THE QUANTUM THEORY OF AMPLIFICATION OF SOUND (ACOUSTIC PHONONS) BY LASER WAVE IN NON- DEGENERATE SEMICONDUCTOR

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Abstract. *Based on the quantum transport equation for the electron- phonon system of semiconductors, the amplification of sound (acoustic phonons) by Laser wave with multiphoton absorption process is theoretically studied. The analytic expressions for the coefficient of amplification of sound (acoustic phonons) and the conditions of amplification of sound (acoustic phonons) in non-degenerate semiconductors and in the case with summation over all values with* $\ell\Omega$ *(*Ω*− a frequency of Laser wave,* $\ell = 0;\ \pm1;\ \pm2...$*) are obtained. The difference of the coefficient of amplification of sound (acoustic phonons) and the conditions of amplification of sound (acoustic phonons) in the case with multiphoton absorption from the case with monophoton absorption is discussed.*

I. INTRODUCTION

The theory of amplification of sound (acoustic phonons) by Laser wave $\vec{E}_0\mathrm{Sin}(\Omega t)$ (Ω - the frequency of Laser wave) in semiconductors has been studied [1,2,3]. In [1,2] the physics problem was restricted for degenerate semiconductors in the case of monophoton absorption. The results of works [1,2] indicate that the absorption coefficient of sound (acoustic phonons) can be negative in some regions of values of acoustic wave vector $\vec{q}$. That is the absorption coefficient of sound (acoustic phonons) changes into the coefficient of amplification of sound (acoustic phonons). In [3] the analytic expressions for the absorption coefficient of sound (acoustic phonons) have been obtained for the case of non-degenerate semiconductors with multiphoton absorption process, but in restricted values: $\lambda^2 \ll kTq^2/2m$, $\omega_{\vec{q}} \gg q^2/2m$, $\omega_{\vec{q}} \ll kT$ and $\lambda \gg kT$ (m and e - the mass and charge of electron ; $\lambda \equiv eE_0/m\Omega$; k - the Boltzman constant ; T - the temperature; $\omega_{\vec{q}}$ - the energy of acoustic phonons ($\hbar = 1$)) and the condition of amplification of sound has not been obtained .

In order to continue the ideas of [1,2,3], in this paper we consider theoretically the amplification of sound (acoustic phonons) by Laser wave in non-degenerate semiconductors with multiphoton absorption process (in the case with summation over all values $\ell\Omega$, $\ell = 0, \pm1, \pm2, ...$) for the value $\omega_{\vec{q}}$, λ which is not restricted by the condition in [3] and we obtain the condition of amplification of sound.

II . THE QUANTUM TRANSPORT EQUATION FOR PHONONS

Hamiltonian of electron-phonon system of semiconductors with the presence of Laser wave $\vec{E}_0\sin(\Omega t)$ has the form

$$H(t) = \frac{1}{2m}\sum_{\vec{p}}\left[\vec{p} - \frac{e}{c}\vec{A}(t)\right]^2 a_{\vec{p}}^+ a_{\vec{p}} + \sum_{\vec{q}}\omega_{\vec{q}}\, b_{\vec{q}}^+ b_{\vec{q}} + \sum_{\vec{p},\vec{q}} C_{\vec{q}}\, a_{\vec{p}}^+ a_{\vec{p}+\vec{q}}(b_{\vec{q}} + b_{-\vec{q}}^+), \qquad (1)$$

where: $a_{\vec{p}}^+$ and $a_{\vec{p}}$ ($b_{\vec{q}}^+$ and $b_{\vec{q}}$) - the creation and annihilation operators of electrons (phonons); $\vec{p}$ - the vector of momentum of electron; c - the velocity of light ; $C_{\vec{q}}$ the interaction constant of electron- acoustic phonon scattering; $\vec{A}(t)$ - the vector potential $\left(-\frac{1}{c}\frac{d\vec{A}(t)}{dt} = \vec{E}_0\text{Sin }(\Omega t)\right)$.

