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RESEARCH USING THE 2-D MODEL TO EVALUATE THE CHANGES OF RIVERBED

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I. Introdution

Bed erosion problem was studied and researched in many places all over the world. Many methods and bed deformation models were built to solve the practical problems. In Vietnam, some models such as HEC-6, MIKE11... were used to analyze and compute the river erosion. But most of them were 1-D models, only computing bed erosion with the assumption of constant erosion depth over the cross-section that couldn't investigate the sediment transportation and non-regular erosion processes in orthogonal direction. Some 2-D hydraulic models as TELEMAC or MIKE21 have only focused on the distribution of water flow velocity but the sediment processes.

Recently rivers of Vietnam have been strongly scoured in both stream-wise and orthogonal directions, in many regions, the river bank erosion is very important, because they have affected on many terms of social and human life. On Red river system erosion was serious, especially after the Hoa Binh Hydropower Plant the river bed erosion becomes more serious. Thus it is necessary to understand and simulate this process using 2-D model.

The Two-dimensional Riverbed Evolution Model- TREM - was constructed in the non-orthogonal curvilinear coordinate system by N. Izumi and N. T. Giang. Model used Finite Control Volume (FCV) method and implicit scheme of Crank-Nicolson. The results of the model are the values of bed elevation, velocity field and sediment concentration at the grid nodes, respectively with each computation time step. Then by using bank stability analysis the riverbank erosion and bank line shift can be determined.

II. Theoritical base of model

1. Basic Equations

a. Fluid flow equations

In Cartesian coordinate, the depth-averaged two-dimensional shallow-water equations include the continuity equation and 2 momentum equations:

$$\frac{\partial h}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0 \tag{1.1}$$

$$\frac{\partial M}{\partial t}+\frac{\partial uM}{\partial x}+\frac{\partial vM}{\partial y}=-gh\frac{\partial Z_s}{\partial x}-\frac{\tau bx}{S}+\frac{\partial}{\partial x}\left(-\overline{u'^2}h\right)+\frac{\partial}{\partial y}\left(-\overline{u'v'h}\right) \tag{1.2}$$

$$\frac{\partial \dot{N}}{\partial t}+\frac{\partial uN}{\partial x}+\frac{\partial vN}{\partial y}=-gh\frac{\partial Z_s}{\partial x}-\frac{\tau by}{S}+\frac{\partial}{\partial y}\left(-\overline{v'^2}h\right)+\frac{\partial}{\partial x}\left(-\overline{u'v'h}\right) \tag{1.3}$$

where: t: time; x,y: the streamwise and lateral coordinates, respectively

h: the water depth; z_s: the water level,

ρ:the water density,

g: gravity acceleration(=9.81 m/s^2),

M,M: x,y components of discharge flux vector,

u,v: x,y components of the depth-averaged velocity vectors,

τ_{bx}, τ_{by} : x,y components of the bed shear stress respectively,

$-\overline{u'^2},-\overline{u'v'},-\overline{v'^2}$: x,y components of depth-averaged Reynolds stress tensors,

$$-\overline{u'^2}=2D_h\left(\frac{\partial u}{\partial x}\right)-\frac{2}{3}K \tag{1.4}$$

$$-\overline{u'v'}=D_h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}\right) \tag{1.5}$$

$$-\overline{v'^2}=2D_h\left(\frac{\partial u}{\partial y}\right)-\frac{2}{3}K \tag{1.6}$$

$$D_h=\alpha hu\ , \tag{1.7}$$

where: D_h: the eddy viscosity; k: depth-averaged turbulent energy,

α: constant; u.: the friction velocity($u_* = \sqrt{\frac{\tau}{\rho}}$, τ : the bed shear stress).

