# ΓHE ABSORPTION OF A WEAK ELECTROMAGNETIC WAVE BY FREE ELECTRONS IN SEMICONDUCTOR SUPERLATTICES IN THE PRESENCE OF A QUANTIZING MAGNETIC FIELD

*Nguyen Quang Bau, Nguyen The Toan, Chhoum Navy*
*College of Natural Sciences - VNU*

## 1. INTRODUCTION

n ordinary semiconductors, the influence of quantizing magnetic field on the absorption of a ' has been investigated in detail by many authors [1-6].

n semiconductor superlattices, the quantum theory of absorption of EMW by free electrons ut the presence of a quantizing magnetic field has been investigated in [7,8]. In [7], the um transport equation is used to obtain the absorption coefficient only in the quantum ency region ($\hbar\omega \gg k_0T$, $\omega$ is the frequency of the EMW; $k_0$ - the Boltzmann constant; he crystal temperature). In [8], the Kubo-Mori method is used to obtain the absorption ient in the whole frequency region from classical frequency ($\hbar\omega \ll k_0T$) to quantum frequency $\gg k_0T$).

o make a further step on the problem, in this work, using Kubo-Mori method we study the nce of a quantizing magnetic field on the absorption coefficient of a semiconductor supers in the case when electrons scatter on nonpolar optical phonons. We shall calculate both erse and longitudinal components of the conductivity tensor and the absorption coefficient.

Ve proceed from Kubo's formula for the conductivity tensor [9, 10]

$$\sigma_{\mu\nu}(\omega) = \lim_{\delta\to+0}\int_0^\infty dt\, e^{i\omega t-\delta t}(J_\mu, J_\nu(t))\,, \tag{1}$$

$J_\mu$ - the $\mu$-component of current density operator ($\mu = x, y, z$) and $J_\mu(t)$ - Heisenberg ntation of $J_\mu$

$$J_\mu(t) = e^{\frac{iHt}{\hbar}} J_\mu e^{\frac{iHt}{\hbar}}\,, \tag{2}$$

amiltonian of the system.

he Hamintonian of the electron-nonpolar optical phonon system takes the form:

$$H = H_0 + U\,, \tag{3}$$

$$H_0 = \sum_\alpha \varepsilon_\alpha a_\alpha^+ a_\alpha + \sum_{\vec q} \hbar\omega_{\vec q} b_{\vec q}^+ b_{\vec q}\,, \tag{4}$$

$$U = \sum_{\alpha\alpha'\vec q} U_{\alpha\alpha'\vec q}\, a_\alpha^+ a_\alpha\, (b_{\vec q} + b_{-\vec q}^+)\,, \tag{5}$$

where $\alpha = (k_x, k_z, n, s)$ (with $n$ - the number of the Landau subband, $s$ - the number of minibend, $k_x$ and $k_z$ are the $x$- and $z$-component of electron's wave vector) is the set of qua number characterizing electron's status in $SL$; $\hbar\omega_{\vec{q}}$ is the energy of phonon with wave vector $\vec{q}$ the energy spectrum of electron; $a^+_\alpha$ and $a_\alpha$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation ope of electron (phonon) respectively; $U_{\alpha\alpha'\vec{q}}$ is the matrix element of electron - nonpolar optical pl interaction.

The time correlation function used in (1) is defined by the following formula

$$(A, B) = \int_0^\beta d\lambda \langle e^{\lambda H} A e^{-\lambda H} B \rangle ,$$

here $\beta = \frac{1}{k_0 T}$. The symbol $\langle \cdots \rangle$ means the averaging of operators with hamiltonian $H$ system.

In [11] Mori pointed out that the Laplace's transformation of the time correlation functi can be represented in the form of infinite continued fraction. One of advantages of this rep tation is the the function will converge faster than that represented in a power series [11].

Using Mori's method, in the second order approximation of interaction, we get the foll formula for the components of the conductivity tensor

$$\sigma_{\mu\nu}(\omega) = \lim_{\delta \to +0} (J_\mu, J_\nu) \Big[ \delta - i(\omega + \eta) + \Big(\frac{i}{\hbar}\Big)^2 (J_\mu, J_\nu)^{-1} \int_0^\infty dt\, e^{i\omega t - \delta t} \big( [U, J_\mu], [U, J_\nu]_{\rm int} \big) \Big]^{-1}$$

with

$$\eta = \langle [J_\mu, J_\nu] \rangle (J_\mu, J_\nu)^{-1} .$$

Here $G_{\rm int}$ is the interacting representation of the operator $G$; $[A, B] = AB - BA$.

