THEOREM OF STANDARD FORM FOR SELF-MODIFYING SOME CLOSURE PROPERTIES AND DECISION PROBL

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The construction of a standard form for self-modifying nets is interesting, as be used in different cases, e.g. to prove the closure properties with respect tc and concatenation. As in ordinary Petri-nets, λ-transitions play an important rol paper gives a construction of standard form for self-modifying nets with terminal m Some closure properties of associated languages are shown and some decision pr are solved.

1. NOTIONS AND DEFINITIONS.

Definition 1. $I\!N$ is a set of nonnegative integers. A *self-modifying net* is a 5-tuple:

$$N = \{P, T, \text{Pre}, \text{Post}, M_o\}$$

where $P = \{p_1, p_2, ..., p_{|P|}\}$ (Set of places)
$T = \{t_1, t_2, ..., t_{|T|}\}$ (Set of transitions)
$P \cap T = \emptyset$
M_o is a $|P|$-dimensional vector (the initial marking)
$\text{Pre} : P \times P_1 \times T \to I\!N$
$\text{Post} : T \times P_1 \times P \to I\!N$, where $P_1 = P \cup \{1\}$ $(1 \notin P)$.
N is called post-self-modifying net, if

$$\text{Pre} : P \times \{1\} \to I\!N \quad \text{and} \quad \text{Post} : T \times P_1 \times P \to I\!N.$$

The marking M which is a mapping from P into $I\!N$ is denoted by a $|P|$-dimensional
$M = (M(p_1), M(p_2), ..., M(p_{|p|})) \in I\!N^{|P|}$ where $M(p_i)$ means the token in place p_i
Given $M \in I\!N^{|P|}$ being a marking, then the function
$V_M : P_1 \to I\!N$ defined by $V_M(q) :=$ IF $q \in P$ THEN $M(q)$ ELSE 1.

Definition 2. A transition $t \in T$ is said to be *firable at* M, if for all $p \in P$

$$M(p) >= \sum_{q \in P_1} \text{pre}(p, q, t).V_M(q).$$

Definition 3. A transition $t \in T$ is *firable* from the marking M to M'

$$M \xrightarrow{t} M' :\Longleftrightarrow t \text{ is firable at } M$$

and

$$\forall p \in P : M'(p) = M(p) - \sum_{q \in P_1} \text{pre}(p, q, t).V_M(q) + \sum_{q \in P_1} \text{post}(t, q, p).V_M(q)$$

tion 4. For any word $W = t_{i1}t_{i2}...t_{in}$ and two marking M and M', the firing relation M' will be defined by the following recursion

$$M \xrightarrow{\lambda} M \quad (\lambda - empty\ word)$$

$$M \xrightarrow{W_t} M' \iff \exists M" \in \mathbb{N}^{|P|} : M \xrightarrow{W} M" \quad \text{and} \quad M" \xrightarrow{t} M'.$$

tion 5. For any transition $t \in T$ and marking $M \in \mathbb{N}^{|P|}$, we define two marking t_{M^-},

$$t_{M^-} := \sum_{q \in P_1} \text{pre}(p,q,t).V_M(q) \quad (p \in P)$$

$$t_{M^+} := \sum_{q \in P_1} \text{post}(t,q,p).V_M(q) \quad (p \in P)$$

ition 6. Let $N = (P, T, \text{pre}, \text{post}, M_o)$ be a self-modifying-net (post-modifying-net) $: T \to X \cup \{\lambda\}$ be a labelled function, among them X is a labelled alphabet. Then $\mathcal{N}, h, X, M_f)$ is called a *labelled net with terminal marking* M_f.

ition 7. Let $A = (P, T, \text{pre}, \text{post}, M_o, h, X, M_f)$ be a labelled SM-net with terminal ng. Then A is called *standard form*, if

$$M_o = (1, 0, ..., 0) \in \mathbb{N}^{|P|} \qquad M_f = (0, 0, ..., 0) \in \mathbb{N}^{|P|}.$$

ions. Let $N = (P, T, \text{pre}, \text{post}, M_o)$ be a self-modifying net, M_f a terminal marking, $: T \to X \cup \{\lambda\}$ a labelling function (in the case $h : T \to X \cup \{\lambda\}$ is called λ-*free*, if $\lambda \quad \forall t \in T$).

