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ON A CLASS OF MONOGENIC FUNCTIONS IN CLIFFORD ALGEBRA

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1. INTRODUCTION

Let $\mathcal{A}$ be an universal algebra induced by a m-dimensional real linear space with a $\{e_1, e_2, ..., e_m\}$. Let A_p be a linear subspace of $\mathcal{A}$ spanned by the $\binom{m}{p}$ products e_A, $\#A = p$ have showed that $L_1 = lin\{e_0, e_1, ..., e_m, e_{A_1}, e_{A_2}, ..., e_{A_s}\}$ is invertible if and only if $m = 4$ $(p \in N)$ and $s = 1$, $e_{A_1} = e_{12...m}$ (see [7]). Based on these results for the case $m = 4p+2$ $(p$ the generalized Cauchy-Rieman operator $D = \sum_{i=0}^{m+1} e_i \partial_{x_i}$, $e_{m+1} = e_{12...m}$ can be construct R^{m+2}. Furthermore we get the fundamental results for the monogenic functions induced D.

2. MONOGENIC FUNCTIONS

Let $\{e_1, e_2, ..., e_m\}$ be a basis of R^m. Consider universal Clifford algebra $\mathcal{A}$ with basis

$$\{e_0, e_1, ..., e_m, e_{12}, ..., e_{m-1m}, ..., e_{12...m}\}.$$

The multiplication in $\mathcal{A}$ is given by the rule

$$e_ie_j + e_je_i = 0 \text{ for } i \neq j \; e_j^2 = -1 \; (i, j = 1, 2, ..., m).$$

So that the multiplication on basis vectors is defined by formula

$$e_Ae_B = (-1)^{\#(A\cap B)}(-1)^{P(A,B)}e_{A\Delta B},$$

where

$$P(A, B) = \sum_{j\in B} P(A, j), \quad P(A, j) = \#\{i \in A : i > j\}.$$

Every element $a = \sum_A a_Ae_A$ $(a_A \in R)$ is called a Clifford number. A product of two Cli numbers $a = \sum_A a_Ae_A$, $b = \sum_B b_Be_B$ is defined by the formula

$$ab = \sum_A \sum_B a_Ab_Be_Ae_B.$$

The involution for basic vectors is given by

$$\overline{e}_A = \overline{e}_{k_1k_2...k_n} = (-1)^{\frac{n(n+1)}{2}} e_{k_1k_2...k_n}.$$

y $a = \sum_A a_A e_A$ we write $\overline{a} = \sum_A a_A \overline{e}_A$.

ition 1. Let $m = 4p + 2$ $(p \in N)$. Consider the differential operator

$$D = \sum_{i=0}^{m+1} e_i \frac{\partial}{\partial x_i} \quad \text{where } e_{m+1} = e_{12\ldots m}$$

he conjugate operator of D

$$\overline{D} = \sum_{i=0}^{m+1} \overline{e}_i \frac{\partial}{\partial x_i}.$$

ns of D and $\overline{D}$ on functions from the left and from the right are governed by the rules (see r all $f(x) = \sum_A e_A f_A(x)$, $f_A(x)$ are real-valued, we have

$$Df = \sum_{i,A} e_i e_A \frac{\partial f_A}{\partial x_i}, \quad fD = \sum_{i,A} e_A e_i \frac{\partial f_A}{\partial x_i}$$

$$\overline{D}f = \sum_{i,A} \overline{e}_i e_A \frac{\partial f_A}{\partial x_i}, \quad f\overline{D} = \sum_{i,A} e_A \overline{e}_i \frac{\partial f_A}{\partial x_i}.$$

ma 1. $D\overline{D} = \overline{D}D = \Delta_{m+2} e_0$, *where* Δ_{m+2} *denotes Laplacien in* R^{m+2}.

