \Psi, \sigma, \vec{\pi}^i, V_\mu, A_\mu$ respectively, in the presence of external sources are defined by:

$$
\begin{aligned}
\frac{\delta W}{\delta\overline{\eta}(x)} &= \psi(x), & \frac{\delta W}{\delta\eta(x)} &= \overline{\psi}(x), \\
\frac{\delta W}{\delta J_\sigma(x)} &= \sigma(x) & \frac{\delta W}{\delta J_i(x)} &= \pi^i(x), \qquad (2.7) \\
\frac{\delta W}{\delta J_\mu^\nu(x)} &= V_\mu(x), & \frac{\delta W}{\delta J_\mu^A(x)} &= A_\mu(x),
\end{aligned}
$$

and the propagators corresponding are determined from:

$$\frac{\delta^2 W}{\delta\overline{\eta}(x)\delta\eta(y)} = S(x,y), \qquad \frac{\delta^2 W}{\delta J_i(x)\delta J_k(y)} = D^{ik}(x,y),$$
$$\frac{\delta^2 W}{\delta J_\sigma(x)\delta J_\sigma(y)} = D_\sigma(x,y), \qquad \frac{\delta^2 W}{\delta J_\mu^{\nu(A)}(x)\delta J_\nu^{\nu(A)}(y)} = \Delta_{\mu\nu}^{\nu(A)}(x,y). \tag{2.8}$$

The CJT effective action $\Gamma[\phi, D]$ is defined as the double Legendre transformatio of the generating functional W[J,K]:

$$\begin{aligned}\Gamma[\phi, D] = W[J,K] - \\ - \int d^4x[\,\overline{\psi}(x)\eta(x) + \overline{\eta}(x)\psi(x) + \sigma(x)J(x) + \pi_i(x)J_i(x) + V_\mu(x)J_\mu^\nu(x) + A_\mu(x)\ldots \\ - \frac{1}{2}\int d^4x d^4y[2\overline{\psi}(x)K_{\overline{\Psi}\Psi}(x,y)\psi(y) + \sigma(x)K_\sigma(x,y)\sigma(y) + \\ + \pi_j(x)K_{jK}(x,y)\pi_K(y) + V_\mu(x)K_{\mu\nu}(x,y)V_\nu(y) + A_\mu(x)K_{\mu\nu}^A A_\nu(x)] \\ - \frac{1}{2}\iint d^4x d^4y[2S(x,y)K_{\overline{\Psi}\Psi}(y,x) + D_\sigma(x,y)K_\sigma(y,x) + \\ + D_{jK}(x,y)K_{Kj}(y,x) + \Delta_{\mu\nu}^V(x,y)K_{\nu\mu}^V(y,x) + \Delta_{\mu\nu}^A(x,y)\Delta_{\nu\mu}^A(y,x)],\end{aligned}$$

Starting from (2.9) one obtains:

$$\begin{aligned}\frac{\delta\Gamma}{\delta\overline{\psi}(x)} &= -\eta(x) - \int d^4y\overline{\Psi}(y)K(y,x),\\ \frac{\delta\Gamma}{\delta\psi(x)} &= -\overline{\eta}(x) - \int d^4yK(x,y)\Psi(y),\\ \frac{\delta\Gamma}{\delta\Psi(x)} &= -J_\sigma(x) - \int d^4yK(x,y)\sigma(y),\\ \frac{\delta\Gamma}{\delta\pi(x)} &= J_i(x) - \int d^4yK_{ik}(x,y)\pi_K(y),\\ \frac{\delta\Gamma}{\delta V_\mu(x)} &= -J_\mu^\nu(x) - \int d^4yK_{\mu\nu}^\nu(x,y)V_\mu(y),\\ \frac{\delta\Gamma}{\delta A_\mu(x)} &= -J_\mu^A(x) - \int d^4yK_{\mu\nu}^A(x,y)A_\mu(y),\end{aligned} \tag{2.10}$$

and

$$\begin{aligned}\frac{\delta\Gamma}{\delta S(x,y)} &= -K_{\overline{\Psi}\Psi}(x,y).\\ \frac{\delta\Gamma}{\delta D_\sigma(x,y)} &= -\frac{1}{2}K_\sigma(x,y),\\ \frac{\delta\Gamma}{\delta D_{jK}(x,y)} &= -\frac{1}{2}K_{jK}(x,y);\\ \frac{\delta\Gamma}{\delta\Delta_{\mu\nu}^{\nu(A)}(x,y)} &= -\frac{1}{2}K_{\mu\nu}^{\nu(A)}(x,y).\end{aligned} \tag{2.11}$$

It is evident that when all the external sources vanish, the systems (2.10) and (2.11) e equal to zero. This is the stationary condition for the physical solutions.