$$R_N(M_o) := \{M \in \mathbb{N}^{|P|} | \exists W \in T^* : M_o \xrightarrow{W} M\} (\textit{reachability set}).$$

ollowing families of languages are defined

$_o(N) := \{W \in T^* | \exists M \in \mathbb{N}^{|P|} : M_o \xrightarrow{W} M\}$ (*firing sequences of* N)

$(N, M_f) := \{W \in T^* | M \xrightarrow{W} M_f\}$

$(N, h) := \{h(W) | \exists M \in \mathbb{N}^{|P|} : M_o \xrightarrow{W} M\}$

$(N, h, M_f) := \{h(W) | \exists M \in \mathbb{N}^{|P|} : M_o \xrightarrow{W} M_f\}$

$_{SM(PSM)}(N) := \{L_o(N) | N$: self-modifying net (post-self-modifying net)$\}$

$_{SM(PSM)}(N, M_f) := \{L_o(N, M_f) | N$:self-modifying net (post-self-modifying net, M_f ter- marking)$\}$

$_{SM(PSM)}(N, h) := \{L(N, h) | N : SM(PSM)$-net and h : λ-free labelling function$\}$

$_{SM(PSM)}(N, h, M_f) := \{L(N, h, M_f) | N : SM(PSM)$-net and h : λ-free labelled function M_f terminal marking$\}$

$^{\lambda}_{SM(PSM)}(N, h, M_f) := \{L(N, h, M_f) | N : SM(PSM)$-net and h: labelled function and rminal marking$\}$.

3. SOME RESULTS.

em 1. (*Standard form*) For any labelled SM-net (PSM-net) with terminal marking

$$A = (P, T, \text{pre}, \text{post}, M_o, h, X, M_f)$$

there exists a labelled SM-net (PSM-net):

$$A' = (P', T', \text{pre}', \text{post}', M'_o, h', X', M'_f),$$

which is equivalent to A in the sense of the languages, such that

$$M'_o = (1, 0, ..., 0) \quad \& \quad M'_f = (0, 0, ..., 0).$$

Proof.

a) Let $A = (P, T, \text{pre}, \text{post}, M_o, h, X, M_f)$ be a SM-net with terminal marking M_f,

$$P := \{p_1, p_2, ..., p_n\}, \quad T := \{t_1, t_2, ..., t_m\}$$

$$h : T \to X \cup \{\lambda\} \quad \text{(labelling function)}$$

$$M_o := (X_1, X_2, ..., X_n) \quad \text{(initial marking)}$$

$$M_f := (Y_1, Y_2, ..., Y_n) \quad \text{(terminal marking)}$$

Addition $(n+1)$ new places : p_o (*start-place*), p_{n+1}, p_{n+2}, ..., p_{2n} (*end-place*), and tw transitions: t_o (*first-transition*), t_{m+1} (*stop-transition*).

Notation. $f|_B$:= restricted function on B.

Also, then the net $A' = (P', T', \text{pre}', \text{post}', M'_o, h', X', M'_f)$ will be defined as follow