. Since $e_{m+1} = e_{12\ldots m}$ and $m = 4p + 2$ $(p \in N)$, we find $\overline{e}_{m+1} = -e_{m+1}$. For every 1, 2, ..., $m\}$, we have

$$\begin{aligned} e_i \overline{e}_{m+1} + e_{m+1} \overline{e}_i &= -e_i e_{m+1} - e_{m+1} e_i \\ &= -((-1)^{i+1} e_{12\ldots i-1 i+1\ldots m} + (-1)^{m-i} e_{12\ldots i-1 i+1\ldots m}) \\ &= ((-1)^{i+1} + (-1)^{4p+2-i}) e_{12\ldots i-1 i+1\ldots m} = 0 \end{aligned}$$

at

$$\begin{aligned} D\overline{D} &= \Big(\sum_{i=0}^{m} e_i \frac{\partial}{\partial x_i} + e_{12\ldots m} \frac{\partial}{\partial x_{m+1}}\Big)\Big(\sum_{j=0}^{m} \overline{e}_j \frac{\partial}{\partial x_j} + \overline{e}_{12\ldots m} \frac{\partial}{\partial x_{m+1}}\Big) \\ &= \sum_{i,j=0}^{m} e_i \overline{e}_j \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} + \sum_{i=0}^{m} (e_i \overline{e}_{12\ldots m} e_{12\ldots m} \overline{e}_i) \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_{m+1}} + e_{12\ldots m} \overline{e}_{12\ldots m} \frac{\partial^2}{\partial x_{m+1}^2} \\ &= \sum_{i=0}^{m} e_i \overline{e}_i \frac{\partial^2}{\partial x_i^2} + e_{12\ldots m} \overline{e}_{12\ldots m} \frac{\partial^2}{\partial x_{m+1}^2} = \Delta_{m+2} e_0. \end{aligned}$$

larly, one can check the equality $D\overline{D} = \Delta_{m+2} e_0$.

nition 2. Let be given open $\Omega \subset R^{m+2}$. A function $f \in C^1(\Omega, \mathcal{A})$ is said to be left (right) ogenic in Ω if and only if $Df = 0$ $(fD = 0)$ in Ω.

ark. If $p = 0$, then $D = \frac{\partial}{\partial x_0} e_0 + \frac{\partial}{\partial x_1} e_1 + \frac{\partial}{\partial x_2} e_2 + \frac{\partial}{\partial x_3} e_{12}$. In this case, from $Df = 0$ one the generalized Cauchy - Rieman 0 system in 4-dimensional space (see [6]).

Definition 3. The function $E(x) = \frac{1}{\omega_{m+2}} \frac{\overline{x}}{r^{m+2}}$ $(x \neq 0)$, where ω_{m+2} is the area of unit ba R^{m+2} and $r = \sqrt{\sum_{j=0}^{m+1} x_j^2}$, is called the Cauchy kernel.

Lemma 2. $DE = ED = 0$.

Proof. We find the following equalities:

$$DE = \sum_{i=0}^{m+1} \sum_{j=0}^{m+1} \frac{1}{\omega_{m+2}} \frac{\partial}{\partial x_i} e_i \frac{1}{r_{m+2}} x_j \bar{e}_j = \sum_{i,j=0}^{m+1} \frac{1}{\omega_{m+2}} e_i \bar{e}_j \frac{\partial}{\partial x_i} \left(\frac{x_j}{r^{m+2}}\right)$$

$$= \sum_{i,j=0}^{m+1} \frac{1}{\omega_{m+2}} e_i \bar{e}_j \left(\frac{\delta_{ij}}{r^{m+2}} - \frac{x_j \frac{\partial}{\partial x_i} r^{m+2}}{r^{2m+4}} \right)$$

$$= \sum_{i,j=0}^{m+1} \frac{1}{\omega_{m+2}} e_i \bar{e}_j \left(\frac{\delta_{ij}}{r^{m+2}} - \frac{(m+2) x_j r^{m+1} \frac{\partial r}{\partial x_i}}{r^{2m+4}} \right)$$

$$= \sum_{i,j=0}^{m+1} \frac{1}{\omega_{m+2}} e_i \bar{e}_j \left(\frac{\delta_{ij}}{r^{m+2}} - \frac{(m+2) x_j r^{m+1} x_i}{r^{2m+5}} \right)$$