Transformation of the above three equations into non-orthogonal curvilinear coordinate can be found in Nagata (2000).

b. Sediment continuity equation

The sediment continuity equation in 2-D written for the layer extended from the bottom to bed surface in general coordinate system can expressed by:

$$J\frac{\partial(\eta)}{\partial Z}+\frac{1}{1-\lambda}\left[\frac{\partial\left(Jq_b^{\psi}\right)}{\partial\psi}+\frac{\partial\left(Jq_b^{\varphi}\right)}{\partial\varphi}+J(E_R-D_R)\right]=0 \tag{1.8}$$

where: η: bed elevation(water surface elevation subtract water depth),

ψ, φ: general coordinate axis,

λ: porosity of the bed material,

J: Jacobian of the transformation from Cartesian coordinate to non-orthogonal curvilinear coordinate system. It is computed by:

$$J=\begin{vmatrix} x_{\psi} & x_{\varphi} \\ y_{\psi} & y_{\varphi} \end{vmatrix}, \tag{1.9}$$

where: x_{ψ}, x_{φ} ,y_{ψ},y_{φ}: first partial derivatives of x, y,

q_b^{ψ} , q_b^{φ} : Bed load discharge per unit of width in ψ and φ, respectively.

They are calculated from the bed load discharge in s (streamwise direction), and n (the direction orthogonal to the streamwise direction) Processes of converting is presented as follows:

$$q_b^{\psi}=\frac{\partial_{\psi}}{\partial_s}q_b^s+\frac{\partial_{\psi}}{\partial n}q_b^n=\left(\psi_x\frac{\partial x}{\partial S}+\psi_y\frac{\partial y}{\partial S}\right)q_b^s+\left(\psi_x\frac{\partial x}{\partial n}+\psi_y\frac{\partial y}{\partial n}\right)q_b^n \tag{1.10}$$

$$q_b^{\varphi}=\frac{\partial_{\varphi}}{\partial_s}q_b^s+\frac{\partial_{\varphi}}{\partial n}q_b^n=\left(\varphi_x\frac{\partial x}{\partial S}+\varphi_y\frac{\partial y}{\partial S}\right)q_b^s+\left(\varphi_x\frac{\partial x}{\partial n}+\varphi_y\frac{\partial y}{\partial n}\right)q_b^n \tag{1.11}$$

where: ψ_x, ψ_y, φ_x, φ_y: first partial derivatives of ψ and φ.

After change, obtains:

$$q_b^{\psi}=\left(\psi_x\frac{u}{V}+\psi_y\frac{v}{V}\right)q_b^s+\left(-\psi_x\frac{v}{V}+\psi_y\frac{u}{V}\right)q_b^n=\frac{1}{VJ}\left(Ju_{\psi}^{\psi}q_b^s-u_{\varphi}q_b^n\right) \tag{1.12}$$

$$q_b^{\varphi}=\left(\varphi_x\frac{u}{V}+\varphi_y\frac{v}{V}\right)q_b^s+\left(-\varphi_x\frac{v}{V}+\varphi_y\frac{u}{V}\right)q_b^n=\frac{1}{VJ}\left(Ju_{\varphi}^{\varphi}q_b^s-u_{\psi}q_b^n\right) \tag{1.13}$$

c. Bed transport equations:

In order to accomplish the sediment transport continuity equation, the component of bed load transport in streamwise direction (s) and the direction orthogonal to (s) direction must be specified before hand. In the study, Ikeda's equations for sediment transport rate which couples the effect of spiral flow and the longitudinal slope of riverbed are adopted. Those equations have the form of:

$$q_b^{s*} = \frac{a^{1/2}}{\mu_C}\left[\tau^* - \tau_{co}(1 + \frac{1}{\mu_C}\frac{\partial\eta}{\partial s})\right]\left[\tau_{co}^{*1/2}(1| + \frac{1}{2\mu_C}\frac{\partial\eta}{\partial s})\right](1 - \frac{1}{\mu_C}\frac{\partial\eta}{\partial s}) \quad (1.14)$$

$$q_b^{n*} = \frac{a^{1/2}}{\mu_c}(\tau^* - \tau_{co}^*)(\tau^{*1/2} - \tau_{co}^{*1/2})\left[\frac{v_b^*}{u_b^*} - \frac{1}{\mu_c}(\frac{\tau_{co}^*}{\tau^*})^{1/2}\frac{\partial\eta}{\partial n}\right], \quad (1.15)$$

where: q_b^{s*}, q_b^{n*} : non-dimensional bed load sediment transport rate in (s) and (n) directions in the curvilinear coordinate system.

