OPTICAL BISTABILITY EFFECT IN HOMOGENEOUSLY BROADENED FABRY-PEROT LSA

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Abstract: *Stationary properties of optical bistability (OB) in single-mode homogeneously broadend Fabry-Perot lasers with saturable absorber (LSA) have been investigated on the basis of the rate equation approximation (REA) with allowance for the spatial hole burning. Two typical aspects of hysteresis curves as well as their characteristics have thoroughly discussed and clearly illustrated. The presented results can be valid when the photon intensity is not to high.*

Considerable progress has taken place in the last two decades in the research on the OB effec [1-5, for a forthcoming review]. As a sole active system displaying absorptive OB, LSA has been extensively studied and improved in recent years. In the preceding paper [6] we have presented the OB operation of LSA with dominant inhomogeneous broadening. As for homogeneously broadened LSA, the OB study reported till now has not been completed yet. The aim of this paper is to investigate the OB effect after developing the REA for the analysis of the steady-state and stability properties of single-mode homogeneously broadened Fabry-Perot LSA.

We consider a Fabry-Perot resonator of length L, directed along the x-axis, into which the amplification and absorption cells with the same length l are inserted at the coordinaes x_a and x_b, respectively. This LSA with a homogeneously broadened line of half-width Γ can sustain only one mode at the circular frequency $\Omega_j = \pi m_j c/L$ (m_j integer, c velocity of light), which is at a distance $\Delta_j = |\Omega_j - \Omega_0|$ from the gain spectrum center Ω. In the rate equation approach (REA), such a Fabry-Perot LSA obeys the followingequations:

$$\frac{dn_j}{dt} = -\chi_j n_j + (n_j+1)Bg_j \left\{ \int_{x_a-\frac{l}{2}}^{x_a+\frac{l}{2}} n_a(x,t)\sin^2\left(\frac{\pi m_j x}{L}\right)dx - \int_{x_b-\frac{l}{2}}^{x_b+\frac{l}{2}} n_b(x,t)sin^2\left(\frac{\pi m_j x}{L}\right)dx \right\} \quad (1.a)$$

$$\frac{\partial n_a(x,t)}{\partial t} = \frac{R_a}{l} - n_a(x,t)\left\{Bg_j n_j \sin^2\left(\frac{\pi m_j x}{L}\right) + \gamma_a\right\} \quad (1.b)$$

$$\frac{\partial n_b(x,t)}{\partial t} = \frac{R_b}{l} - n_b(x,t)\left\{Bg_j n_j \sin^2\left(\frac{\pi m_j x}{L}\right) + \gamma_b\right\} \quad (1.c)$$

Here n_j, χ_j denote the photon number and the losses of the lasing mode.. B is Einstein coefficient. g_j stands for the gain profile assumed to be a Lorentzian:

$$g_j = \frac{\Gamma^2}{\Gamma^2 + 4\Delta\Omega_j^2}$$

$n_a(x,t)$ and $n_b(x,t)$ are the densities of population differences between the atomic u and lower levels in both media. The standing wave patterns and the spatial hole bournin Fabry-Perot cavity are taken into consideration by the $\sin^2$-factors. R_a, γ_a and R_b, γ_b the pumping rates and the decay constants in the amplifier and the absorber, resspecti

Let's introduce new quantities of the form [7,8]:

$$N_i = \int\limits_{x_i-\frac{L}{2}}^{x_i+\frac{L}{2}} n_i dx \text{and} N_{ij} = \int\limits_{x_i-\frac{L}{2}}^{x_i+\frac{L}{2}} n_i \cos\left(\frac{2\pi m_j x}{L}\right) dx (\text{here } i \quad \text{stand for } a \quad \text{or} \quad b)$$

The related equations for N_a and N_b are found from (1.b) and (1.c) by integra their both sides with respect to x. As for N_{aj} and N_{bj}, the same procedure is ca out after multiplying the both sides of (1.b) and (1.c) by $\cos(2\pi m_j x/L)$. Regroouping obtained equations yields:

$$\frac{dn_j}{dt} = -\chi_j n_j + \frac{1}{2}Bg_j(n_j+1)[N_a - N_{aj} - N_b + N_{bj}] \qquad ($$

$$\frac{dN_a}{dt} = R_a - N_a\left(\frac{1}{2}Bg_j n_j + \gamma_a\right) + \frac{1}{2}Bg_j n_j N_{aj} \qquad ($$

$$\frac{dN_{aj}}{dt} = -N_{aj}\left(\frac{1}{2}Bg_j n_j + \gamma_a\right) + \frac{1}{4}Bg_j n_j N_a \qquad ($$

$$\frac{dN_b}{dt} = R_b - N_b\left(\frac{1}{2}Bg_j n_j + \gamma_b\right) + \frac{1}{2}Bg_j n_j N_{bj} \qquad ($$

$$\frac{dN_{bj}}{dt} = -N_{bj}\left(\frac{1}{2}Bg_j n_j + \gamma_b\right) + \frac{1}{4}Bg_j n_j N_b \qquad ($$