Proceed from (1) and used method of [4,5,6] the quantum transport equation for phonons in semiconductors with the presence of laser wave has the form

$$\frac{\partial\langle b_{\vec{q}}\rangle_t}{\partial t} + i\omega_{\vec{q}}\langle b_{\vec{q}}\rangle_t = \left|C_{\vec{q}}\right|^2\sum_{\vec{p}}(n_{\vec{p}} - n_{\vec{p}-\vec{q}})\int_{-\infty}^{t} dt_1\langle b_{\vec{q}}\rangle_{t_1}\times$$

$$\times\sum_{\ell,s=-\infty}^{\infty}\exp\left\{i(\varepsilon_{\vec{p}} - \varepsilon_{\vec{p}-\vec{q}})(t_1 - t) - i\ell\Omega t_1 + is\Omega t\right\}J_\ell\,(\vec{a}\vec{q})J_s(\vec{a}\vec{q}), \qquad (2)$$

where: the symbol $\langle x\rangle_t$ means the averaging of statistics of operator x; $n_{\vec{p}}$ the distribution function of electron; $\varepsilon_{\vec{p}}$ the energy of electron; $\vec{a} = (e\vec{E}_0/m\Omega^2)$; $J_\ell(z)$ - the Bessel function with real argument.

III. AMPLIFICATION OF SOUND (ACOUSTIC PHONONS) IN NON-DEGENERATE SEMICONDUCTORS IN MONOPHOTON ABSORPTION PROCESS

By using the Fourier transformation from (2) we find

$$(\omega - \omega_{\vec{q}})B_{\vec{q}}(\omega) = \left|C_{\vec{q}}\right|^2\sum_{\ell,s=-\infty}^{\infty} J_\ell(\vec{a}\vec{q})J_s(\vec{a}\vec{q})\Pi_{\vec{q}}(\omega + \ell\Omega)B_{\vec{q}}(\omega - \ell\Omega + s\Omega), \qquad (3)$$

where

$$\Pi_{\vec{q}}(\omega + \ell\Omega) = \sum_{\vec{p}}\frac{n_{\vec{p}-\vec{q}} - n_{\vec{p}}}{\varepsilon_{\vec{p}-\vec{q}} - \varepsilon_{\vec{p}} + (\omega + \ell\Omega) + i\delta}\,, (\delta \to +0). \qquad (4)$$

In the case of $\ell = s$ from (3) we have dispersion equation

$$(\omega - \omega_{\vec{q}}) - \left|C_{\vec{q}}\right|^2\sum_{\ell=-\infty}^{\infty} J_\ell^2(\vec{a}\vec{q})\Pi_{\vec{q}}(\omega + \ell\Omega) = 0, \qquad (5)$$

and absorption coefficient of sound (acoustic phonons)

$$\alpha(\vec{q}) = \pi\left|C_{\vec{q}}\right|^2\sum_{\ell=-\infty}^{\infty} J_\ell^2(\vec{a}\vec{q})\sum_{\vec{p}}(n_{\vec{p}} - n_{\vec{p}+\vec{q}})\,\delta\left(\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} - \omega_{\vec{q}} - \ell\Omega\right), \qquad (6)$$

with $\delta(x)$- the Dirac delta function.

Suppose that the argument of the Bessel function is small so that $\vec{a}\vec{q} \equiv \lambda/\Omega \ll 1$. Passing from summation to integration over $\vec{p}$ in (6) the absorption coefficient of sound (acoustic phonons) for monophoton absorption process (in the case with summation over only values $\ell\Omega$ with $\ell = 0, \pm 1$) has the form

$$\alpha(\vec{q}) = \frac{n_0 \xi^2}{\rho s}\left(\frac{\pi m}{2kT}\right)^{1/2}\left\{\left(\frac{\lambda}{2\Omega}\right)^2\left[\exp(2S_{\vec{q}}\omega_{\vec{q}}\Omega)\mathrm{sh}\left(\frac{\omega_{\vec{q}}-\Omega}{2kT}\right) + \exp(-2S_{\vec{q}}\omega_{\vec{q}}\Omega)\right.\right.$$
$$\left.\left.\times\, \mathrm{sh}\left(\frac{\omega_{\vec{q}}+\Omega}{2kT}\right)\right]\exp\left(-S_{\vec{q}}(\omega_{\vec{q}}^2 + \Omega^2 + \frac{q^4}{4m^2})\right)\right\}, \quad (7)$$

where $S_{\vec{q}} = m/(2q^2KT)$; n_0-the density of electron; ξ - the constant of deformation potential; s -velocity of sound ; ρ - the density of crystal.

