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FILTERING FROM LEVY ANNEALING NOISES

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1. INTRODUCTION.

In this paper we consider a problem of state estimation of a stochastic dynamical sys represented by an evolution equation from the non-equilibrium mechanics. The observatio this system is driven by an annealing process that is simulated from a phenomenon consid in Condensed Matter Physics to find the ground state where the energy of the system attain minimum value.

But what is the best state estimation of a stochastic dynamical system? The syste expressed by a stochastic process X. We like to know about X but we can not observe it dire We can only observe it by another process θ. This process carries all informations about X could get. But these informations are perturbed by some noise Y. The problem state estima is that of finding a way to estimate true values of X on the basis of data given by the observa θ. And from a mathematical point of view, the best estimation can be taken as the conditi expectation $E(X/\theta)$.

And the annealing is known as a thermal process for obtaining low energy states of a sol a heat bath. This process contains two steps :

- Increase the temperature of the heat bath to a maximum value at which the solid melt

- Decrease carefully the temperature of the heat bath until the particles arrange thems in the ground state X of the solid where the energy $U(X)$ is minimum.

One simulates this process to study problems from global optimization. Among various proaches to Simulated Annealing that of diffusion lead us to a stochastic differential equatic which the distribution of the solution reaches a Gibbs distribution at state where U is minin when the temperature τ decreases to zero.

A Lévy process Y is a stochastic process of stationary independent increments. Brow motions and Poisson processes are typical Lévy processes. Problems of state estimation : Brownian or Poissonian noises are well investigated.

The noise Y considered in our state estimation problem is at the same time a diffusion nealing and a Lévy process. This problem arises from Simulated Annealing Theory applie Filtering.

2. DYNAMICS

Let Ω be the space of all functions ω from the interval $[0,T]$ $(0 < T < \infty)$ into a lo compact Hausdorff space S :

$$\Omega = \{\omega : [0,T] \to S\}$$

denote $\omega(t) = X_t(\omega)$.

t W be the space of all real-valued functions defined on $[0, 1]$

$$W = \{w : [0, T] \to \mathbf{R}^1\}.$$

e denote the value of w at $t \in [0, T]$ by

$$w(t) = \theta_t(w).$$

by $\mathcal{F}_t$ the σ-field generated by $(X_s; 0 \leq s \leq t)$ and by $\mathcal{W}_t$ the σ-field generated by $(\theta_s; 0 \leq$

ppose that P is a probability measure defined on the measurable space $(\Omega \times W, \mathcal{F}_T \times \mathcal{W}_T)$. ochastic dynamical system considered here is the following process :

$$\{\Omega \times W, \mathcal{F}_T \times \mathcal{W}_T, P, (X_t, \theta_t)\} \tag{2.1}$$

ler an element Y_t of W such that $(W, \mathcal{W}, Q; Y_t)$ is a Lévy process, where $\mathcal{W} = \bigvee_{t=0}^{T} \mathcal{W}_t$ and probability on $(W, \mathcal{W})$. According the Lévy decomposition we have :

$$Y_t = M_t + \int_0^t b(s)ds + \int_0^t \int_{|u| \leq 1} uJ_c(ds, du) + \int_0^t \int_{|u|>1} J(ds, du) \tag{2.2}$$

M_t is a continuous martingale such that

$$d < M_t, M_t > = a(t)dt,$$

u) a Poisson random measure with density λ

$$E[J(dt, du)] = \lambda(t, du)dt$$

(dt, du) the compensator of $J(dt, du)$:

$$J_c(dt, du) = J(dt, du) - \lambda(t, du)dt.$$

3. DIFFUSION ANNEALING PROCESS

assume that we are given a Gibbs distribution g on $\mathbf{R}$ with a density with respect to the ne measure dx, given by

$$g(y) = \frac{1}{Z_\tau} \exp\{-\frac{1}{\tau} U(y)\} \tag{3.1}$$

$$Z_\tau = \int_{-\infty}^{\infty} \exp\{-\frac{1}{\tau} U(y)\} dx \tag{3.2}$$

$U(y)$ is a real-valued function on $\mathbf{R}$ so that $Z_\tau < \infty$; T is a positive constant and considered emperature in an annealing process.

seek a diffusion process Y_t on $\mathbf{R}$ having g as its unique invariant measure. This process specified via a stochastic differential diffusion equation of the form

$$dY(t) = b(Y(t))dt + \sigma(Y(t))dB(t).$$

where $b(y)$ and $\sigma(y)$ are the drift and the diffusion coefficients, respectively, and $B(t)$ is the st Brownian motion on $\mathbf{R}$.

According to Langevin and Smoluchowski, the corresponding diffusion equation is

$$dY(t) = U'(Y(t))dt + \sqrt{2\tau}dB(t).$$

And one can prove that under some assumptions opposed on U, this equation has a s $Y(t)$ whose density $p(t,y)$ weakly tends to $g(y)$ when $t \to \infty$. The solution $Y(t)$ of (3.. course a diffusion process expressing the state of the annealing material in the temperatur at the time t. It is called a diffusion annealing process. On the other hand, one observe tha temperature τ tends to 0 the Gibbs distribution (3.1) concentrates on the global minima expressing the minimum energy of the system.

