THE RELATIVISTIC OPERATOR QUANTIZATION OF LINEARIZED GRAVITY THEORY

Nguyen Suan Han
Faculty of Physics - College of Natural Sciences - VNU

1. Quantization of the linearized gravity gives an evidence about the importance of physically motivated assumptions for the small metric - tensor components to be neglected, which concerns with the existence of gravitational waves in the conventional understanding of this problem.

Gravitational wave in gravity theory are considered as quantum excitations of weak classical fields. In this context, the construction of a gravity quantization scheme which is adequate to the problem of elementary excitations is important. From such a point of view the relativistic operator quantization method(*) with an explicit solution of the constraint equation [1, 2] is distinguished among the large variety of gravitational field quantization approaches [3, 4].

2. Consider the action for Einstein gravity theory

$$S = \int \sqrt{-g} d^4x. \tag{1}$$

Where R is scalar curvature $g = det\sqrt{-g}\ g^{\mu\nu}(x)$; $g^{\mu\nu}(x)$ is the inverse of the tensor $g_{\mu v}(x)$. In a weak field approximation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu v}, \quad h_{\mu\nu} << l. \tag{2}$$

The variables $h_{\mu\nu}(x)$ discribe the linearzed gravitational field, and $\mu\nu$ is the Minkovski metric tensor with diagonal $(1, -1, -1, -1)$. The Lagrangian then takes the form (up to $O(h^3)$ - terms)

$$\begin{aligned} L^2 &= \frac{1}{2}(h_{\mu\mu}\ h_{\nu\nu} - h_{\mu\nu}\ h_{\mu\nu}) + h_{\mu\nu}\partial_{nu}\partial_\sigma h_{\mu\sigma} - h_{\mu\mu}\partial_\nu\partial_\sigma h_{\nu\sigma} \\ L^{(2)}(h_{00}) &= h_{00}(\partial_k\partial_i h_{ki} - \partial_k^2 h_{jj}), \\ L^{(2)}(h_{0i}) &= -2\partial_0 h_{0i}(\partial_k h_{ki} - \partial_i h_{jj}) - \partial_k h_{0i}(\partial_i h_{0k} - \partial_k h_{0i}). \end{aligned} \tag{3}$$

This action contains constraints which introduce a transverse structure

$$\frac{\delta L^{(2)}}{\delta h_{00}} = 0 \Rightarrow \partial_k A_k^T = 0, \quad A_k^T = \partial_i h_{ki} - \partial_k h_{ij} \tag{4}$$

with the corresponding equation of motion

$$\frac{\delta L^{(2)}}{\delta h_{0i}} = 0 \Rightarrow \partial_0 A_k^T = \partial_i(\partial_i h_{0k} - \partial_k h_{0i}). \tag{5}$$

(*) For brevity this method can be called "minimal", because it is concerned with the quantization only of minimal number of physical degrees of freedom remaining after the explicit solution of a constraint on the classical level [1],[2] On the solution of the constraints (4) (5), Lagrangian (3) reads

$$L^{(2)} = \frac{1}{2}\partial^{\mu} h_{ik}\Lambda(ik \mid lm)\partial_{\mu} h_{lm}, \tag{6}$$

$$\Lambda(ik \mid lm) = \delta_{ik}\delta_{lm} + \delta_{il}\delta_{km} - \frac{\partial_i\partial_l}{\Delta}\delta_{km} - \frac{\partial_k\partial_m}{\Delta}\delta_{il},$$

where the projection operator $\Lambda(ik|lm)$ can be considered as defining the distance in the space of dynamical field h_{ik} orbits with respect to infinitesimal gauge transformations

$$h_{ik} \to h_{ik} + \partial_i\lambda_k + \partial_k h_i. \tag{7}$$

From Lagrangian (6), canonical momenta is obtained

$$\rho_{rs} = \Lambda(rs|lm)\partial_0 h_{lm}(x) \tag{8}$$

which obey the following commutation relations

$$\left[h_{lm}(x), \rho_{rs}(y)\right] = \Lambda(lm|rs)(x)\delta(x-y). \tag{9}$$

The energy - momentum tensor is obtained to be symmetric amd gauge invariant

$$T^{(c)}_{\mu\nu} = \partial_\mu h_{ik}\Lambda(ik|lm)\partial_\nu h_{lm} - \frac{1}{2}\eta_{\mu\nu}\partial^\sigma h_{ik}\Lambda(ik|lm)\partial_\sigma h_{lm}. \tag{10}$$

It does not represent a full derivative and gives rise to a set of Poincaré - group generators in which boost generators induce and additional gauge transformation of the dynamical fields

$$i\left[M_{i0}, h_{rs}(x)\right] = (x_0\partial_i - x_i\partial_0)\Lambda(rs|lm)h_{lm}(x) + \left(\partial_r\frac{1}{\Delta}\rho_{is} + \partial_m\frac{1}{\Delta}\rho_{rl}\right). \tag{11}$$

This additional gauge transformation leads to a time - axis rotation that ensure the relativistic covariance of this manifestly non-covariant quantization procedure.