$$= \sum_{i,j=0}^{m+1} \left(\frac{1}{\omega_{m+2}} e_i \bar{e}_j \left(\frac{\delta_{ij}}{r^{m+2}} - \frac{(m+2) x_j x_i}{r^{m+4}} \right) \right.$$

$$= \sum_{i,j=0}^{m+1} \frac{1}{\omega_{m+2}} e_0 \left(\frac{1}{r^{m+2}} - \frac{(m+2) x_i^2}{r^{m+4}} \right) - \sum_{i<j} \frac{1}{\omega_{m+2}} (e_i \bar{e}_j + e_j \bar{e}_i) \frac{(m+2) x_i x_j}{r^{m+4}}$$

$$= \sum_{i=0}^{m+1} \frac{1}{\omega_{m+2}} e_0 \left(\frac{1}{r^{m+2}} - \frac{(m+2) x_i^2}{r^{m+4}} \right)$$

$$= \frac{1}{\omega_{m+2}} \left(\frac{m+2}{r^{m+2}} - \frac{(m+2) r^2}{r^{m+4}} \right) = 0,$$

which was to be proved.

Similarly one can check the equality $ED = 0$.

3. CAUCHY'S INTEGRAL AND ITS APPLICATIONS

Let M be a $(m+2)$-dimensional differentiable and oriented manifold contained in some subset Ω of R^{m+2}. By means of the $(m+1)$-forms:

$$d\hat{x}_i = dx_0 \wedge dx_1 \wedge \cdots \wedge dx_{i-1} \wedge dx_{i+1} \wedge \cdots \wedge dx_{m+1} \quad i = 0, 1, \ldots, m+1.$$

An $\mathcal{A}$-valued $(m+1)$-forms is introduced by putting

$$d\sigma = \sum_{i=0}^{m+1} (-1)^i e_i d\hat{x}_i .$$

and the volume-element by $dx = dx_0 \wedge dx_1 \wedge \cdots \wedge dx_{m+1}$.

rem 1. *If f, $g \in C^1(\Omega, \mathcal{A})$ then for and $(m+2)$-chain C on $M \subset \Omega$ we have*

$$\int_{\partial C} f\, d\sigma\, g = \int_C (fD.g + f.Dg)\, dx\,.$$

. Let $f = \sum_A f_A\, e_A$, then by Stokes's Theorem for real-valued functional we have

$$\begin{aligned}\int_{\partial C} f\, d\sigma\, g &= \int_{\partial C} \sum_{A,i,B} (-1)^i e_A\, e_i\, e_B\, f_A\, g_B\, d\hat{x}_i \\ &= \sum_{A,i,B} (-1)^i e_A\, e_i\, e_B \int_C (-1)^i \frac{\partial}{\partial x_i}(f_A\, g_B) dx \\ &= \int_C \sum_{A,i,B} e_A\, e_i\, e_B \Big(\frac{\partial f_A}{\partial x_i}\, g_B + f_A\, \frac{\partial g_B}{\partial x_i} \Big) dx \\ &= \int_C (fD.g + f.Dg) dx\,.\end{aligned}$$

llary 1. *If f is a left monogenic function and g is a right monogenic function in Ω for the 1) chain C on $M \subset \Omega$, we have $\int_{\partial C} f\, d\sigma\, g = 0$.*

ark. Putting $f = 1$, for any $g \in C^1(\Omega, \mathcal{A})$, we have

$$\int_{\partial C} d\sigma\, g = 0 \text{ if } Dg = 0\,.$$

result is nothing else but Cauchy's theorem.

ma. 3. *If $f \in C^1(\Omega, \mathcal{A})$, then for any $x \in \Omega$ we have*

$$\lim_{x \to +0} \int_{\partial B} f(y)\, d\sigma\, E(y-x) = f(x)$$

$$\lim_{x \to +0} \int_{\partial B} E(y-x)\, d\sigma\, f(y) = f(x)\,,$$

B is the ball having the center x and radius e being sufficiency small.