If $\omega_{\vec{q}}/\Omega\rangle 1$ we have $\alpha(\vec{q})\rangle 0$. If $\omega_{\vec{q}}/\Omega \ll 1$ we have $\alpha(\vec{q})\langle 0$ and which has the form

$$\alpha(\vec{q}) = -\frac{n_0 \xi^2}{\rho s}\left(\frac{\pi m}{2kT}\right)^{1/2}\left\{\left(\frac{\lambda}{2\Omega}\right)^2 \mathrm{sh}\left(\frac{\Omega}{2kT}\right)\mathrm{sh}(2S_{\vec{q}}\omega_{\vec{q}}\Omega)\right.$$
$$\left.\times\, \exp\left[-S_{\vec{q}}\left(\omega_{\vec{q}}^2 + \Omega^2 + \frac{q^4}{4m^2}\right)\right]\right\}. \quad (8)$$

It means that we have the coefficient of amplification of sound (acoustic phonons) .

IV. AMPLIFICATION OF SOUND (ACOUSTIC PHONONS)IN NON-DEGENERATE SEMINCONDUCTORS IN MULTIPHOTON ABSORPTION PROCESS

From (6) we have

$$\alpha(\vec{q}) = \pi\left|C_{\vec{q}}\right|^2 \sum_{\ell=-\infty}^{\infty} J_\ell^2(\vec{a}\vec{q})\sum_{\vec{p}} n_{\vec{p}}\{\delta(\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} - \omega_{\vec{q}} - \ell\Omega) - \delta(\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} + \omega_{\vec{q}} - \ell\Omega)\}\,. \quad (9)$$

In the case of $\lambda\rangle\Omega$ used changing formula in [6,7] :

$$\sum_{\ell=-\infty}^{\infty} J_\ell^2\left(\frac{\lambda}{\Omega}\right)\delta(\varepsilon_{\mp} - \ell\Omega) = \frac{\theta(\lambda^2 - \varepsilon_{\mp}^2)}{\pi(\lambda^2 - \varepsilon_{\mp}^2)^{1/2}}\,] \,, \quad (10)$$

where

$$\varepsilon_{\mp} \equiv \varepsilon_{\vec{p}\pm\vec{q}} - \varepsilon_{\vec{p}} \mp \omega_{\vec{q}} \quad ; \quad \theta(z) = \begin{cases} 1 & \text{if} \quad z\rangle 0 \\ 0 & \text{if} \quad z\langle 0, \end{cases}$$

we have

$$\alpha(\vec{q}) = \left|C_{\vec{q}}\right|^2 \sum_{\vec{p}} n_{\vec{p}}\left\{\frac{\theta(\lambda^2 - \varepsilon_-^2)}{(\lambda^2 - \varepsilon_-^2)^{1/2}} - \frac{\theta(\lambda^2 - \varepsilon_+^2)}{(\lambda^2 - \varepsilon_+^2)^{1/2}}\right\}. \quad (11)$$

Passing from summation to integration over $\vec{p}$ in (11) the analytic expression for the absorption coefficient of sound (acoustic phonons) by Laser wave for multiphoton absorption process (in the case with summation over all values $\ell\Omega$ with $\ell = 0, \pm 1, \pm 2, ...$) is following

$$\alpha(\vec{q}) = \frac{\xi^2 n_0}{2\rho s}\left(\frac{m}{2kT}\right)^{1/2}\exp\left(-\frac{m\lambda^2}{2q^2kT}\right)\exp\left[-\frac{m}{2q^2kT}\left(\frac{q^2}{2m}-\omega_{\vec{q}}\right)^2\right]\sum_{j=0}^{\infty}\frac{\Gamma\left(j+\frac{1}{2}\right)}{J!}\times$$
$$\times\left\{A(\omega_{\vec{q}}) - \exp\left(-\frac{\omega_{\vec{q}}}{kT}\right)A(-\omega_{\vec{q}})\right\}, \quad (12)$$

where : $A(\omega_{\vec{q}}) = \left(\frac{\lambda}{\frac{q^2}{2m}-\omega_{\vec{q}}}\right)^j I_j\left[\frac{\lambda m}{q^2kT}\left(\frac{q^2}{2m}-\omega_{\vec{q}}\right)\right]$; $I_j(z)$ - the Bessel function with complex argument; $\Gamma(z)$ - the Gamma function.