The basis in this space can be defined as

$$\begin{aligned} \Lambda(ik|lm) &= \varepsilon^a_{ik}\varepsilon^a_{lm}, \\ \varepsilon^a_{ik}\Lambda(ik|lm)\varepsilon^b_{lm} &= \delta^{ab}, \\ \varepsilon^a_{ik}P_{ik} &= 0, \quad a,b = 1,2. \end{aligned} \tag{12}$$

where P_{ik} is the three - dimensional projection operator

$$P_{ik} = (\delta_{ik} - \frac{\partial_i\partial_k}{\Lambda}), \tag{13}$$

$$P_{ik} = e^\alpha_i e^\alpha_k, \quad e^\alpha_i P_{ik} e^\beta_k = \delta^{\alpha\beta}, \quad \alpha,\beta = 1,2.$$

Thus, the relevant polarizations are found to be

$$\varepsilon^1_{ik} = e^1_i e^1_k - e^2_i e^2_k, \quad \text{and} \quad \varepsilon^2_{ik} = e^1_i e^2_k, \tag{14}$$

and for the independent physical variables

$$h^a = \varepsilon^a_{ik}\Lambda(ik|lm)h_{lm}, \tag{15}$$

the free two - component scalar field Lagrangian is obtained

$$L^{(2)} = \frac{1}{2}\partial^\mu h^a \partial_\mu h^a; \tag{16}$$

hence, plane waves are present in the excitation spectrum of the linearized gravity theory.

Therefore, minimal quantization of weak gravitational fields reproduces the radiational - gauge results together with corresponding additional conditions which are in fact generated by the equations of motion for the nondynamical fields h_{00} and h_{0i}.

For the original theory (1) without any additional assumptions about the fields minimal quantization consists of excluding nonphysical degrees of freedom through the exact solutions of their equations of motion (constraints). However, the coincidence of the linearized expansion of the action obtained with the one considered above (in the naive linearization scheme) is by no means obvious, the reason being the distinguished role of the Newton component g_{00}. Thus, assuming that condition (2) concerns only dynamical fields h_{ik}, we are forced to consider also components h_{i0} as small variables because of the constraints, but no restriction is imposed on the Newton component h_{00}. In the minimal quantization method, a component h_{00} is considered as a classical one and assumption $h_{00} << 1$ [5] is by means motivated because we don't know the strength of the Newton potential the gravitational wave is interacting with.

With the help of the relativistic operator quantization method the theory of linearized gravitational field is formulated in a manifest relativistic - covariant form providing its straightforward quantization with same transformation properties of the quantized fields with respect to the Lorentz - group action as classical theory. The Lagrangian (16) obtained describes an unconstrained hamiltonian system.

The author would like to thank Profs V.N. Pervushin, L. Litov, and Ilieva N. For stimulating discussions. The financial support of the National Basic Research Program in Natural Sciences KT-04 is highly appreciated.

REFERENCES

[1] Nguyen Suan Han, V.N Pervushin, *Mod. Phys. Lett.* A N$_o$**2**(1987); *Fortschr. Phys.* N$_o$**8**(1989)611; *Can.J. Phys.* **69**(1991)pp.684 - 691.
[2] Nguyen Suan Han. *ICTP, IC/95/17, Trieste* (1995)pp.1.
[3] P.A.M. Dirac. *Proc. Roy. Soc.A*, **246**(1958)pp.333.
[4] R. Arnowitt, S. Deser, C. Misner. *Phys. Rev.***117**(1960)pp.1596.
[5] N. Ilieva, L.Litov, V.N.Pervushin. *JINR*. E2-90-507.

TẠP CHÍ KHOA HỌC ĐHQGHN, KHTN, t.XV, n^{0}2 - 1999

ÁP DỤNG PHƯƠNG PHÁP LƯỢNG TỬ HÓA TOÁN TỬ TƯƠNG ĐỐI TÍNH CHO TRƯỜNG HẤP DẪN TUYẾN TÍNH

Nguyễn Xuân Hãn

Khoa Vật lý - Đại học KH Tựnhiên - ĐHQG HàNội

Áp dụng phương pháp lượng tử hóa toán tử tương đối tính cho trường hấp dẫn tuyến tính ở gần đúng trường yếu. Bằng việc giải chính xác phương trình liên kết để cho thành phần Newton qua các thành phần vật lý khác chúng ta đã thu được Lagrangian (16) mô tả hệ gồm những thành phần vật lý độc lập với nhau.