In the calculating process, we assumed that phonons are in equilibrium state and ele are present in the lowest $SL$-miniband. The EMW is assumed polarized perpendicularly magnetic field $\vec{E} \perp \vec{H}$ ($\vec{E}$ is the electric vector of EMW and $\vec{H}$ the magnetic field vector). case $\vec{H} \parallel Oz$, we choose the potential vector $\vec{A} = \vec{A}(-Hy, 0, 0)$. With these assumptions, strong coupling approximation, the energy spectrum of electrons takes the form [12]

$$\varepsilon_\alpha = \Big(n + \frac{1}{2}\Big)\hbar\Omega - \Delta \cos k_z d = \varepsilon_\perp + \varepsilon_\parallel ,$$

here $\Omega = eH/m_e c$ is the cyclotron frequency; $e$ and $m_e$ are the charge and the mass of el respectively; $2\Delta$ - the width of the lowest energy miniband and $d$ - the $SL$-period.

Our results of calculation are as follows.

## 2. THE CONDUCTIVITY TENSOR

$$\sigma_{xx}(\omega) = \sigma_{yy}(\omega) = \frac{1}{4} C_\perp \big\{ [i(\omega - \Omega) + \Gamma_\perp(\Omega)]^{-1} + [i(\omega + \Omega) + \Gamma_\perp(\Omega)]^{-1} \big\} ,$$

$$\sigma_{xy}(\omega) = \sigma_{yx}(\omega) = \frac{1}{4i} C_\perp \big\{ [i(\omega - \Omega) + \Gamma_\perp(\Omega)]^{-1} - [i(\omega + \Omega) + \Gamma_\perp(\Omega)]^{-1} \big\} ,$$

$$\sigma_{zz}(\omega) = C_\parallel [i\omega + \Gamma_\parallel(\Omega)]^{-1} .$$

$$C_\perp = \frac{n_e e^2}{d}\left(\frac{2\pi\beta}{m_e^3}\right)^{1/2}\Omega\,, \tag{13}$$

$$C_\parallel = \frac{m_2 e^2\Delta^2 d\beta e^{\beta\mu}}{8\pi\hbar^4}\,\frac{\Omega}{Sh(\beta\hbar\Omega/2)}\,, \tag{14}$$

$$\Gamma_{\perp(\parallel)} = (i/\hbar)^2 C^{-1}_{\perp(\parallel)}[\Gamma^+_{\perp(\parallel)}(\Omega) + \Gamma^-_{\perp(\parallel)}(\Omega)]\,, \tag{15}$$

$$\Gamma^\pm_\perp = \frac{2E^2_{op}e^2 n_e\hbar\omega_0}{c\rho u^2\chi^{1/2}\Delta d^2}\left(\frac{2}{m_e\pi\beta}\right)^{\frac{1}{2}}\frac{\Omega^2(1-e^{\beta\hbar\omega})}{\omega^2}Sh\left(\frac{\beta\hbar\Omega}{2}\right)N_0\sum_{n,n'}(n+n'+1)$$
$$\times e^{-\beta\hbar\Omega(n+\frac{1}{2})}\left[e^{\beta\Delta}\sum_{m=1}^{\infty}\frac{(2\beta\Delta)^{2m}}{(2m)!\,2m} + A^\pm\right], \tag{16}$$

$$\Gamma^\pm_\parallel = \frac{E^2_{op}e^2 n_e\Delta\omega_0}{c\rho u^2}\left(\frac{2m_e}{\pi\chi\beta}\right)^{\frac{1}{2}}\frac{1-e^{\beta\hbar\omega}}{\hbar^2\omega^2}N_0\left\{\frac{2}{\beta\Delta} + \frac{1-e^{\beta\hbar\Omega}}{2}e^{\beta\hbar(\omega\mp\omega_0)}\Omega^3\right.$$
$$\times\sum_{n,n'}e^{-n'\beta\hbar\Omega}\left\{\pi M^2_\pm I_0(\beta\Delta) + \pi(M_\pm\beta\Delta + \frac{2}{\beta\Delta})I_1(\beta\Delta)\right.$$
$$-2M_\pm\sum_{m=1}^{\infty}\frac{(2m-1)!!\,(m+2)}{(2m)!!}\beta\Delta\,{}_1F_2\left(\frac{2m+3}{2};\,\frac{3}{2},\,m+2;\,\frac{(\beta\Delta)^2}{4}\right)$$
$$-2\sum_{m=1}^{\infty}\frac{(2m-1)!!}{(2m)!!}\,{}_1F_2\left(\frac{2m+3}{2};\,\frac{1}{2},\,m+2;\,\frac{(\beta\Delta)^2}{4}\right)$$
$$\left.\left.+\sum_{m=1}^{\infty}\sum_{l=0}^{l<2m}\frac{[(2m-1)!!]^2}{(2m-l)!l!}M_\pm^{2m-l}\left[2-M^2_\pm-\frac{2(2m+1)lM^2_\pm}{(2m+1-l)(2m+2-l)}\right]A^\pm\right\}\right\}, \tag{17}$$