τ^* : non dimensional bed shear stress.

τ_{co}^*: non-dimensional critical bed shear stress, it can be computed by any method, in this study, the Iwagaki's formula (1958) is used,

μ_c: Coulomb friction factor, value of 0.7 was taken for computation,

u_b^*, v_b^*: the dimensionless slip velocity component in streamwise and transverse directions in the curvilinear (s,n) coordinate system.

All other symbols have been defied previously.

d. Transformation of bed load equations

In solving the continuity equation in the general non-orthogonal coordinate system, equation (1.14) and (1.15) should be transformed accordingly to (ψ,φ) coordinate instead of (s, n) coordinate. Each term in those equations are transformed subsequently as follows:

(1).Term $\frac{\partial\eta}{\partial s}$:
$$\frac{\partial\eta}{\partial s} = \frac{1}{V}(\frac{\partial\eta}{\partial\psi}U^\psi + \frac{\partial\eta}{\partial\varphi}U^\varphi) \quad (1.16)$$

(2).Term $\frac{\partial\eta}{\partial n}$:
$$\frac{\partial\eta}{\partial n} = \frac{1}{JV}(\frac{\partial\eta}{\partial\varphi}U^\psi - \frac{\partial\eta}{\partial\psi}U^\varphi)\ \frac{\partial\eta}{\partial n} = \frac{1}{JV}(\frac{\partial\eta}{\partial\varphi}U^\psi - \frac{\partial\eta}{\partial\psi}U^\varphi) \quad (1.17)$$

(3). Term $\frac{1}{r}$:
$$\frac{1}{r} = \frac{1}{V^3}\left[U^\psi\left(\frac{\partial v}{\partial\psi}u - \frac{\partial u}{\partial\psi}v\right) + U^\varphi\left(\frac{\partial v}{\partial\varphi}u - \frac{\partial u}{\partial\varphi}v\right)\right] \quad (1.18)$$

e. The continuity equation of suspended sediment

In Cartesian coordinate system, the continuity equation of suspended load has the form as described in following equation:

$$\frac{\partial(Ch)}{\partial t} + \frac{\partial(Q^xC)}{\partial x} + \frac{\partial(Q^yC)}{\partial y} - \frac{\partial}{\partial x}\left(h\varepsilon_x\frac{\partial C}{\partial x}\right) - \frac{\partial}{\partial y}\left(h\varepsilon_y\frac{\partial C}{\partial y}\right) - (E_R - D_R) = 0 \quad (1.19)$$

Using the assumption of locally constant diffusion coefficient in horizontal direction, resulting in a transformed equation:

$$J\frac{\partial}{\partial t}(Ch)+\frac{\partial}{\partial\psi}(JCQ^{\psi})+\frac{\partial}{\partial\varphi}(JCQ^{\varphi})-$$
$$-\frac{\partial}{\partial\psi}h\varepsilon_h(\frac{g_{22}}{J}\frac{\partial C}{\partial\psi}-\frac{g_{12}}{J}\frac{\partial C}{\partial\varphi})-\frac{\partial}{\partial\psi}h\varepsilon_h(\frac{g_{22}}{J}\frac{\partial C}{\partial\psi}-\frac{g_{12}}{J}\frac{\partial C}{\partial\psi})-,$$
$$-\frac{\partial}{\partial\varphi}h\varepsilon_h(\frac{g_{11}}{J}\frac{\partial C}{\partial\varphi}-\frac{g_{21}}{J}\frac{\partial C}{\partial\varphi})-J(E_R-D_R)=0,$$

where: C: suspended concentration at level z.

