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CHOOSING THE $SU(3)_L \times U(1)_N$ MODELS BASED ON THE ANOMALY CANCELLATION

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1. Recently, there are some different $SU(3)_L \times U(1)_N$ models appeared in the litera [1-7]. These models have the triplets and the antitriplets representations including also the ex particles which, in some cases, have strange charge such as 5/3, -4/3. The question is put ou there a general way to find out these models? How many models can one propose? This p aims at resolving these problems.

The plan of the paper is as follows. Section 2 is devoted to present the relations between charges of the exotic particles in a triplet (or in an antitriplet) and the hypercharge Y of the Sl group. In section 3, basing on the requirement of the anomaly-free, we investigate different mo classified according to the choice of the triplets and the antitriplets. The last section is devote our conclusion and comments. The Gell-Mann matrices used here λ_a, $\overline{\lambda}_a$ $(a = 1, 2, ..., 8)$ have expression of [4] concordant to the triplets and antitriplets respectively.

2. It is well-known that the model is anomaly-free if we have equal number of triplets and titriplets, counting the color of $SU(3)$. The general total of these two multiplets is 12 consi of 3 neutrinolepton multiplets and 9 quark multiplets. For convenience, we consider only one trino-lepton multiple and three quark multiplets in each of which there are three generation neutrino-lepton) or three colors (of quark).

In the fundamental representation of $SU(3)$ of 3-dimension, the form of the hypercharge as follows:

$$Y = a\lambda + 2N$$

in which a is belong to each model, N is the subscript of $U(1)_N$ and has a value belong to triplet. The general form of the neutrino-lepton triplet and the quark triplet is as follows:

$$\Psi_a = \begin{pmatrix} v_a \\ e_a \\ v_a^{ex} \end{pmatrix}_L \sim (3, N); \quad Q_L = \begin{pmatrix} u_i \\ d_i \\ d_i' \end{pmatrix}_L \sim (3, N'),$$

where $a = 1, 2, 3$, the generation index, $i = 1, 2, 3$, the flavour index. Here v_a^{ex} and d_i' are exotic particles whose charge must be defined. It is easy to find out the relation between charge of the exotic particle and the hypercharge Y throughout the factor a and N by mea the well-known Gell-Mann - Nishijiman relation. We have:

$$\frac{1}{2}\frac{a}{\sqrt{3}} + N = -\frac{1}{2} \quad (3)$$

$$-\frac{a}{\sqrt{3}} + N = Q_v \quad (4)$$

the neutrino-lepton triplet)

$$\frac{1}{2}\frac{a}{\sqrt{3}} + N' = \frac{1}{6} \quad (5)$$

$$-\frac{a}{\sqrt{3}} + N' = Q_{d'}. \quad (6)$$

the quark triplet).

e Q_v and $Q_{d'}$ are the charges of the exotic neutrino (lepton) and the exotic quark respectively. m these relations, we have the relation between these exotic charges:

$$Q_{d'} - Q_v = 2/3 \quad (7)$$

There is an other fundamental representation of $SU(3)$ of 3-dimension, e.g. the antitriplet resentation (see [8] or Appendix of [4]). In this representation, the relation of hypercharge Y is follows: $Y = a\overline{\lambda}_8 + 2\overline{N}$ where $\overline{\lambda}_8 = \text{diag}\, 1/\sqrt{3}(2,-1,-1)$. The form of the neutrino-lepton and rk antitriplet is

$$\Psi_{a_L} = \begin{pmatrix} l_a^{ex} \\ v_a \\ e_a \end{pmatrix}_L \sim (3^*, \overline{N}); \quad Q_{i_L} = \begin{pmatrix} u_i' \\ u_i \\ d_i \end{pmatrix}_L \sim (3^*, \overline{N}') \quad (8)$$

$$a=1,2,3, \; i=1,2,3$$

re l_a^{ex} and u_i' are the exotic particles and have the charge Q_ℓ and $Q_{u'}$ respectively. The analogue tions are:

$$-\frac{1}{2}\frac{a}{\sqrt{3}} + \overline{N} = -\frac{1}{2} \quad (9)$$

$$\frac{a}{\sqrt{3}} + \overline{N} = Q_\ell \quad (10)$$

$$-\frac{1}{2}\frac{a}{\sqrt{3}} + \overline{N}' = \frac{1}{6} \quad (11)$$

$$\frac{a}{\sqrt{3}} + \overline{N}' = Q_{u'} \quad (12)$$

We have also the following relation:

$$Q_{u'} - Q_\ell = 2/3 \quad (13)$$

According to the choice of the triplets and the antitriplets and to the way of the anomaly cellation, we have a further relation of the charges of the particles in the multiplets and then can find out the value of a, N, $\overline{N}$ and Q of the exotic particles. Thus, we can obtain all different $(3)_L \times U(1)_N$ models. Here we do not pay attention to the singlets of $U(1)_N$ concerning the t-handed particle as it is easy to find them.

3. Now, let us consider the choice of the model. There are two ways of choosing the triplet the antitriplets as follow:

1) Two triplets in which, one is the neutrino-lepton multitriplet, and the other is the multiplet (the any one of the three quark multiplets), two quark antitriplets.

2) Two quark multiplets, two antitriplets in which one is the neutrino-lepton and the ot the quark multiplets.

By taking into account the three generations of neutrino-lepton and the three colors of qu there is in both cases, the equal number of triplets and of antitriplets, i.e. 6 multiplets for type.

To ensure the anomaly-free, there are two cases:

a. The algebraic sum of the charges of all the particles in all triplets (or in all antitri must be vanished.

b. The algebraic sum of the charges of the particles in all triplets and antitriplets mu vanished.