$$A = e^{-\beta\Delta M_\pm}\left\{\pi I_0(\beta\Delta) + \frac{1}{2}\sum_{m=1}^{\infty}\sum_{l=0}^{l<2m}\frac{[(2m-1)!!]^2}{(2m-l)!\,l!}M_\pm^{2m-l}\right.$$
$$\times[(-1)^l+1]\,B\left(\frac{l+1}{2},\frac{1}{2}\right){}_1F_2\left(\frac{l+1}{2};\,\frac{1}{2},\,\frac{l+2}{2};\,\frac{\beta^2\Delta^2}{4}\right)$$
$$+\frac{\beta\Delta}{2}\sum_{m=1}^{\infty}\sum_{l=0}^{l<2m}\frac{[(2m-1)!!]^2}{(2m-l)!\,l!}M_\pm^{2m-l}$$
$$\left.\times[(-1)^l+1]\;B\left(\frac{l+2}{2},\frac{1}{2}\right){}_1F_2\left(\frac{l+2}{2};\,\frac{3}{2},\,\frac{l+3}{2};\,\frac{\beta^2\Delta^2}{4}\right)\right\}, \tag{18}$$

$$M_\pm = [(n'-n)\hbar\Omega - \hbar\omega \pm \hbar\omega_0]/\Delta\,, \tag{19}$$

is the Bessel function of complex variable; $B(r,s)$ - the Beta function; $\alpha_1, ..., \alpha_p; \beta_1, ..., \beta_q; z)$ - the generalized hypergeometric function; $n_e$ - the electron concen- n; $\chi$ - the electric constant; $\omega_0$ - the limit frequency of optical phonon; $N_0$ - the phonon ntration; $E_{op}$ - the deformation potential for nonpolar optical phonon scattering; $u$ - the l velocity; $\rho$ - the crystal density; $\mu$ - the chemical potential and $c$ - the light velocity.

The + sign goes with phonon emission and the - sign goes with phonon absorption in the ss of absorption of the EMW.

## 3. THE ABSORPTION COEFFICIENT

Knowing complex conductivity tensor, the absorption coefficient of EMW in semicond superlattices can be found by the common formula

$$\alpha(\omega) = \mathrm{Re}\sigma(\omega)\,.$$

So we get:

$$\alpha_{xx}(\omega) = \alpha_{yy}(\omega) = \frac{1}{2\hbar^2}\frac{\omega^2+\Omega^2}{(\omega^2-\Omega^2)^2}[\Gamma_\perp^+(\Omega)+\Gamma_\perp^-(\Omega)]\,,$$

$$\alpha_{yx}(\omega) = \frac{n_e e^2}{d}\frac{\pi\beta^{1/2}}{m_e^3}\frac{\Omega^2}{\Omega^2-\omega^2}$$

$$\alpha_{zz}(\omega) = \frac{1}{\hbar^2\omega^2}[\Gamma_\parallel^+(\Omega)+\Gamma_\parallel^-(\Omega)]\,.$$

Expressions for $\Gamma_\perp^\pm(\Omega)$ and $\Gamma_\parallel^\pm(\Omega)$ are (16) and (17).

In our calculation, the law of conservation of energy imposes a constrain on the energy trum of electron

$$\left|M_\pm - \frac{\varepsilon_\parallel}{\Delta}\right| < 1\,.$$

This constrain leads to some new points in the process of absorption of a EMW in semicond superlattices:

- Only electrons which satisfy the inequality

$$(n'-n)\hbar\Omega - \hbar\omega \pm \hbar\omega_0 - \Delta < \varepsilon_\parallel < (n'-n)\hbar\Omega - \hbar\omega \pm \hbar\omega_0 + \Delta$$

can take part in the process of absorption.