2. *Numerical solutions*

a. Concept of discretization in FVM

The basic of finite volume method (FVM) is based on the conservation rule applied for finite control volume. The genetic conservation equation for a scalar ϕ transported by the flow has the integral form of:

$$\underset{(1)}{\frac{\partial}{\partial t}\int_{\Omega}\rho\phi d\Omega}+\underset{(2)}{\int_{S}\rho\phi Vn dS}=\underset{(3)}{\int\Gamma grad\phi n dS}+\underset{(4)}{\int_{\Omega}q_{\phi}d\Omega} \quad (1.21)$$

In the equation (1.21), Ω and S are the volume of and surface enclosing CV, respectively,

n: unit vector orthogonal to surface S and direction outward, v is fluid velocity vector,

ρ: the density of mixture of water and suspended sediment,

Term (1) is the rate of change of the property within the control volume,

Term (2) is net flux of the quantity ϕ transported through the CV boundary by convective mechanism,

Term (3) is net flux of the quantity ϕ transported through the CV boundary by diffusive mechanism,

Term (4) is total sources or sinks of quantity ϕ occur within the CV.

The FVM's discretization involves in tow steps. The first step is approximation of integrals in equation (1.21) and the second step is the interpolation. The final outcome of discretization process is an algebraic system that needed to be solved by any conventional methods.. Generally speaking, FVM is an advanced approach of finite different method (FDM), where the mass conservative characteristic is strictly reserved for each CV surrounding a computational node.

Continuity equation of suspended sediment concentration in the general coordinate system has form:

$$J\frac{\partial}{\partial t}(Ch)+\frac{\partial}{\partial\psi}\left(JCQ^{\psi}\right)+\frac{\partial}{\partial\varphi}\left(JCQ^{\varphi}\right)-\frac{\partial}{\partial\psi}h\varepsilon_h\left(\frac{g_{22}}{J}\frac{\partial C}{\partial\psi}-\frac{g_{12}}{J}\frac{\partial C}{\partial\varphi}\right)-$$
$$-\frac{\partial}{\partial\varphi}h\varepsilon_h\left(\frac{g_{11}}{J}\frac{\partial C}{\partial\varphi}-\frac{g_{12}}{J}\frac{\partial C}{\partial\psi}\right)-J(E_R-D_R)=0 \quad (1.22)$$

Using Crank-Nicolson scheme in the integral, the corresponding form of equation (1.22) can be presented as:

$$J(Ch)_{i,j}^{n+1}+\frac{\Delta t}{2}\frac{\partial}{\partial\psi}\left(JCQ^{\psi}\right)_{i,j}^{n+1}+\frac{\Delta t}{2}\frac{\partial}{\partial\varphi}\left(JCQ^{\varphi}\right)_{i,j}^{n+1}-$$
$$-\frac{\Delta t}{2}\frac{\partial}{\partial\psi}h\varepsilon_h\left(\frac{g_{22}}{J}\frac{\partial C}{\partial\psi}-\frac{g_{12}}{J}\frac{\partial C}{\partial\varphi}\right)_{i,j}^{n+1}-$$
$$-\frac{\Delta t}{2}\frac{\partial}{\partial\varphi}h\varepsilon_h\left(\frac{g_{11}}{J}\frac{\partial C}{\partial\varphi}-\frac{g_{12}}{J}\frac{\partial C}{\partial\psi}\right)_{i,j}^{n+1}- \quad (1.23)$$
$$-\Delta tJ(E_R-D_R)_{i,j}^{n+1}-\frac{\Delta t}{2}\frac{\partial}{\partial\psi}(JCQ^{\psi})_{i,j}^{n+1}-\frac{\Delta t}{2}\frac{\partial}{\partial\varphi}(JCQ^{\varphi})_{i,j}^{n+1}+$$
$$+\frac{\Delta t}{2}\frac{\partial}{\partial\psi}h\varepsilon_h\left(\frac{g_{22}}{J}\frac{\partial C}{\partial\psi}-\frac{g_{12}}{J}\frac{\partial C}{\partial\varphi}\right)_{i,j}^{n+1}+\frac{\Delta t}{2}\frac{\partial}{\partial\varphi}h\varepsilon_h\left(\frac{g_{11}}{J}\frac{\partial C}{\partial\varphi}-\frac{g_{21}}{J}\frac{\partial C}{\partial\psi}\right)_{i,j}^{n+1}+J(Ch)_{i,j}^{n+1}=0\,.$$