. Let $f(y) = \sum_A f_A(y)\, e_A$. Then by Stokes's theorem one gets

$$\begin{aligned}\int_{\partial B} f(y)\, d\sigma\, E(y-x) &= \frac{1}{\omega_{m+2}} \frac{1}{e^{m+2}} \sum_{A,i,j} e_A\, e_i\, \bar{e}_j \int_{\partial B} (-1)^i (y_j - x_j)\, f_A(y)\, d\hat{y}_i \\ &= \frac{1}{\omega_{m+2} e^{m+2}} \sum_{A,i,j} e_A\, e_i\, \bar{e}_j \int_B \frac{\partial}{\partial y_i}((y_j - x_j) f_A) dy \\ &= \frac{1}{\omega_{m+2} e^{m+2}} \sum_{A,i,j} e_A\, e_i\, \bar{e}_j \int_B \Big(\delta_{ij} f_A + (y_j - x_j) \frac{\partial f_A}{\partial y_i} \Big) dy\,.\end{aligned}$$

$I_1 = \int_B (y_j - x_j) \frac{\partial f_A}{\partial y_i}\, dy$. By the average value's theorem one gets $I_1 = (y_j^* - x_j) \frac{\partial f_A}{\partial y_i}(y^*) V_B$, e $y^* \in B$ and V_B is the volume of B. Then

$$\lim_{\epsilon\to 0^+} \frac{I_1}{\epsilon(m+2)} = \lim_{\epsilon\to 0^+} \frac{(y_j^* - x_j)\dfrac{\partial f_A}{\partial y_i}(y^*)V_B}{\epsilon^{m+2}}$$

$$= \lim_{\epsilon\to 0^+} \frac{(y_j^* - x_j)\dfrac{\partial f_A}{\partial y_i}(y^*).c.\epsilon^{m+2}}{\epsilon^{m+2}} \quad (c \text{ is a constant})$$

$$= \lim_{c\to 0^+} c.(y_j^* - x_j)\,\frac{\partial f_A}{\partial y_i}(y^*) = 0 \quad (y_j^* \to x_j \text{ when } \epsilon \to 0^+).$$

Put $I_2 = \int_B \delta_{ij} f_A\, dy$. If $i \neq j$ then $I_2 = 0$.

If $i = j$ then

$$\frac{I_2}{\epsilon_{m+2}} = \frac{\int_B f_A\, dy}{\epsilon_{m+2}}.$$

By the average value's theorem one gets

$$\frac{I_2}{\epsilon^{m+2}} = \frac{1}{\epsilon^{m+2}} f_A(y_A^*)\, V_B \quad (\text{where } y_A^* \in B,\ V_B \text{ is the volum of } B)$$

$$= \frac{1}{\epsilon^{m+2}} f_A(y_A^*)\frac{\omega_{m+2}}{m+2} = \frac{\omega_{m+2}}{m+2} f_A(y_A^*) \quad (\omega_{m+2} \text{ is the area of } B).$$

Hence

$$\lim_{\epsilon\to 0^+} f(y)\, d\sigma\, E(y-x) = \lim_{\epsilon\to 0^+} \frac{1}{\omega_{m+2}} \sum_{i,A} e_A \frac{\omega_{m+2}}{m+2} f_A(y_A^*)$$

$$= \sum_{i=0}^{m+1} \frac{1}{m+2} f(x) = f(x) \quad (\text{Because if } \epsilon \to 0^+ \text{ then } y_A^* \to x).$$

Similarly, one can check the equality

$$\lim_{\epsilon\to 0^+} \int_{\partial B} E(y-x)\, d\sigma\, f(y) = f(x).$$

Theorem 2. *Let $S \subset \Omega$ be a $(m+2)$-dimensional differentiable and oriented compact man with-boundary. IF $f \in C^1(\Omega, \mathcal{A})$ then*

$$\int_{\partial S} E(y-x) d\sigma\, f(y) - \int_S E(y-x).Df(y)\, dy = \begin{cases} f(x) & \text{if } x \in S^0 \\ 0 & \text{if } x \in \Omega \setminus S \end{cases}$$