- The number of the Laudau subband $n'$ which electron can move to after the absorptio in the interval

$$n'_{\max} > n' > n'_{\min}$$

with

$$n'_{\max} = n + \frac{\varepsilon_\parallel + \hbar(\omega \mp \omega_0)}{\hbar\Omega} + \frac{\Delta}{\hbar\Omega} \text{ and } n'_{\min} = n + \frac{\varepsilon_\parallel + \hbar(\omega \mp \omega_0)}{\hbar\Omega} - \frac{\Delta}{\hbar\Omega}\,.$$

The width of this interval is $\Delta n' = n'_{\max} - n'_{\min} = 2\Delta/\hbar\Omega$.

## 4. CONCLUSION

Analytic expressions (10-12), (21-23) for the conductivity tensor and the absorption ficient of EMW in semiconductor superlattices, in the case when the electron-nonpolar o phonon scattering is dominant, are obtained for the first time. They show complicate depe cies on the temperature ($T$), the frequency ($\omega$) of the EMW, $SL$-parameters ($\Delta$, $d$) and c magnetic field ($H$).

The law of conservation of energy leads to the condition (25) for the electron energy spec It also limits the number of Laudau subband, which electron can move to after the absorpti the interval (26).

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## REFERENCES

H. Fan. Photon - electron interaction in crystals. Mir, Moscow, 1969.

E. M. Epshtein. Interaction of intensive electromagnetic waves on electron properties of semiconductors. Communication of HEE of USSR, Ser. Radiophysics, **18** (1975), 785.

E. P. Pokatilo, V. M. Fomin. Multiphoton Absorption by free charge carriers in semiconductors. Phys. Stat. Sol. B, **73** (1976), 553.

V. M. Fomin, E. P. Pokatilov. Nonlinear absorption of electromagnetic wave by band charge carriers in a laser field. Phys. Stat. Sol. B., **78** (1976), 831.

G. M. Shmelve, Nguyen Quang Bau, Nguyen Hong Shon. Absorption of light by free carriers in the presence of a laser wave. Sov. Phys. Tech. Semicond., **15** (1981), 1999.

Nguyen Quang Bau. Influence of intensive modulated electromagnetic waves on the absorption of a wake electromagnetic wave in semiconductors. Communications in Physics, Vol. 13, No. 3+4 (1988), 28.

V. V. Pavlovich, E. M. Epshtein. Quantum Theory of absorption of electromagnetic waves by free carriers in semiconductor superlattices. Sov. Phys. Sol. Stat., **19** (1977), 1760.

G. M. Shmelev, I. A. Chaikovski, Nguyen Quang Bau. High frequency conduction in semiconductor superlattice. Sov. Phys. Tech. Semicond., **12** (1978), 1932.

R. J. Kubo. Statistical - mechanical theory of irrivesible processes. Phys. Soc. Japan, **12** (1957), 570.

D. K. Ferry, Carlo Jacoboni. Quantum transport in semiconductors. New York - London, 1992, p. 18 - p. 37.

H. Morri. A continued-fraction representation of the time-correlation functions. Progr. Theor. Phys., **34** (1965), 399.

G. M Shmelve, Nguyen Hong Shon, Vo Hong Anh. Sound Absorption in semiconductors with a superlattices in a quantizing magnetics field. Sol. Stat. Communications, **48** (1983), 239.

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# SỰ HẤP THỤ SÓNG ĐIỆN TỪ YẾU BỞI ĐIỆN TỬ TỰ DO TRONG BÁN DẪN SIÊU MẠNG KHI CÓ MẶT TỪ TRƯỜNG

*Nguyễn Quang Báu, Nguyễn Thế Toàn, Chhoum Navy*
*Trường ĐH Khoa học tự nhiên - ĐHQG Hà Nội*

Phương pháp Kubo - Mori được sử dụng để tính toán tenxơ độ dẫn cao tần về hệ số hấp thụ ; điện từ yếu (EMW) bởi điện tử tự do trong bán dẫn siêu mạng không suy biến khi có mặt từ trường lượng tử hướng dọc theo trục siêu mạng. Cơ chế tán xạ điện tử - phonon quang a cực được coi là chủ yếu. Các tác giả xem xét sự phụ thuộc của tenxơ độ dẫn và hệ số hấp vào hiện độ, vào các tham số của siêu mạng và vào cường độ từ trường. Việc xem xét cho ; số mức phân vùng từ Landau mà điện tử có thể nhảy tới sau khi hấp thụ bị giới hạn bởi ng độ từ trường và độ rộng của các mini vùng của siêu mạng.