b. Nine-diagonal coefficient matrix solver

From previously derivations, the suspended sediment transport equation in non-orthogonal coordinate system in discretized form written for control volume (i,j) is:

$$a_1C_{i-1,j}^{n+1}+a_2C_{i,j}^{n+1}+a_3C_{i+1,j}^{n+1}+a_4C_{i,j-1}^{n+1}+a_5C_{i,j+1}^{n+1}+$$
$$+a_6C_{i-1,j-1}^{n+1}+a_7C_{i-1,j+1}^{n+1}+a_8C_{i+1,j-1}^{n+1}+a_9C_{i+1,j+1}^{n+1}=b\,, \quad (1.24)$$

where: a_1-a_9 : are coefficients.

Equation (1.24) is generalized as:

$$\sum_{k=1}^{9}a_KC_K=b \quad (1.25)$$

The resulted system of equations involves unknown for single equation in each time step and has the form of band matrix. The algorithms for solving that system of equation can be any integration method. Hereby, the research adopted the line-by-line technique to solve those relevant equations.

c. Discretization of Exner's equation

Rewrite Exner's equation in the form:

$$J\frac{\partial \eta}{\partial t} = -\frac{1}{1-\lambda}\left[\frac{\partial\left(Jq_b^{\psi}\right)}{\partial \psi} + \frac{\partial\left(Jq_b^{\varphi}\right)}{\partial \varphi} + J\left(E_R - D_R\right)\right] \tag{1.26}$$

Applying the same rule of discretization, we have:

$$\left.\frac{\partial\left(Jq_b^{\psi}\right)}{\partial \psi}\right|_{(i,j)} = A_{i,j} = \frac{0.25}{\Delta\psi}\left\{\left(J_{i+1,j+1} + J_{i+1,j}\right)\left(q^{\psi}_{b\,i+1,j+1} + q^{\psi}_{b\,i+1,j}\right) - \left(J_{i,j+1} + J_{i,j}\right)\left(q^{\psi}_{b\,i,j+1} + q^{\psi}_{b\,i,j}\right)\right\} \tag{1.27}$$

$$\left.\frac{\partial\left(Jq_b^{\varphi}\right)}{\partial \varphi}\right|_{(i,j)} = B_{i,j} = \frac{0.25}{\Delta\varphi}\left\{\left(J_{i+1,j+1} + J_{i,j+1}\right)\left(q^{\varphi}_{b\,i+1,j+1} + q^{\varphi}_{b\,i,j+1}\right) - \left(J_{i+1,j} + J_{i,j}\right)\left(q^{\varphi}_{b\,i+1,j} + q^{\varphi}_{b\,i,j}\right)\right\}. \tag{1.28}$$

Substitute equation (1.27) , (1.28) into equation (1.26), obtains:

$$\Delta\eta = -\frac{\Delta t}{\left(1-\lambda\right)J^{\wedge}_{i,j}}\left[A_{i,j} + B_{i,j} + J^{\wedge}_{i,j}\left(E_{Ri,j} - D_{Ri,j}\right)\right]. \tag{1.29}$$

Equation (1.29) is the final descretized form of Exner'r equation. It is solved by explicit scheme. The outcome is the change in riverbed elevation at each time step at center of each computation grid. The new bed elevation is updated, and flow module is started computing for the next time step.