Proof. If $x \in \Omega \setminus S$ then by Theorem 1 and Lemma 2, it follows

$$\int_{\partial S} E(y-x) d\sigma\, f(y) = \int_S (E(y-x)D.f(y) + E(y-x).Df(y))dy = \int_S E(y-x).Df(y)\, dy$$

Hence

$$\int_{\partial S} E(y-x) d\sigma\, f(y) - \int_S E(y-x).Df(y)\, dy = 0.$$

Now take $x \in S^0$ to be fixed and choose $R > 0$ such that $B(x, R) \subset S^0$. Let $V = S \subset B$. by Theorem 1 we have

$$\int_{\partial V} E(y-x)d\sigma\, f(y) = \int_V \{E(y-x)D.f(y) + E(y-x).Df(y)\}dy$$
$$= \int_V E(y-x).Df(y)\, dy\,.$$

nown that

$$\int_{\partial S} E(y-x)d\sigma\, f(y) - \int_{\partial B} E(y-x)d\sigma\, f(y) = \int_{\partial V} E(y-x)d\sigma\, f(y)\,.$$

$$\int_{\partial S} E(y-x)d\sigma\, f(y) - \int_{\partial V} E(y-x)D\, f(y)\, dy = \int_{\partial B} E(y-x)d\sigma\, f(y)\,.$$

$R \to 0^+$ then V tends to S and Lemma 3 follows

$$\int_{\partial S} E(y-x)d\sigma\, f(y) - \int_S E(y-x).Df(y)\, dy = f(x)\,.$$

llary 2. *Let $S \subset \Omega$ be as in Theorem 2. If f is a left monogenic function, then*

$$\int_{\partial S} E(y-x)d\sigma\, f(y) = \begin{cases} f(x) & \text{for } x \in S^0 \\ 0 & \text{for } x \in \Omega \setminus S \end{cases}$$

llary 3. *If f is left monogenic function in Ω and $a \in \Omega$ then*

$$f(a) = \frac{1}{R^{m+2}V_{m+2}} \int_{B(a,\,R)} f(u)\, du$$

ch $R > 0$ such that $B(a,\,R) \subset \Omega$, where V_{m+2} is the volume of unit ball in R^{m+2}.

Take $R > 0$ such, that $B(a,\,R) \subset \Omega$. Applying Corollary 2 for the ball $B(a,\,R)$, we have

$$f(a) = \frac{1}{\omega_{m+2}} \int_{\partial B(a,\,R)} \frac{u-a}{|u-a|^{m+2}}\, d\sigma\, f(u) = \frac{1}{r^{m+2}\omega_{m+2}} \int_{\partial B(a,\,R)} (u-a)d\sigma\, f(u).$$

ıeorem 1 one gets

$$f(a) = \frac{m+2}{R^{m+2}\omega_{m+2}} \int_{B(a,\,R)} f(u)\, du = \frac{1}{R^{m+2}V_{m+2}} \int_{B(a,\,R)} f(u)\, du\,.$$

ition 4. If $\mu = \sum_A \mu_A e_A$, then $|\mu|_0 = 2^{m/2}\big(\sum_A \mu_A^2\big)^{1/2}$.

na 4. *Suppose that there exists a point $a \in \Omega$ such $|f(x)|_0 \le |f(a)|_0$ for all $x \in \Omega$. Put $\{x \in \Omega : |f(x)|_0 = |f(a)|_0 = \lambda\}$. Then $\Omega_\lambda = \Omega$.*

First, we show that Ω_λ is closed in Ω. Indeed $\Omega \neq \emptyset$, because $a \in \Omega_\lambda$. If $\Omega_\lambda = \Omega$, then ·oof is termined. So let $y \in \Omega \setminus \Omega_\lambda$. Hence $|f(y)|_0 < \lambda$. As $|f(\cdot)|_0$ is continuous in Ω, it is possible to find an $R > 0$, such that $|f(u)|_0 < \lambda$ for all $u \in B(y, R)$ i.e. such $B(y, R) \subset \Omega$ This means that Ω_λ is closed in Ω.