So we have four following types of models:

Type a1 This type consists of two triplets (neutrino-lepton and quark triplets) and two antitr of quarks. The sum of charges in the two triplets vanishes (so as in the two antitriplets).

For the triplets, we have the following relation:

$$Q_v + Q_{d'} = 2/3$$

The eqs [7] and [14] give use; $Q_v = 0$; $Q_{d'} = 2/3$. And from these, we have:

$$a = -\frac{\sqrt{3}}{3};\ N = -\frac{1}{\sqrt{3}};\ N' = \frac{1}{3}$$

for the antitriplets, we have

$$Q_{U'} = -\frac{1}{3};\ a = -\frac{\sqrt{3}}{3};\ \overline{N} = 0$$

Finally, we have a $SU(3)_L \times U(1)_N$ model with the following multiplets:

$$\Psi_{aL} = \begin{pmatrix} v_a \\ e_a \\ v_a^{ex} \end{pmatrix}_L \sim (3, -\frac{1}{3});\ Q_{\ell_L} = \begin{pmatrix} u_e ll \\ d_\ell \\ d'_\ell \end{pmatrix}_L \sim (3, \frac{1}{3});\ Q_{i_L} = \begin{pmatrix} u'_i \\ u_i \\ d_i \end{pmatrix}_L \sim (3^*, 0),\ i = 1$$

It is just the model suggested by R. Foot, H. N. Long and T. A. Tuan (see [7]).

Type a2 This type satisfies the two points a and 2 above.

By the same way, we have the following result:

$$\Psi_{aL} = \begin{pmatrix} \ell_a^{ex} \\ v_a \\ e_a \end{pmatrix}_L \sim (3^*, \frac{1}{3});\ Q_{\ell_L} = \begin{pmatrix} u'_\ell \\ u_\ell \\ d_\ell \end{pmatrix}_L \sim (3^*, -\frac{1}{3});\ Q_{i_L} = \begin{pmatrix} u_i e \\ d_i \\ d'_i \end{pmatrix}_L \sim (3, 0),\ i = 1$$

It is just the model I of F. Pisano and V. Pleitez [4].

b1 This type satisfies the two points b and 1 above.

'he point b gives us:

$$2Q_{u'} + Q_1 + Q_{d'} = 0 \quad (19)$$

eqs (5), (6) and (9) - (12), we have:

$$2Q_{d'} - 1/3 = -(2Q_{u'} - 1/3) = 1 + 2Q_1 \quad (20)$$

qs (19) and (20) are not linear-independent. So we consider one of three quantities Q_v, $Q_{u'}$ $Q_{d'}$ as a parameter and give it an arbitrary value.

'hoosing $Q_1 = 1$, we have $Q_{d'} = 5/3$ and $Q_{u'} = -4/3$ and then:
$a = -\sqrt{3}$, $N = 0$, $N' = 2/3$ and $N = -1/3$.

'he triplets and antitriplets are:

$$= \begin{pmatrix} v_a \\ e_a \\ \ell_a^{ex} \end{pmatrix}_L \sim (3, 0); \; Q_{\ell_L} = \begin{pmatrix} u_\ell \\ d_\ell \\ d'_\ell \end{pmatrix}_L \sim (3, \frac{2}{3}); \; Q_{i_L} = \begin{pmatrix} u'_i \\ u_i \\ d_i \end{pmatrix}_L \sim (3^*, -\frac{1}{2}), \; i = 1, 2. \quad (21)$$

Ve have the model II of F. Pisano and V. Pleitez [2]. If we give Q_ℓ a positive multiplet of e, ve the Model III of F. Pisano and V. Pleitze [4]. So we have a set of models of type b1.

b2 Similarly, we have the following relations:

$$2Q_{d'} + Q_{u'} + Q_\ell = 0 \quad (22)$$

$$-(2Q_{d'} - 1/3) = 2Q_{u'} - 1/3 = 1 + 2Q\ell \quad (23)$$

e eqs (22) and (23) are not linear independent, we choose $Q\ell$ the parameter.

'or $Q_\ell = 1$ we have $Q_{u'} = 5/3$ and $Q_{d'} = -4/3$, then $a = -\sqrt{3}$, $N' = 2/3$, $\overline{N} = -1$, $-1/3$.

'he triplets and the antitriplets are:

$$= \begin{pmatrix} l_a^{ex} \\ v_a \\ e_a \end{pmatrix}_L \sim (3^*, -1); \; Q_{\ell_L} = \begin{pmatrix} u'_\ell \\ u_\ell \\ d_\ell \end{pmatrix}_L \sim (3^*, -\frac{1}{3}); \; Q_{i_L} = \begin{pmatrix} u_i \\ d_i \\ d'_i \end{pmatrix}_L \sim (3, \frac{2}{3}), \; i = 1, 2. \quad (24)$$

'hese multiplets belong to a new model not yet appeared up to now in the literature. We ılso a set of models of type b2 corresponding to different values of Q_ℓ.

merising, we have found out and classified all the $SU(3)_L \times U(1)_N$ models into four different The models a1 and a2 are conjugated each other. The types b1 and b2 do not only include model but according to the value of the parameter Q_ℓ each type consists of a set of models. are conjugated each other, too. The found models do not only coincide to the well-known ›ut also include of the new ones. We know that the 331 model is the extending model of the There are many interesting effects of this model: the appearance of a new boson Z', the Z ıixing angle, the existence of the FCNC (flavour - changing neutral currents) etc. All these ›ms are the consequence of the choice of the model. So we hope that the classification of the s will be useful in studying them. In the next paper we will concern with this problem.

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CHỌN CÁC MẪU $SU(3)_L \times U(1)_N$ DỰA TRÊN KHỬ DỊ THƯỜNG

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