3. Determining measure of stream-bank erosion

Cross sections, after scouring, will create a new roof with greater slope. Using slip computation method of soil mechanics can determine measure of stream-bank erosion. Slip form can be flat slip or slip curve, under effect of slip and anti-slip forces:

$$\eta = \frac{\sum c_i l_i + \sum N_i tg\varphi_i}{\sum T_i}, \tag{1.30}$$

where: Numerator is anti-slip force, and denominator is slip force,

c_i: sticky force; l_i: length of i^{th} slip curve,

N_i: anti-slip force(shear direction),

T_i: slip force (normal direction)

In approximability form, applying principle of strong balance of Coulomb caan compute the needed slope to guarantee steadying strong balance:

$$tg\beta = tg\varphi + C/\gamma.H \quad (1.31)$$

where: β: slope of steadying strong balance,

φ: inside friction angle; C: sticky force,

H: Height of slope roof; γ: specific weight of sandy soil.

III. Test model to curved bend of Red river

The model was developed and applied for testing in SonTay curved bend of Red River (fig.1). In order to determine the upstream and downstream boundaryy, river network is routed by HEC-6 model. Topographical data is adopted fronm measured data during the end of 1997 and beginning 1998. The grid system werre generated by software GenGrid95 of CAFLAB (Yuengnam Univ.- South Koreaa). Time step for computation flow was 0.25 second and for sediment computation waas 2 second. Results of distribution of bed elevation, velocity field and depth of thne segment were shown in table 1 and fig. 2. Rating curve and bed change afteer computation are coincided with observed results at SonTay station that locateed within the segment. This is the preliminary test so the erosion-crumbling of bankks was not computed yet.