The derivation of the equations for N_{aj} and N_{bj} has been done under the condi that one may neglect the terms of the form:

$$\int\limits_{x_i-\frac{L}{2}}^{x_i+\frac{L}{2}} n_i \cos\left(\frac{4\pi m_j x}{L}\right) dx \quad (\text{ here } \quad i \quad \text{stands for} \quad a \quad \text{or} \quad b)$$

in comparison with N_i and N_{ij}.The range of validity can be estimated by evaluatin relevant integrals in resonance. The calculations show that the used approxiimati justified if the LSA photon number is not larger than 4.10^{11}.

Setting the time derivatives in Eqs.(4) equal to zero, we obtain the steady-s equation:

$$Q_j - \frac{1}{2}g_j\left(Q_j + \frac{B}{\gamma}\right)\left[\frac{\sigma_a\left(\frac{1}{2}g_jQ_j+1\right)}{\left(\frac{1}{2}g_jQ_j+1\right)^2 - \frac{1}{8}g_j^2Q_j^2} - \frac{\sigma_b\left(\frac{1}{2}g_jQ_j+\xi\right)}{\left(\frac{1}{2}g_jQ_j+\xi\right)^2 - \frac{1}{8}g_j^2Q_j^2}\right] = 0$$

where

$$Q_j = \frac{B}{\gamma} n_j \quad - j^{th} \text{ mode intensity with } \quad \gamma = \gamma_a,$$

$$\sigma_a = \frac{B}{\gamma} \frac{R_a}{\chi_j} \quad \text{and} \quad \sigma_b = \frac{B}{\gamma} \frac{R_b}{\chi_j} \quad \text{- laser and absorber pumping rates,}$$

$$\xi = \frac{\gamma_b}{\gamma_a} \quad \text{- saturation coefficient.}$$

With a relative error not more than 10%, Eq. (5) is approximated by a cubic equation:

$$Q_j^3 + a_1 Q_j^2 + a_2 Q_j + a_3 = 0, \tag{6}$$

where

$$a_1 = \frac{c + b\xi}{bcg_j} - \frac{a}{2bc}(c\sigma_a - b\sigma_b)$$

$$a_2 = \frac{\xi}{bcg_j^2} - \frac{a}{2bcg_j}(\xi\sigma_a - \sigma_b)$$

$$a_3 = -\frac{aB}{2bcg_j\gamma}(\xi\sigma_a - \sigma_b) \quad \text{with} \quad a \approx 0,980, b \approx 0,592 \quad \text{and} \quad c \approx 0,550.$$

The OB occurrence in the LSA under consideration implies that it's steady-state equation must have three distinct positive solutions. To this end, one can solve either (6) by variational method or (5) implicitly by numerical calculation. The analytical results show that the OB may occurs in a certain range of laser pumping rate σ_a provided the LSA parameters ξ and σ_b satisfy the follwing conditions:

$$\xi < \xi_{\max} \equiv \frac{-abg_j\sigma_b + \sqrt{abg_j\sigma_b(abg_j\sigma_b + 8c)}}{4b} \tag{7}$$

or

$$\sigma_b > \sigma_{b\,\min} \equiv \frac{2b\xi^2}{ag_j(c - b\xi)} \tag{8}$$

The OB onset and the OB-off values are then determined by:

$$\sigma_{a1} = \frac{2(c - b\xi)}{acg_j} + \frac{b\sigma_b}{c} + 2\sqrt{\frac{2b\sigma_b(c - b\xi)}{ac^2 g_j}} \tag{9}$$

$$\sigma_{a2} = \frac{\sigma_b}{\xi} + \frac{2}{ag_j} \tag{10}$$

Just as σ_b goes beyond a critical value σ_{bt}, a portion of the OB lower branch becomes negative, thus physically meaningless, and hence the OB curve is partly truncated away. By increasing further σ_b, the truncated OB curve is always displaced towards higher σ_a, the OB height grows continually, whereas the effective OB width remains constant. The OB phase diagrams at $\Delta\Omega_j = 0$ and $\Delta\Omega_j = 0,3\Gamma$ are represented in Fig. 1. The (ξ, σ_b)

parameter plane is always divided into 3 domains (from left to right): monostable, bistable and truncated bistable. Fig. 2 shows the OB width variation for possible values of ξ and σ_b in resonance.