The CJT effective action can be derived directly basing on [6]:

$$\begin{aligned}\Gamma = & S + iTrln[S_0^{-1}S] - iTr[S_0^{-1}(\overline{\sigma}, \omega, A_0)S] + iTrl \\ & - \frac{i}{2}Trln[D_\delta^{(0)-1}D_\sigma] + \frac{i}{2}Tr[D(\overline{\sigma})]^{-1}D_\sigma - \frac{i}{2}Trl \\ & - \frac{i}{2}Trln[D_{ij}^{(0)-1}D_{ij}] + \frac{i}{2}Tr[D_{ij}^{(0)}(\overline{\sigma})]^{-1}D_{ij} - \frac{i}{2}Trl \\ & - \frac{i}{2}Trln[\Delta_{\mu\nu}^{(0)\nu}\Delta_{\mu\nu}^{\nu}] + \frac{i}{2}Tr[\Delta_{\mu\nu}^{(0)\nu}]^{-1}\Delta_{\mu\nu}^{\nu} - \frac{i}{2}Trl \\ & - \frac{i}{2}Trln[\Delta^{(0)A}\Delta_{\mu\nu}^{(0)A}]^{-1}\Delta_{\mu\nu}^{A} - \frac{i}{2}Trl + \Gamma_2.\end{aligned} \tag{2.12}$$

here the free propagators in the momentum representation have the following formation

$$\begin{aligned}S_0(p) &= \frac{1}{\hat{p} - M_N^{(0)} - i\in}, & M_N^{(0)} &= g\sigma_0, \\ D_\sigma^{(0)} &= \frac{1}{p^2 - m_\sigma^{(0)2} - i\in}, & m_\sigma^{(0)2} &= \mu^2 + 3\lambda^2\sigma_0^2, \\ D_{ij}^{(0)}(p) &= \frac{1}{p^2 - m_\pi^{(0)2} - i\in}\delta_{ij}, & m_\pi^{(0)2} &= \mu^2 + \lambda^2\sigma_0^2, \\ \Delta_{\mu\nu}^{\nu}(p) &= \frac{1}{p^2 - m_\omega^2 - i\in}\delta_{\mu\nu}, \\ \Delta_{\mu\nu}^{A}(p) &= \frac{i(g^{\mu\nu} - p^\mu p^\nu/p^2)}{p^2 - i\in}, & & \text{(Landau gauge)}\end{aligned} \tag{2.13}$$

d the functional operators are determined by:

$$\begin{aligned}[D_\sigma^{(0)}(p, \overline{\sigma})]^{-1} &= p^2 - m_\sigma^2, \\ [D_{ij}^{(0)}(p, \overline{\sigma})]^{-1} &= \delta_{ij}(p^2 - m_\pi^2), \\ S_0^{-1}(p; \overline{\sigma}, \omega, A_0) &= \hat{p} - M_N - g_\omega\gamma_\omega^0 - eQ\gamma^0 A_0,\end{aligned} \tag{2.14}$$

Γ_2 is given by all the two-particle irreducible vacuum graphs with the vertices termined by S_{int} and the propagators set equal to S(x,y) [10].