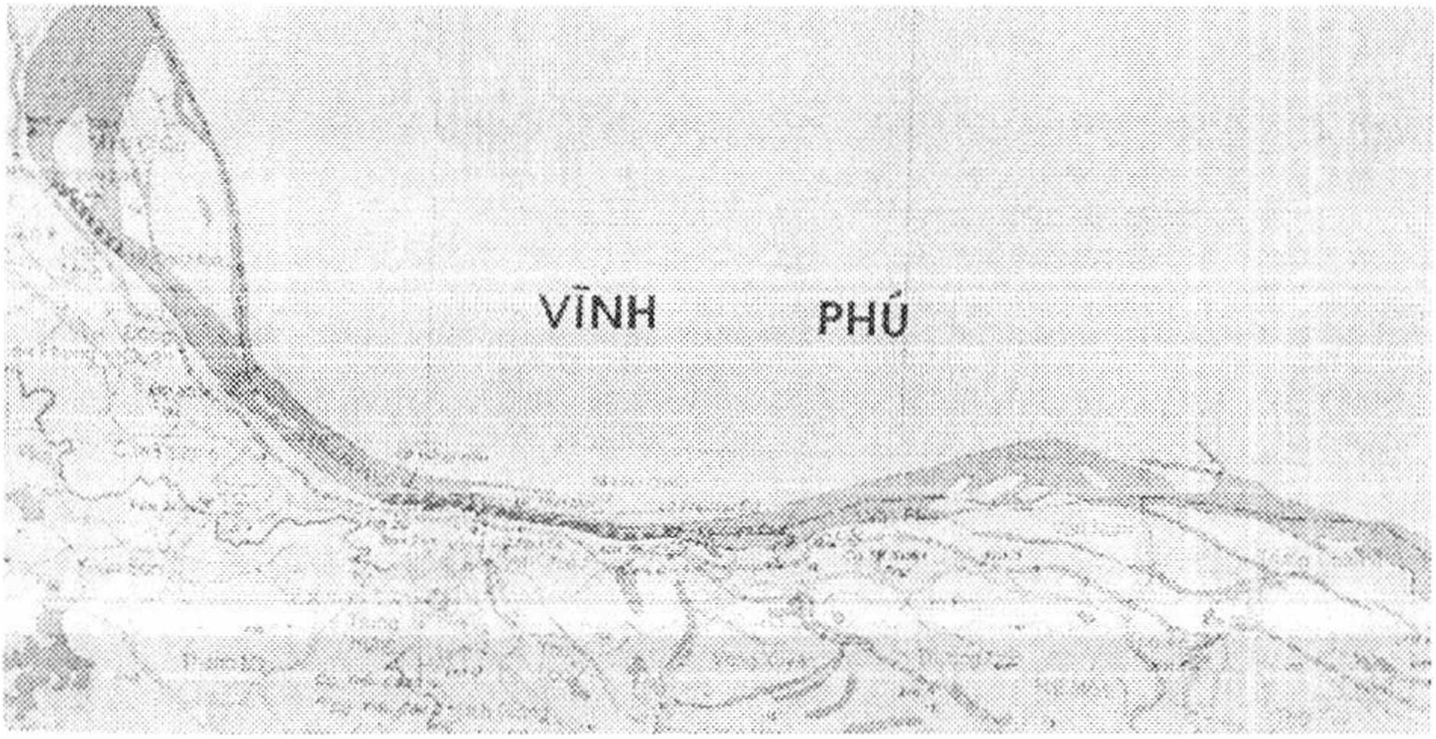

Figure 1. SonTay curved bend of Red River

Table 1. Computation results of distribution of velocity, depth and bed elevation of river reach

I	J	X	Y	Z	U	V	H
1	1	-8212.75	2989.2	8.526	0.434	-0.573	4.132
1	2	-8163.5898	3033.6699	2.273	0.604	-0.822	10.385
1	3	-8114.4399	3078.1399	0.808	0.801	-1.123	11.85
1	4	-8065.2798	3122.6001	1.516	0.8	-1.157	11.141
1	5	-8016.1299	3167.0701	2.489	0.741	-1.107	10.169
1	6	-7966.9702	3211.54	2.835	0.692	-1.068	9.822
1	7	-7917.8198	3256.01	2.872	0.666	-1.062	9.786
1	8	-7868.6602	3300.47	2.868	0.647	-1.067	9.79
1	9	-7819.5098	3344.9399	2.952	0.627	-1.072	9.705
1	10	-7770.3501	3389.4099	3.599	0.594	-1.051	9.058
1	11	-7721.2002	3433.8701	3.947	0.556	-1.02	8.711
1	12	-7672.04	3478.3401	4.762	0.516	-0.981	7.896
1	13	-7622.8901	3522.8101	6.355	0.45	-0.889	6.302
1	14	-7573.73	3567.28	7.228	0.383	-0.788	5.43
1	15	-7524.5801	3611.74	10.133	0.282	-0.603	2.525
1	16	-7475.4199	3656.21	10.132	0.102	-0.228	2.525