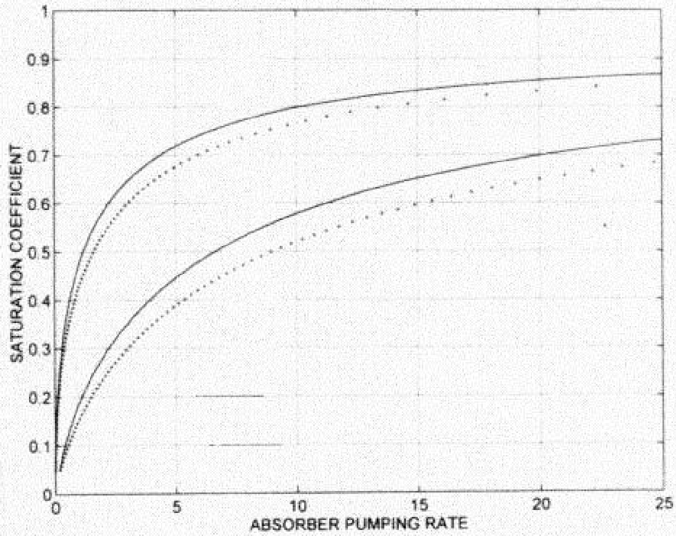

Fig. 1. *Optical bistable phase diagrams for resonant (solid lines) and nonresonant homogeneously broadened Fabry-Perot LSA*

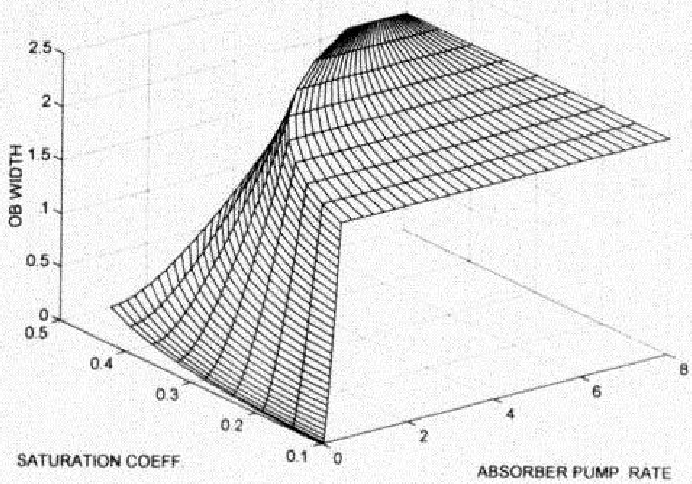

Fig. 2. *OB width variation in resonance*

Let us now perform the linear stability analysis of the steady-state solutions of Eqs. (4) for an OB set of ξ and σ_b-values. We let:

$$n_j(t) = n_{js} + \eta_j e^{-\lambda t};$$
$$N_a(t) = N_{as} + \eta_a e^{-\lambda t}; \quad N_{aj}(t) = N_{ajs} + \eta_{aj} e^{-\lambda t}$$
$$N_b(t) = N_{bs} + \eta_b e^{-\lambda t}; \quad N_{bj}(t) = N_{bjs} + \eta_{bj} e^{-\lambda t}$$

then linearize these equations with respect to the assumedly real fluctuations $\eta_j, \eta_a, \eta_{aj}$ and arrive at a system of linear homogeneous algebraic equations with a secular equat of the form:

$$\det(A + \lambda I) = 0 \tag{12}$$

where I is the unity matrix and A- a matrix with the following elements a_{ij}:

$$a_{11} = -\gamma_j + \frac{1}{2} B g_j [N_{as} - N_{ajs} - N_{bs} + N_{bjs}];$$

$$a_{12} = -a_{13} = -a_{14} = a_{15} = \frac{1}{2} B_{gj} \left(Q_{js} + \frac{B}{\gamma} \right);$$

$$a_{21} = -\frac{1}{2}\gamma g_j (N_{as} - N_{ajs}); \quad a_{22} = -\gamma \left(\frac{1}{2} g_j Q_{js} + 1 \right); \quad a_{23} = \frac{1}{2}\gamma g_j Q_{js}; \quad a_{24} = a_{25} = 0;$$

$$a_{31} = \frac{1}{4}\gamma g_j (N_{as} - 2N_{ajs}); \quad a_{32} = \frac{1}{4}\gamma g_j Q_{js}; \quad a_{33} = a_{22}; \quad a_{34} = a_{35} = 0;$$