For translatetional invariance the quantum effective potential is defined by:

$$\Gamma[\phi, D] = -V[\phi, D]\int d^4x \tag{2.15}$$

Starting (2.12) - (2.15) we can arrive at the expression for effective potential in omentım space:

$$
\begin{aligned}
&V(\overline{\sigma}, \omega, A_0; S, D_\sigma, D_{ij}, \Delta^\nu_{\mu\nu}, \Delta^A_{\mu\nu}] = \\
&= -\frac{\lambda^2}{4}\left(\overline{\sigma}^2 - \sigma_0^2\right)^2 + \frac{m_\pi^{(0)2}}{2}\left(\overline{\sigma}^2 - \sigma_0^2\right) - c(\overline{\sigma} - \sigma_0) - \frac{m_\omega^2}{2}\omega^2 \\
&- i\int \frac{d^4p}{(2\pi)^4} Tr\Big\{ln\big[S_0^{-1}(p)S(p) - S_0^{-1}(p;\overline{\sigma}, \omega, A_0)S(p) + 1\big]\Big\} \\
&+ \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\Big\{ln\big[\big(D_\sigma^{(0)}(p)\big)^{-1}D_\sigma(p)\big] - \big[D_\sigma^{(0)}(p;\overline{\sigma})\big]^{-1}D_\sigma(p) + 1\Big\} \\
&+ \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\Big\{\big[\big(D_{ij}^{(0)}(p)\big)^{-1}D_{ij}(p)\big] - \big[D_{ij}^{(0)}(p;\overline{\sigma}, A_0)\big]^{-1}D_{ij}(p) + 1\Big\} \\
&+ \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\Big\{ln\big[\big(\Delta_{\mu\nu}^{(0)\nu}(p)\big)^{-1}\Delta^\nu_{\mu\nu}(p) - \big[\Delta_{\mu\nu}^{(0)\nu}(p)\big]^{-1}\Delta^\nu_{\mu\nu}(p) + 1\Big\} \\
&+ \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\Big\{ln\big[\big(\Delta_{\mu\nu}^{(0)A}(p)\big)^{-1}\Delta^A_{\mu\nu}(p) - \big[\Delta_{\mu\nu}^{(0)A}(p)\big]^{-1}\Delta^A_{\mu\nu}(p) + 1\Big\} \\
&+ \frac{1}{2}g\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} Tr\Big\{S(p)\Gamma(p,k)S(k)D_\sigma(p-k)\Big\} \\
&+ \frac{1}{2}g\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} Tr\Big\{\gamma^5\tau^i S(p)\Gamma^j(p,k)S(k)D_{ij}(p-k)\Big\} \qquad (2.\\
&+ \frac{i}{2}g_\omega\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} Tr\Big\{\gamma^\mu S(p)\Gamma^\nu(p,k)S(k)\Delta^\nu_{\mu\nu}(p-k)\Big\} \\
&+ \frac{i}{2}eQ_\pi\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}.Tr\big\{D_\pi(p)\Gamma^\nu_A(p,k)D_\pi(k)\Delta^A_{\mu\nu}(p-k)\big\} \\
&- \frac{1}{2}eQ_N\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} Tr\Big\{\gamma^\mu S(p)\Gamma^\nu(p,k)S(k)\Delta^A_{\mu\nu}(p-k)\Big\} \\
&+ \frac{3i}{2}\lambda^2\overline{\sigma}\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\pi(p)D_\pi(p+k)D_\sigma(k)T(k;p,-p-k) \\
&+ \frac{i}{2}\lambda^2\overline{\sigma}\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\pi(p)D_\pi(p+k)D_\sigma(k)T(p,k-p-k;) \\
&+ \frac{15}{4}i\lambda^2\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\pi(p)D_\pi(k) \\
&+ \frac{3}{2}i\lambda^2\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\pi(p)D_\sigma(k) \\
&+ \frac{3}{4}i\lambda^2\iint \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\sigma(p)D_\sigma(k) \\
&- ieQ_\pi^2\iiint \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\pi(p)D_\pi(q)\Delta^A_\mu(p+q-k)\Delta^{A\mu\nu}(k) \\
&+ \frac{1}{4}i\lambda^2\iiint \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\Big\{3D_\sigma(p)D_\sigma(q)D_\pi(p+q-k)D_\pi(k)T(p,q;-p-k+k, \\
&+ \frac{1}{6}D_\pi(p)D_\pi(q)D_\pi(p+q-k)D_\pi(k)\big(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\big)T^{ijkl}(;p,q,-p,-q+k,- \\
&+ \frac{1}{8}i\lambda^2\iiint \frac{d^4p}{(2\pi)^4}\ \Big\{D_\sigma(p)D_\sigma(q)D_\sigma(p+q-k)D_\sigma(k)T(p,q,-p-q+k,-k;)\Big\}
\end{aligned}
$$

where the mesonic T matrices are given the NN - meson irreducible vertices.