It is important to note that the expression for the absorption coefficient of sound (acoustic phonons) is calculated exactly by direct summation over all values $\ell\Omega$. Futhermore, if

$$A(\omega_{\vec{q}}) > \exp\left(-\frac{\omega_{\vec{q}}}{kT}\right)A(-\omega_{\vec{q}}), \quad (13)$$

we have $\alpha(\vec{q})\rangle 0$ and if

$$A(\omega_{\vec{q}}) < \exp\left(-\frac{\omega_{\vec{q}}}{kT}\right)A(-\omega_{\vec{q}}), \quad (14)$$

we have $\alpha(\vec{q})\langle 0$ and which has the form

$$\alpha(\vec{q}) = -\frac{\xi^2 n_0}{2\rho s}\left(\frac{m}{2kT}\right)^{1/2}\exp\left(\frac{-m\lambda^2}{2q^2kT}\right)\exp\left[-\frac{m}{2q^2kT}\left(\frac{q^2}{2m}-\omega_{\vec{q}}\right)^2\right]\sum_{j=0}^{\infty}\frac{\Gamma\left(j+\frac{1}{2}\right)}{J!}\times$$
$$\times\left\{\exp\left(-\frac{\omega_{\vec{q}}}{kT}\right)A(-\omega_{\vec{q}}) - A(\omega_{\vec{q}})\right\}. \quad (15)$$

It means that we have again the coefficient of amplification of sound (acoustic phonons).

From the analytic formula (12) we make analyze the absorption coefficient of sound in limit cases: $\omega_{\vec{q}} \gg q^2/2m$, $\lambda \gg kT$, $\omega_{\vec{q}} \ll kT$ and $\lambda^2 \ll k_0Tq^2/2m$ then the argument of the Bessel function is bigger than 1 , the summation only value $j = 0$ and $I_0(z) \approx e^z\sqrt{2\pi z}$. Our result for the absorption coefficient of sound in this limit cases is the same result of [3].Not that if the condition of amplification of sound (14) is satisfied then the absorption coefficient of sound changes into the coefficient of amplification of sound.

We numerically evaluate and plot the analytic formula (7) for the case of monophoton absorption (Fig.1) and (12) for the case of multiphoton absorption (Fig.2) in the same condition. The absorption coefficient of sound (acoustic phonons) is plotted as a function of the frequency of Laser wave (Ω) and of the frequency of sound (acoustic phonon) $(\omega_{\vec{q}})$.

From the graphics we can see

- For the monophoton absorption : when $\omega_{\vec{q}}/\Omega > 1$ we have the absorption coefficient of sound (acoustic phonons) $\alpha(\vec{q}) > 0$.

- In the same condition for the case of multiphoton absorption we have again the coefficient of amplification of sound (acoustic phonons) $\alpha(\vec{q}) < 0$.

Note that, the dependence of the coefficients of amplification of sound (acoustic phonons) on the frequency of Laser wave (Ω) and on the frequency of sound (acoustic phonons) $(\omega_{\vec{q}})$ is nonlinear and complicated.

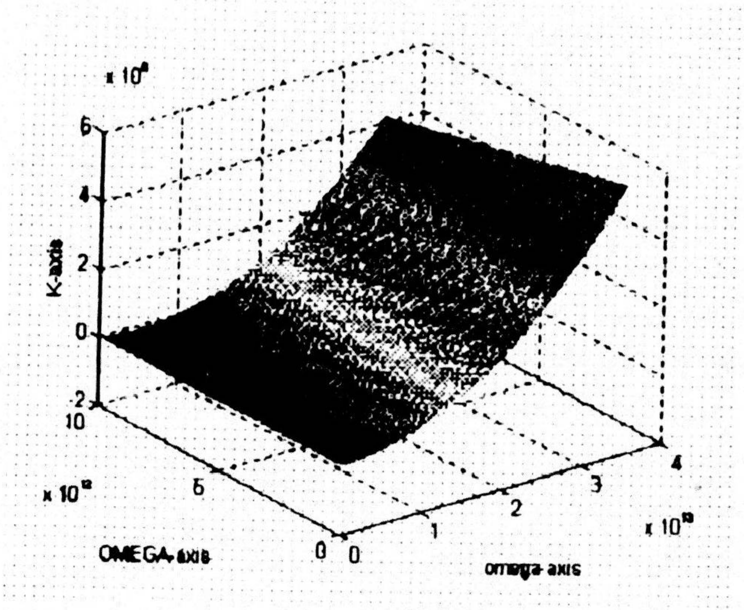

Fig 1: The coefficient of amplification of sound (acoustic phonons)
in the case of monophoton absorption
K- axis $\equiv \alpha(\vec{q})$- axis ; **omega** - axis $\equiv \omega_{\vec{q}}$ - axis ; **OMEGA** - axis $\equiv \Omega$ - axis .