$$a_{41} = -\frac{1}{2}\gamma g_j (N_{bs} - N_{bjs}); \quad a_{42} = a_{43} = 0; \quad a_{44} = -\gamma \left(\frac{1}{2} g_j Q_{js} + \xi \right); \quad a_{45} = a_{23};$$

$$a_{51} = \frac{1}{4}\gamma g_j (N_{bs} - 2N_{bjs}); \quad a_{52} = a_{53} = 0; \quad a_{54} = a_{44};$$

or

$$\lambda^5 - b_1 \lambda^4 + b_2 \lambda^2 + b_4 \lambda - b_5 = 0 \tag{13}$$

where

$$b_1 = -(a_{11} + a_{22} + a_{33} + a_{44} + a_{55})$$

$$\begin{aligned} b_2 =& a_{11}(a_{22} + a_{33} + a_{44} + a_{55}) + a_{22}(a_{33} + a_{44} + a_{55}) - 2a_{23}a_{32} - \dots \\ & \dots - a_{12}(a_{21} - a_{31} - a_{41} + a_{51}) + a_{44}a_{55} \end{aligned}$$

$$\begin{aligned} b_3 =& (a_{11} + a_{22} + a_{33})(a_{44}a_{55} - a_{23}a_{32}) + a_{11}(a_{22} + a_{33})(a_{44} + a_{55}) + \dots \\ & \dots + (a_{11} + a_{44} + a_{55})(a_{22}a_{33} - a_{23}a_{32}) - \\ & - a_{12}[(a_{21} - a_{31})(a_{44} + a_{55}) - (a_{22} + a_{33})(a_{41} - a_{51})] - \dots \\ & \dots - a_{12}[a_{21}(a_{32} + a_{33}) - a_{31}(a_{22} + a_{23}) - a_{41}(a_{32} + a_{55}) + a_{51}(a_{23} - a_{44})] \end{aligned}$$

$$\begin{aligned} b_4 =& a_{11}(a_{22}a_{33} - a_{23}a_{32})(a_{44} + a_{55}) + [a_{11}(a_{22} + a_{33}) + a_{22}a_{33} - a_{23}a_{32}](a_{44}a_{55} - a_{23}a_{32}) - \dots \\ & \dots - a_{12}[(a_{21} - a_{31})(a_{44}a_{55} - a_{23}a_{32}) - (a_{41} - a_{51})(a_{22}a_{33} - a_{23}a_{32})] - \dots \\ & \dots - a_{12}(a_{44} + a_{55})[a_{21}(a_{32} + a_{33}) - a_{31}(a_{22} + a_{23})] + \dots \\ & \dots + a_{12}(a_{22} + a_{33})[a_{41}(a_{32} + a_{55}) - a_{51}(a_{23} + a_{44})] \end{aligned}$$

$$\begin{aligned} b_5 =& a_{11}(a_{22}a_{33} - a_{23}a_{32})(a_{44}a_{55} - a_{23}a_{32}) - \\ & - a_{12}[a_{21}(a_{32} + a_{33}) - a_{31}(a_{22} + a_{23})](a_{44}a_{55} - a_{23}a_{32}) \dots + \\ & \dots + a_{12}(a_{22}a_{33} - a_{23}a_{32})[a_{41}(a_{32} + a_{55}) - a_{51}(a_{23} + a_{44})] \end{aligned}$$

According to the Routh - Hurwitz theorem, all the real parts of the roots λ_i of Eq.(13) are positive, that implies the corresponding steady-state solutions are stable, provided that:

$$b_1 > 0; \quad b_1 b_2 - b_3 > 0; \quad b_3(b_1 b_2 - b_3) - b_1(b_1 b_4 - b_5) > 0;$$

$$(b_3 b_4 - b_2 b_5)(b_1 b_2 - b_3) - (b_1 b_4 - b_5)^2 > 0; \quad b_5 > 0$$

In resonnance, by varying separately ξ and σ_b in their possible ranges, the stabi of the obtained OB curves is numerically checked with $\chi_j = 10^{-2}s^{-1}, B = 10^{-3}s^{-1}$ $\gamma = 10^8 s^{-1}$. The results are summarized in Fig. 3. The point and plus marks repres unstable and stable solutions, respectively. One can see that the lower and upper branc steadily exhibit the stability, whereas the whole middle branch is always unstable for ev appropriate set of parameter values. In the truncated OB region and the results show t except the middle branch, the remained OB curve is always stable. For given values ξ and σ_b in the bistable phase domain, we also check the stability of the non-resonn steady-state solutions. The results show that the more the LSA is detuned, the more OB curves are contracted and shifted towards higher values of σ_a, but still keep the sa stability behaviour as in resonance (Fig. 4).