3. THE GAP EQUATIONS

The stationary condition (2.10) - (2.11) imply that the effective potential must be stationary against variations of S. From (2.16) it is given by

$$\begin{aligned} S^{-1}(p) &= S_0^{-1}(p;\overline{\sigma},\omega,A_0) \\ &+ g^2\int\frac{d^4k}{(2\pi)^4}S(k)\Gamma_\sigma(k,p)D_\sigma(k-p) \\ &+ ig^2\int\frac{d^4k}{(2\pi)^4}\left[\gamma^5\tau^i s(k)\Gamma^j(k,p)D_{ij}(k-p)\right] \\ &+ g_\omega^2\int\frac{d^4k}{(2\pi)^4}\left[\gamma_\mu S(k)\wedge_\nu^\nu(k,p)\Delta_{\mu\nu}^\nu(k-p)\right] \\ &+ (eQ)^2\int\frac{d^4k}{(2\pi)^4}\left[\gamma_\mu S(k)\Gamma_\nu^A(k,p)\Delta_{\mu\nu}^A(k-p)\right], \end{aligned} \tag{3.1}$$

where:

$$\begin{aligned} S_0^{-1}(p;\overline{\sigma},\omega,A_0) &= p - M_N - g_\omega\gamma^0\omega - eQ\gamma^0A_0 \\ &= S_0^{-1}(p) - g_\omega\gamma^0\omega - e(\tau_3+\frac{1}{2})\gamma^0A_0. \end{aligned} \tag{3.2}$$

It showns that the symmetry - breaking part of S^{-1} is proportional to τ_3.

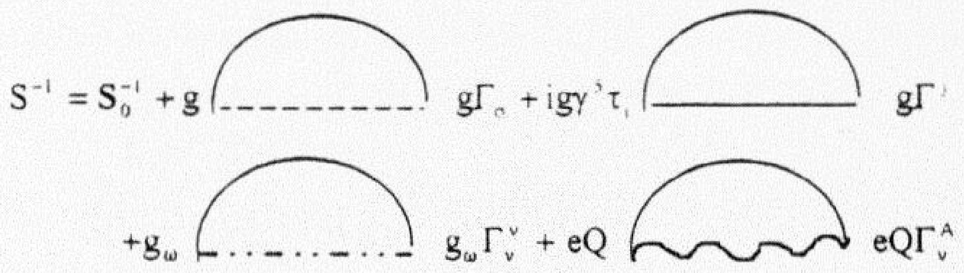

Fig 1. *The nucleon propagator satisfies the equation (3.1). The bold solid line represent the nucleon propagator S, the dashed line notes sigma propagator D_σ, the solid line pion propagator D_{ij}, the dashed - dotted line notes omega propagator $\Delta_{\mu\nu}^\nu$ and the wavy line - photon propagator $\Delta_{\mu\nu}^A$.*

One also obtains the expressions for the boson propagator

$$D_\sigma^{-1}(p) = \left[D_\sigma^{(0)}(p;\overline{\sigma})\right]^{-1} - ig^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[S(k)\Gamma(k,k+p)S(k+p)\right]$$
$$- 3i\lambda^2\overline{\sigma} \int \frac{d^4k}{(2\pi)^4}\left[D_\pi(k)D_\pi(k+p)T(p,k,-p-k) + D_\sigma(p)D_\sigma(p+k)T(p,k,-p-\right.$$
$$- 3i\lambda^2 \int \frac{d^4k}{(2\pi)^4}\left[D_\pi(k) + D_\sigma(k)\right]$$
$$- i\lambda^2 \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\left[3D_\sigma(q)D_\pi(q+p-k)D_\pi(k)T(p,q;-p-q+k,-k)\right. \qquad ($$
$$\left. + D_\sigma(q)D_\sigma(q+p-k)D_\sigma(k)T(p,q,-p-q+k,-k;)\right],$$