To prove that Ω_λ is open in Ω, take $z \in \Omega_\lambda$ and $R > 0$ such that $B(z, R) \subset \Omega$. By Cor 3 one gets

$$f(z) = \frac{1}{R^{m+2}V_{m+2}} \int_{B(z, R)} f(u)\, du .$$

It follows that

$$\lambda^2 = |f(z)|_0^2 = \frac{2^m}{R^{2m+4}V_{m+2}^2} \sum_A \Big(\int_{B(z, R)} f_A(u)\, du \Big)^2 .$$

Using Holder's inequality one gets

$$\begin{aligned} \lambda^2 &\leq \frac{2^m}{R^{2m+4}V_{m+2}^2} \sum_A \Big(\int_{B(z, R)} du \Big) \Big(\int_{B(z, R)} f_A^2(u)\, du \Big) \\ &\leq \frac{1}{R^{m+2}V_{m+2}} \int_{B(z, R)} |f(u)|_0^2\, du . \end{aligned}$$

Hence

$$0 \leq \frac{1}{R^{m+2}V_{m+2}} \int_{B(z, R)} (|f(u)|_0^2 - \lambda^2)\, du \leq 0 .$$

Which yields that $|f(u)|_0 = \lambda$ for all $u \in B(z, R)$. This means that $B^0(z, R) \subset \Omega_\lambda$, which fc that Ω_λ is open in Ω.

Theorem 3. *Let Ω be open, connected and f be a left monogenic function in Ω. If there ex point $a \in \Omega$ such that $|f(x)|_0 \leq |f(a)|_0$ for all $x \in \Omega$ then f must be a constant function on*

Proof. Put $|f(a)|_0 = \lambda$, and $\Omega_\lambda = \{x : |f(x)|_0 = \lambda\}$. By Lemma 4, it follows $\Omega_\lambda = \Omega$ or $|f(x)|$ for all $x \in \Omega$.

If $\lambda = 0$ then clearly $f(x) = 0$ for all $x \in \Omega$.

For $\lambda > 0$ and $x \in \Omega$, we have

$$2^m \sum_A f_A^2(x) = \lambda^2 . \qquad (*)$$

Differentiating twice times both sides of $(*)$, one gets

$$\sum_A \Big(\frac{\partial}{\partial x_i} f_A(x) \Big)^2 + \sum_A f_A(x) \frac{\partial^2}{\partial x_i^2} f_A(x) = 0 \quad i = 0, 1, 2, \ldots, m+1 .$$

Summing up over $i = 0, 1, 2, \ldots, m+1$ yields

$$\sum_{i, A} \Big(\frac{\partial}{\partial x_i} f_A(x) \Big)^2 + \sum_A f_A(x).\Delta_{m+2} f_A(x) = 0 .$$

Or

$$\sum_{i, A} \Big(\frac{\partial}{\partial x_i} f_A(x) \Big)^2 = 0 \text{ for all } x \in \Omega .$$

So that $\frac{\partial}{\partial x_i} f_A(x) = 0$ in Ω for all $i = 0, 1, 2, \ldots, m+1$ and for all A, which means that constant in Ω.

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VỀ MỘT LỚP CÁC HÀM CHỈNH HÌNH TRONG ĐẠI SỐ CLIFFORD

Nguyễn Cảnh Lương

Đại học Khoa học tự nhiên - ĐHQGHN

Để nghiên cứu tính chất chỉnh hình của hàm $f : R^{m+1} \to A$ với A là đại số Clifford 2^m chiều, ri ta xét toán tử vi phân $D_0 = \sum_{j=0}^{m} e_j$ ([1]). Trong bài này khi $m = 4p + 2$ $(p \in N)$ chúng ở rộng toán tử D_0 bởi toán tử $D = \sum_{j=0}^{m+1} e_j$ trong đó $e_{m+1} = e_{12\ldots m}$ tác động lên lớp hàm $I \subset R^{m+2} \to A$. Trong trường hợp này, đối với toán tử D chúng ta cũng nhận được những quả tương tự như đối với toán tử D_0.