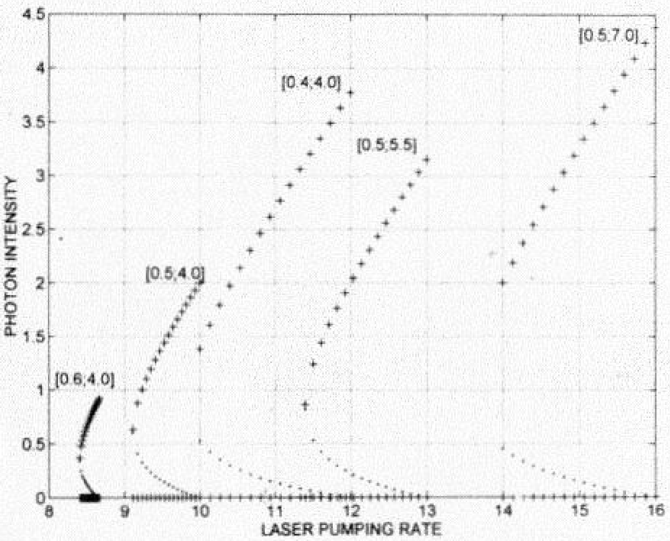

Fig. 3. *Stability of resonant hysteresis curves at various sets of* ξ, σ_b

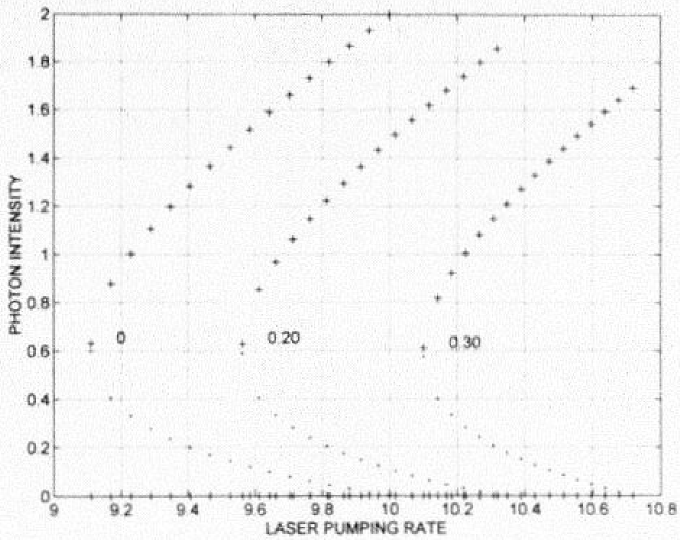

Fig. 4. *Stability of hysteresis curves at various values of* $\Delta\Omega_j/\Gamma$ *for* $\xi = 0.5$ *and* σ_b =

In conclusion, we have demonstrated the occurrence of the OB effect and studied the principal OB characteristics in a homogeneously broadened LSA model by solution of the rate equations for dual two-level atom media in a single-mode, standing wave cavity. Once appeared, depending on LSA parameters, the corresponding hysteresis curve may have either full-shaped or partly truncated form. By increasing the absorber pumping rate and/or decreasing the saturation coefficient, one can get larger and wider full-shaped OB curves. The detuning, if any, not only increase the OB onset pumping rate, but decrease both the OB width and the OB height as well. In the sense of Lyapunov linear stability, no unstable section has been found on the upper OB branch as reported in LSA with inhomogeneous broadening. Thus, one may not observe simultaneously the passive Q-switching (PQS) and the OB in this kind of LSA as far as the used approximation holds.

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HIỆU ỨNG LƯỠNG ỔN ĐỊNH QUANG TRONG LSA FABRY-PEROT MỞ RỘNG ĐỒNG NHẤT

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Những tính chất dừng của hiệu ứng lưỡng ổn định quang học trong chế độ đơn mode của laser Fabry-Perot chứa chất hấp thụ bão hoà có mở rộng đồng nhất được nghiên cứu trên cơ sở gần đúng phương trình tốc độ khi để ý đến sự tạo hốc không gian. Hai dạng đường cong trễ điển hình cũng như những đặc trưng của chúng được xem xét đầy đủ và minh hoạ rõ ràng. Những kết quả trình bày trong bài báo này đúng trong trường hợp cường độ bức xạ laser không quá lớn