where

$$D_\sigma^{(0)}(p;\overline{\sigma}) = p^2 - m_\sigma^2 = D_\sigma^{(0)}(p) - 3\lambda^2(\overline{\sigma}^2 - \sigma_0^2),$$
$$D_{ij}^{-1}(p) = D_{jn}^{(0)}(p;\overline{\sigma}) + g^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5\tau S(k)\Gamma^j(k,k+p)S(k+p)\right]$$
$$\dot{-}\, i\delta_{ij}\lambda^2 \int \frac{d^4k}{(2\pi)^4}\left[D_\sigma(k) + 5D_\sigma(k) + 2\overline{\sigma}D_\sigma(k)D_\pi(k+p)T(-k;k+p,-p)\right]$$
$$- i\lambda^2 \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\left[\Delta_{ij}D_\sigma(q)D_\sigma(k)D_\pi(p+k-p)T(q,k;-q-k+p,-p)\right.$$
$$+ \frac{1}{3}D_\pi(q)D_\pi(k)D_\pi(q+k-p)(\delta_{ia}\delta_{bc} + \delta_{ib}\delta_{ac} + \delta_{ic}\delta_{ab})T(;q,k,-q-k+p,-p)$$
$$- ieQ_\pi \frac{d^4k}{(2\pi)^4} Tr\left[D_\pi(k)\Gamma_A^\nu(k,k+p)\Delta_{\mu\nu}^A(k+p)\right] \qquad (3.$$
$$+ 2e^2Q_\pi^2 \iint \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\pi(q)\Delta_{\mu\nu}^A(k)\Delta^{A\mu\nu}(q+k-p)$$

where $D_{ij}^{(0)}(p,\sigma) = \delta_{ij}(p^2 - m_\pi^2) = D_{ij}^{(0)}(p) - \lambda^2(\overline{\sigma} - \sigma_0^2)$ $D_\pi = \frac{1}{3}\delta^{ik}D_{ik}$ and $Q_\pi^2 =$

The equations (3.1), (3.3) and (3.4) are the homogenous, nonlinear equations. Make linearizing the propagators, i.e., setting:

$$S^{-1} = S_0^{-1} - \sum N \;, \qquad S = S_0 + S\sum NS_0$$
$$D_\sigma^{-1} = \left[D_\sigma^{(0)}\right]^{-1} - \Pi_\sigma \;, \qquad D_\sigma = D_\sigma^{(0)} + D_\sigma\Pi_\sigma D_\sigma^{(0)}, \qquad (3.6$$
$$D_{ij}^{-1} = \left[D_{ij}^{(0)}\right]^{-1} - \Pi_{ij} \;, \qquad D_\pi = D_\pi^{(0)} + D_\pi\Pi_{ij}D_\pi^{(0)}.$$

The symmetry breaking part in (3.1), (3.3) and (3.4) coincides with the ladder Bathe Salpeter equation, which is well known, accquires the masses for nucleon, scalar and vecto meson.

It is evident that the electromagnetic interactions in nuclear matter give the con tributions to self energy of the nucleon and pion meson. It takes the form of the ga equations proposed by Johnson [12]

$$\Delta M = eQ_N\gamma^0 A_0 + eQ_N \int \frac{d^4k}{(2\pi)^4}\left[\gamma^\mu S(k)\Gamma^\nu_A(k,p)\Delta^A_{\mu\nu}(k-p)\right], \tag{3.5}$$

ınd

$$\Delta m_\pi^2 = ieQ_\pi \int \frac{d^4k}{(2\pi)^4} Tr\left[D_\pi(k)\Gamma^\nu_A(k,k+p)\Delta^A_{\mu\nu}(k+p)\right] \qquad (3.6.)$$
$$- 2e^2Q_\pi^2 \iint \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} D_\pi(q)\Delta^A_{\mu\nu}(k)\Delta^{A\mu\nu}(q+k-p).$$

Graphically this is represented in Fig 2.