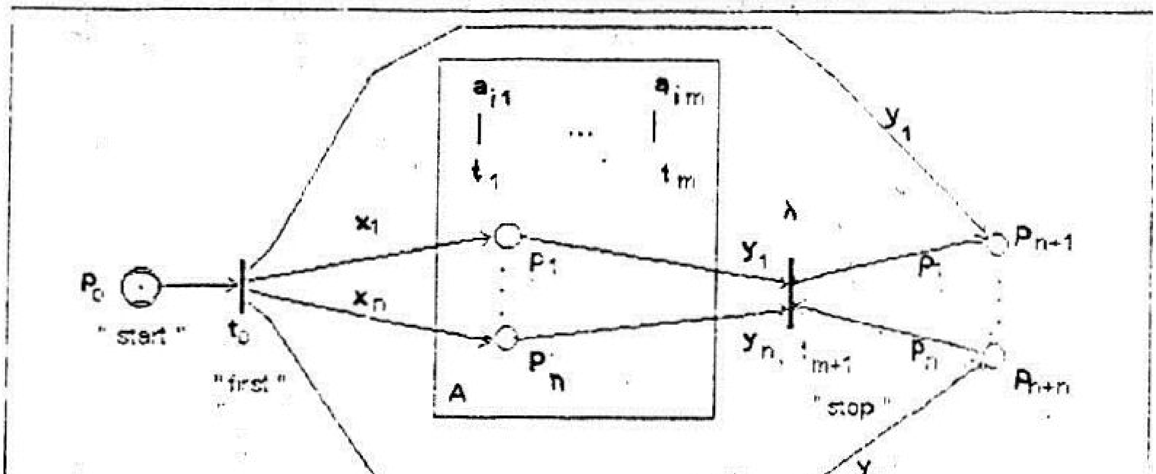

Fig. 1: The net A' in standard form

where $P' := P \cup \{p_o, p_{n+1}, p_{n+2}, ..., p_{2n}\}$, $\quad T' = t \cup \{t_o, t_{m+1}\}$

$X' := X$; $\quad \text{pre}'|_{P \times P_1 \times T} = \text{pre}|_{P \times P_1 \times T}$

$\text{pre}'(p_o, 1, t_o) = 1, \quad \text{pre}'(p_i, 1, t_{m+1}) = Y_i \quad (i = 1, 2, ..., n)$

$\text{pre}'(p_{n+i}, p_i, t_{m+1}) = 1 \quad (i = 1, 2, ..., n)$

$\text{post}'|_{T \times P_1 \times P} = \text{post}|_{T \times P_1 \times P}$

$\text{post}'(t_o, 1, p_i) = X_i \quad (i = 1, 2, ..., n)$.

$\text{post}'(t_o, 1, p_{n+i}) = Y_i \quad (i = 1, 2, ..., n)$

$h'|_T = h|_T; \quad h'(t_o) = h'(t_{m+1}) = \lambda$

$:= (1, 0, ..., 0) \in \mathbb{N}^{|P|}$
$:= (0, 0, ..., 0) \in \mathbb{N}^{|P'|}$

is easy to see that the transition t_o fired only one time to transform the net A into ;ial marking. The transition t_{m+1} also tired only one time when the net A run into minal marking M_f, and finally the transition t_{m+1} fired to transform the net A' into minal marking $M'_f = (0, ..., 0)$.
›reover, $L(N, h, M_f) = L(N', h', M'_f)$ where

$$N = (P, T, \text{pre}, \text{post}, M_o) \quad \& \quad N' = (P', T', \text{pre}', \text{post}', M'_o).$$

Let $A = (P, T, \text{pre}, \text{post}, M_o, h, X, M_f)$ be labelled PSM-net with terminal marking irst, we can transform the PSM-net A as the SM-net A' similar to a). But in the net re exist the arcs $p_{n+1} : 0 \xrightarrow{p_i} I^{\lambda}_{t_{m+1}}$ that are not in form of PSM-net, so by using the ds of R.Valk [2], we transform the arc PSM-net by respected transform one time token based on λ-transition:

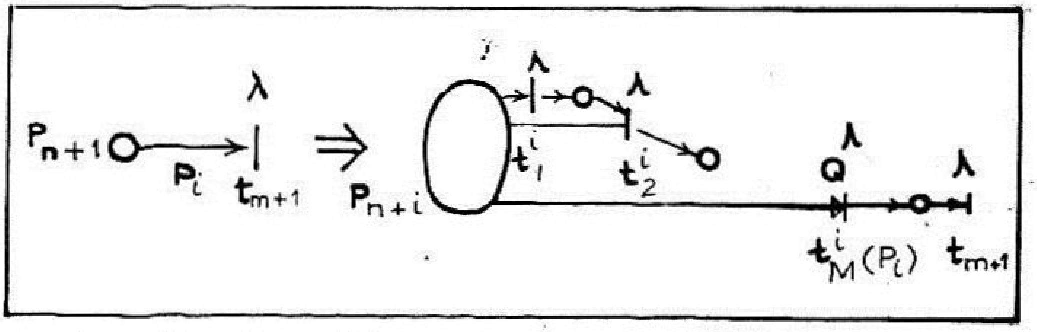

Fig.2: Transform of the arc (p_{n+i}, p_i, t_{m+1}) *in the form of PSM*

en we change back the SM-net A' into PSM-net $A"$ satisfies

$$L(N, h, M_f) = L(N', h', M'_f) = L(N", h". M"_f)$$

$h"$ is defined by h' and Fig. 2, $M" = (1, 0, ..., 0)$, $M"_f = (0, 0, ..., 0)$.

em 2. The $L^{\lambda}_{SM(PSM)}(N, h, M_f)$ is closed under union, concatenation, intersection, ıorphism, inverse homomorphisms and intersection with regular sets.
oof. In this report, we make the proof only for the closure to the union and contion, the others will be seen in R.Valk's works [2].

ıure to the union.
cording to the theorem 1, we suppose that

$$L_1 = L(N_1, h_1, M_{f1}), \quad L_2 = L(N_2, h_2, M_{f2}) \in L^{\lambda}_{SM(PSM)}(N, h, M_f)$$

them the labelled nets with terminal marking
$= L(N_1, h_1, X_1, M_{f1})$ & $A_2 = L(N_2, h_2, X_2, M_{f2})$ are in standard form.
ıen the start-places of these two nets are coincident, we get a labelled net A with al marking, satisfied

$$L_1 \cup L_2 = L(N, h, M_f),$$

where $A = (P, T, \text{pre}, \text{post}, M_o, h, X, M_f)$: $M_o = (1, 0, ..., 0)$ and $M_f = (0, 0, ..., 0)$, function h is defined on the bases of h_1 and h_2:

$$\forall t \in T = T_1 \cup T_2 : \quad h(t) = \begin{cases} h_1(t), & \text{if } t \in T_1 \\ h_2(t), & \text{if } t \in T_2 \end{cases}$$

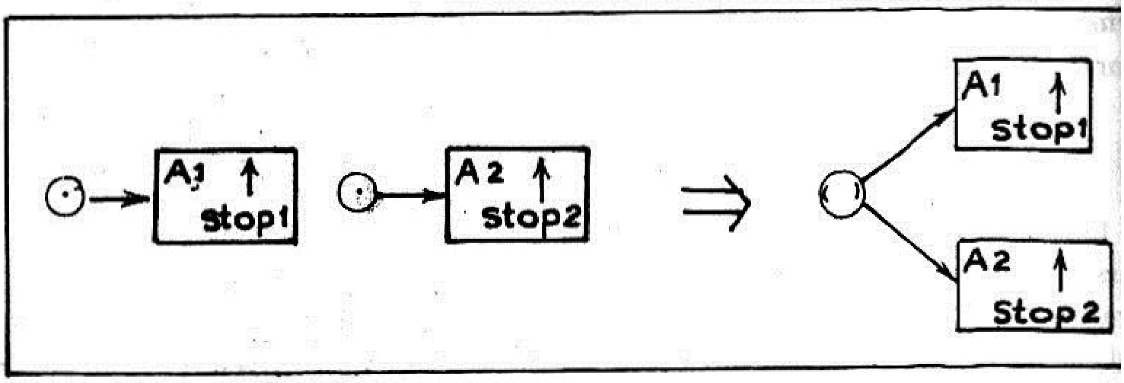

Fig. 3: Union of A_1 and A_2

b) **Closure to concatenation.**

Similarly, let $L_1 = L(N_1, h_1, M_{f1})$, $L_2 = L(N_2, h_2, M_{f2})$ be two languages gene the nets standard form. Then two nets are concatened by the connection of the stop-transition 1 of the net A_1 with start-place 2 of the net A_2 and we have got t satisfied

$$L = L_1.L_2 = L(N, h, M_f) : A = (P, T, \text{pre}, \text{post}, M_o, h, X, M_f) = (N, h, X, M_f), \quad \text{where } M_o = (1, 0, ..., 0)$$