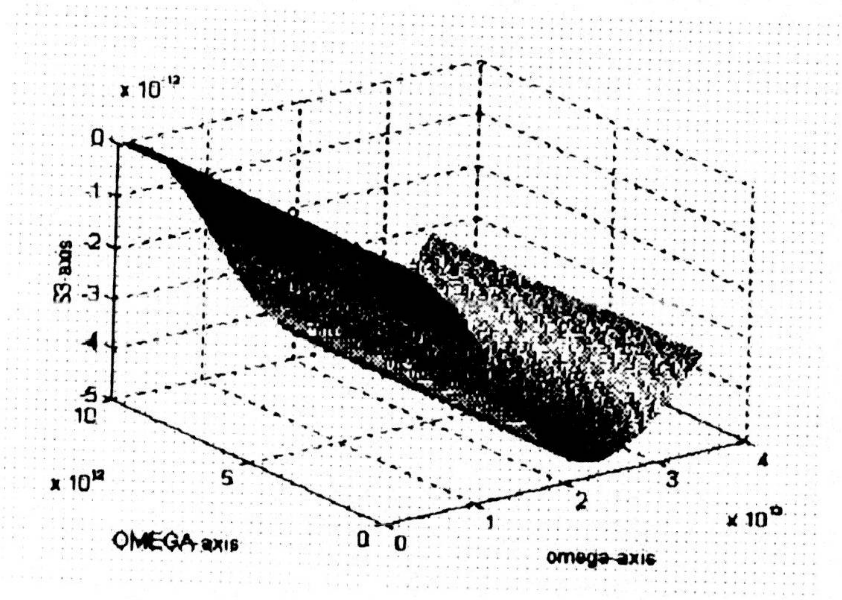

Fig 2: The coefficient of amplification of sound (acoustic phonons)
in the case of multiphoton absorption
S3- axis $\equiv \alpha(\vec{q})$- axis ; **omega** - axis $\equiv \omega_{\vec{q}}$ - axis ; **OMEGA** - axis $\equiv \Omega$ - axis.;

V. CONCLUSION

In the conclusion, we want to emphasize that :

1. The analytic expressions for the condition $\omega_{\vec{q}}/\Omega \ll 1$ and the coefficient of amplification of sound (acoustic phonons) (8) in the case of monophoton absorption and for the condition and coefficient of amplification of sound (acoustic phonons) (14) , (15) in the case of multiphoton absorption are obtained by us first time. In the limit cases $\lambda^2 \ll kTq^2/2m, \omega_{\vec{q}} \gg q^2/2m$; and $\omega_{\vec{q}} \ll kT$, $\lambda \gg kT$ the absorption coefficient of sound with multiphoton absorption process (12) return to the result of [3].

2. In the case of monophoton absorption (Section 3) : our results are different from results of [1,2]. The reason of difference is that results of [1,2] for the case of degenerate semiconductors, but our results for the case of non-degenerate semiconductors.

3. In the multiphoton absorption (Section 4) : the expressions for the condition and the coefficient of amplification of sound (acoustic phonons) (14), (15) show that the different dependencies of that ones in comparison with results of [1,2] and Section 3. The reason of difference is that results of [1,2] and section 3 for the case of monophoton absorption, but the expressions (14) and (15) for the case of multiphoton absorption.

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LÝ THUYẾT LƯỢNG TỬ VỀ SỰ GIA TĂNG SÓNG ÂM (PHONO ÂM) BỞI SÓNG LASER TRONG BÁN DẪN KHÔNG SUY BIẾN

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Trên cơ sở phương trình động lượng tử do hệ điện tử - phonon của bán dẫn, nghiên cứu lý thuyết sự gia tăng sóng âm (phonon âm) bờ sóng Laser có kể đến quá trình hấp thụ nhiều photon. Thu được biểu thức giải tích cho hệ số gia tăng sóng âm (phonon âm) và điều kiện gia tăng sóng âm (phonon âm) trong trường hợp có kể đến tính tổng theo tất cả các đại lượng chứa $\ell\Omega(\Omega$ – tần số sóng Laser, $\ell = 0, \pm1; \pm2; ...)$. Thảo luận về sự khác nhau của hệ số gia tăng sóng âm (phonon âm) trong trường hợp hấp thụ nhiều photon so với trường hợp hấp thụ một photon