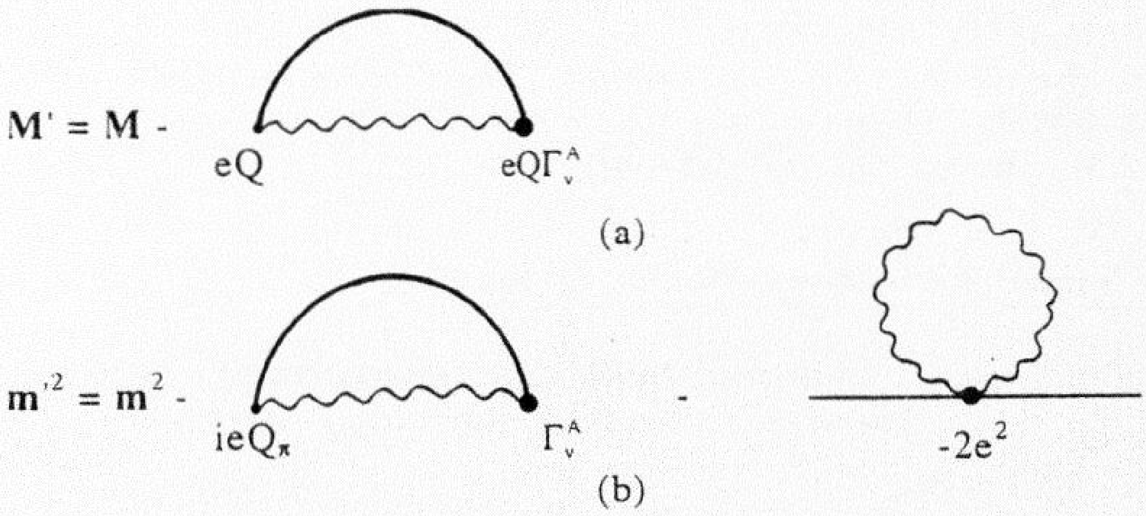

Fig.2. *The gap equations for nucleon and pion meson in $\sigma - \omega$ Model*

4. CONCLUSION

We have presented an effective action approach for the nuclear Sigma - Omega model hen the electromagnetic field is considered. The method used seems persuasive since it llows us to determine consistently the gap equation and the propagators of the fields oncerned. Further more, the theory naturally accounts for all high order corrections. It able to determine the energy density of the nuclear ground state and the phase trasitions nuclear medium.

Aknowledgements

The author would like thank Prof. Tran Huu Phat and Prof. R. Baier for their ıvestigation of this research.

The support of Vietnam National University Hanoi is aknowledgement.

REFERENCES

1. H. Kleinert. *Fortschritte der Physik*, **30**(1982), 351-400.
2. D.W. McKay and H.J. Munczeck. *Phys. Rev.*, **D30**(1988), 195.
3. R. Fukuda, M. Komachiya and M. Ukita. *Phys. Rev.*, **D38**(1988), 3347.
4. A. Hädicke. *Modern Physics*, **A6**(1991), 3321.
5. H.J. Munczeck. *Phys. Rev.*, **D52**(1995), 4736.
6. John M. Cornwall, R. Jackiw and E. Tombouliz. *Phys. Rev.*, **D10**(1974), 2428.
7. T. Banks and S. Raby. *Phys. Rev.*, **D14**(1974), 2182.
8. M. Nakano, A. Hasegawa, H. Kouno and K. Koide. *Phys. Rev*, **C49**(1994), 3061.
9. J.D. Walecka, *Ann. Phys.*, (NY) **83**(1974), 491; B.D. Serot and J.D. Walecka, *Adv. Nucl. Phys.*, **16**(1986), 1.
10. Tran Huu Phat and Phan Hong Lien. *Nuovo Cimento*, **A110**(1997), 847.
11. B.W. Lee. Chiral Dynamics, *Gordon & Breach, N.Y*, 1972.
12. K. Johnson. *Phys. Rev.*, **D10**(1974).

TẠP CHÍ KHOA HỌC ĐHQGHN, KHTN, t.XVI, n^{0}2 - 2000

PHƯƠNG TRÌNH KHE NĂNG LƯỢNG
TRONG MÔ HÌNH HẠT NHÂN SIGMA-OMEGA

Phan Hồng Liên
Học viện Kỹ thuật Quân sự

Tác dụng hiệu dụng CJT và thế hiệu dụng lượng tử cho mô hình hạt nhân sigma-omega đã thu được khi xét đến tương tác của các trường spinor và pion với trường điện từ trong chất hạt nhân. Từ đó trực tiếp rút ra các phương trình khe năng lượng. Kết quả cho thấy phần phá vỡ đối xứng của nghịch đảo hàm truyền nucleon tỷ lệ với thành phần thứ ba của spin đồng vị τ_3.