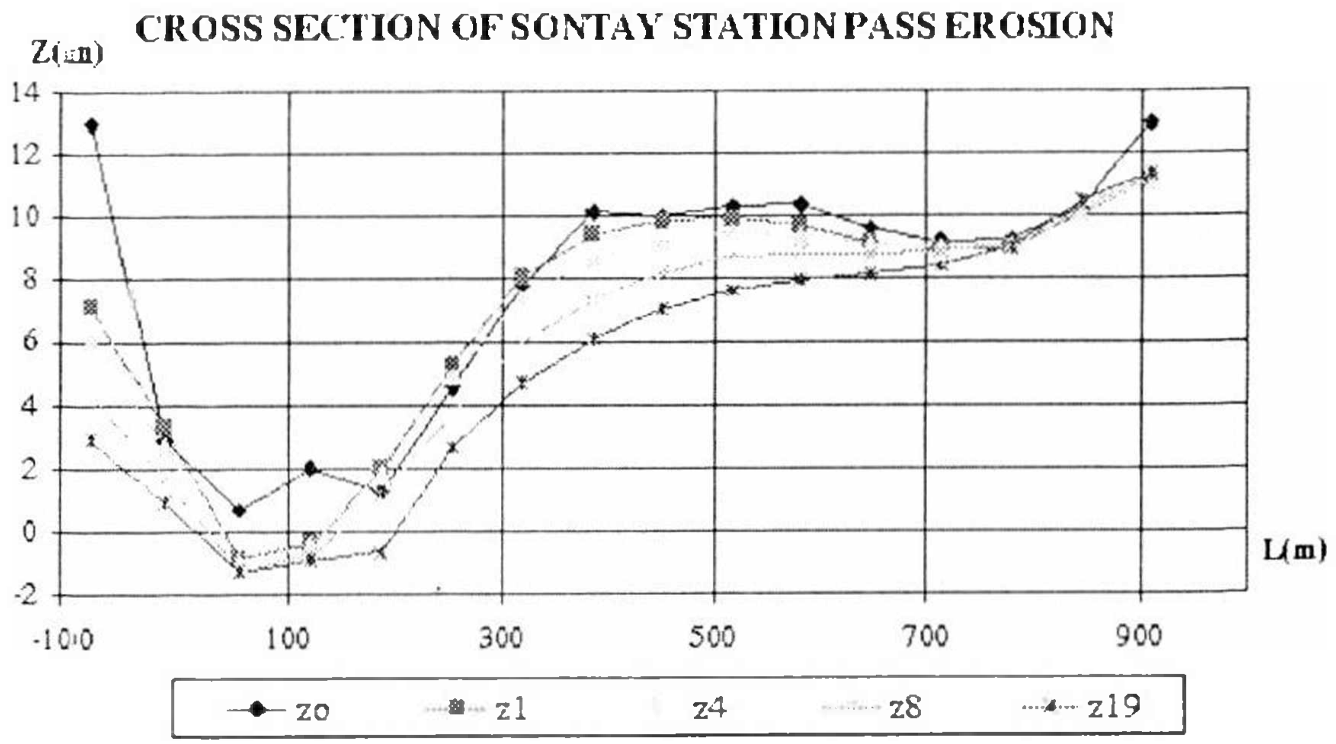

Figure. 2. Cross section of SonTay station pass flood days

IV. Comments

1. Recently, the erosion of riverbed is the hot problem so a two-dimensional model to analyze and simulate those processes is needed.

2. Model computed transport of bed and suspended sediments to orthogonal direction and distribution following depth of suspended sediment, reflecting more adequate resasons and present condition of river erosion and sediment transport balance.

3.Model uses finite control volum and Crank-Nicolson scheme that has effecter to sediment transport.

4.This is begining test,therefore erosion-crumble problem not is investigated yet.

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TẠP CHÍ KHOA HỌC ĐHQGHN, KHTN & CN, T.XIX, $N_0$1, 2003

NGHIÊN CỨU ỨNG DỤNG MÔ HÌNH 2 CHIỀU TÍNH TOÁN BIẾN DẠNG LÒNG DẪN

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Việc nghiên cứu xói lở lòng sông đã được tiến hành ở nhiều nơi trên thế giới. ở nước ta đã sử dụng một số mô hình như HEC-6, MIKE11 để phân tích, tính toán xói lở. Tuy nhiên các mô hình trên mới chỉ giải quyết bài toán 1 chiều. Một số mô hình thủy lực 2 chiều như TELEMAC hay MIKE21 cũng mới chỉ xét ở phạm vi phân bố tốc độ dòng chảy.

Cho đến nay hệ thống sông ngòi Việt Nam bị xói lở theo cả chiều dọc và chiều ngang rất mạnh mẽ và chúng có tác động tương hỗ với nhau. Vì vậy cần thiết có một mô hình 2 chiều để giải quyết bài toán này. Mô hình biến dạng lòng dẫn 2 chiều trong hệ tọa độ phi tuyến không trực giao TREM (Two-dimensional Riverbed Evolution Model constructed in the nun-orthogonal curvilinear coordimate system) cho phép xác định sự phân bố tốc độ cũng như biến đổi đáy sông theo cả hướng dọc và hướng ngang.