4. STATE ESTIMATE OF A SYSTEM WITH LEVY ANNEALING NOISES

In the Simulated Annealing one are interested in problems of state estimation of a dy system perturbed by some noises related to an annealing process. We consider a dynamical (X_t, θ_t) defined as in Section 2, where X_t is the system process and θ_t is the observation by

a. $\theta_t = \int_0^t h(s, X_s)ds + Y_t,$

b. $h(t,x)$ is a positive and non-anticipating function such that

$$E\int_0^t h^2(s, X_s)ds < \infty \qquad \text{for each } t,$$

c. Y_t is a Lévy process.

Lemma 4.1. *The observation process θ_t itself is also a Lévy process.*

Proof. Because Y_t is a Lévy process we have the following decomposition :

$$Y_t = M_t + \int_0^t b(s)ds + \int_0^t \int_{|u|\leq 1} uJ_c(ds, du) + \int_0^t \int_{|u|\geq 1} uJ(ds, du)$$

Then θ_t has also a Lévy decomposition

$$\theta_t = M_t + \int_0^t \bar{b}(s)ds + \int_0^t \int_{|u|\leq 1} J_c(ds, du) + \int_0^t \int_{|u|\geq 1} uJ(ds, du)$$

where $\bar{b} = b + h$. Thus θ_t is a Lévy process.

Lemma 4.2. *The first term M_t in the Lévy decomposition of the observation process martingale. Furthermore it is a Brownian motion.*

Proof. In virtue of the Lévy-Khintchine formula, the Fourier transform of M_t is :

$$\psi_M(u) = \frac{\sigma^2}{2}u^2 - i\alpha u$$

M_t is a Brownian motion which is a martingale.

ition 4.1. The noise Y_t is called a Lévy annealing noise if it satisfies the following diffusion ing equation :

$$dY_t = -U'(Y_t)dt + \mathcal{E}_t dM_t \tag{4.5}$$

$U \in C_b^2(\mathbf{R})$, $\mathcal{E}_t$ is some parameter tending to 0 when $t \to \infty$ and M_t is the first composant Lévy decomposition of θ_t which is a Brownian motion as shown in Lemma 2.

rem 4.1. *If* $U \in C_b^2$ *and* $Z_t = \int_{-\infty}^{\infty} e^{-\frac{1}{\mathcal{E}_t}U(y)}dy < \infty$ *(4.6)*
he Lévy annealing noise Y_t is lightly different from a Gibbs distribution when $t \to \infty$.

Since Y_t is the solution of an annealing equation

$$dY_t = -U'(Y_t)dt + \sqrt{2\tau}dM_t$$

U satisfies conditions for convergence of the annealing process Y_t, the temperature $\tau = \frac{1}{2}\mathcal{E}_t^2$ ses to 0 when $t \to \infty$ and M_t is in fact a Brownian motion, then the distribution of Y_t is uch different from that of the Gibbs distribution defined by

$$g_t(y) = \frac{1}{Z_t}e^{-\frac{1}{\tau_t}U(y)}.$$

ition 4.2. Let $f(x)$ be a bounded measurable function on S; $f : S \to \mathbf{R}$. we say $\pi_t(f)$ the ;ate estimation of the system X via f the following conditional expectation

$$\pi_t(f) = E_P(f(X_t)|\mathcal{W}_t) \tag{4.7}$$

$\mathcal{W}_t$ is the σ-field $\mathcal{W}_t = \sigma(\theta_s; s \leq t)$ which records all information about X up to the t.

em 4.2. *Suppose that $f : S \to \mathbf{R}$ is a bounded measurable function on S such that there is ded measurable and non-anticipating function $g : [0,T] \times W \times S \to \mathbf{R}$ for which*

$$f(X_t) - f(X_0) - \int_0^t g(s, \theta_s, X_s)ds$$

artingale with respect to $(\mathcal{W}_t, P_W)$ then the best estimation is given by the following equation

$$d\pi_t(f) = \pi_t(g)dt + \pi_t(f - \pi(h))d\overline{M}_t + \int \pi_t(f(\frac{\varrho}{\pi(\varrho)} - 1)\overline{J}_c(dt, du) \tag{4.8}$$

ϱ is some non-anticipating function on $[0,T] \times W \times S$.

The observation θ is in fact a Lévy process (Lemma 4.1), so under assumptions of Theorem : can apply results by Grigelionis [1] to obtain the equation (4.8) for best estimation $\pi_t(f)$.

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LỌC TỪ TIẾNG ỒN NUNG LUYỆN LEVY

Trần Hùng Thao
Viện Toán học, Trung tâm Khoa học tự nhiên và Công nghệ quốc gia

Trong bài báo này, chúng tôi đề cập đến bài toán thiết lập phương trình cho ước lượi trị chân thực của quá trình nung luyện kiểu khuếch tán $\{X(t)\}$ dựa trên quá trình quan sát $\{\theta(t)\}$. Phương trình lọc đã được thiết lập và nghiên cứu. Bài báo là sự tổng quát của kê cổ điển bằng cách thay quá trình quan sát kiểu khuếch tán bởi quá trình Lévy.