$M_f = (0, 0, ..., 0)$ and h is defined on the base of h_1 and h_2:

$$\forall t \in T = T_1 \cup T_2 : \quad h(t) = \begin{cases} h_1(t), & \text{if } t \in T_1 \\ h_2(t), & \text{if } t \in T_2 \end{cases}$$

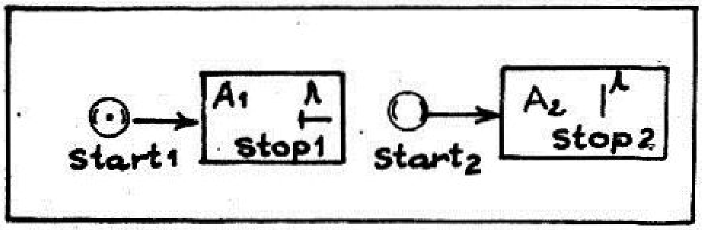

Fig. 4: Concatenation of A_1 and A_2

Notes. In the above proof, we considere $T_1 \cap T_2 = \emptyset$ as when $T_1 \cap T_2 <> \emptyset$, rename the transition of T_2, and the functions pre, post are defined according to the of the given nets.

4. Some decision problems.

1) *The empty problems.*

em 3.

The empty problem for the $L_{SM(PSM)}(N)$ is decidable.

The empty problem for the $L_{SM(PSM)}(N, M_f)$ is undeciable.

oof.

Let $L = L_o(N) \in L_{SM(PSM)}(N)$ be a language generated by

$$N = (P, T, \text{pre}, \text{post}, M_o).$$

$; = \emptyset$ will be decided after the following steps.

ep 1. Caculate all $t^-_{iM_o}$ $(i = 1...|T|)$

ep 2. Find out, if there exists $t^-_{jM_o}$ such that $M_o >= t^-_{jM_o}$

ep 3. Conclusion, if it exists so $L <> \emptyset$; if not, so $L = \emptyset$.

Let $L = L(N, M_f) \in L_{SM(PSM)}$ a language generated by

$$N = (P, T, \text{pre}, \text{post}, M_o)$$

;he terminal marking M_f. Then $L = \emptyset$ if and only if $M_f \in R_n(M_o)$. Because the ıbility problem is undecidable for the $SM(PSM)$-nets, so the empty problem for $_{SM)}(N, M_f)$ is undecidable too (see R.Valk [2]).

| *The membership problem.*

:em 4. The membership problem for the

$$L_{SM(PSM)}(N), \quad L_{SM(PSM)}(N, M_f), \quad L_{SM(PSM)}(N, h), \quad L_{SM(PSM)}(N, M_f)$$

|dable.

roof.

| Let $L = L_o(N) \in L_{SM(PSM)}(N)$ be a language generated by

$$N = (P, T, \text{pre}, \text{post}, M_o)$$

$= t_{i1}t_{i2}...t_{ik} \in T^*$. Then $W = t_{i1}t_{i2}...t_{ik}$ belong to $L_o(N)$, if there are markings M_o, , M_k with

$$M_o \xrightarrow{t_{i1}} M_1, M_1 \xrightarrow{t_{i2}} M_2, ..., M_{k-1} \xrightarrow{t_{ik}} M_k$$

ı the contrary $W \notin L_o(N)$.

milarly, let $L = L(N, M_f) \in L_{SM(PSM)}(N, M_f)$, and $W = t_{i1}t_{i2}...t_{in} \in T^*$. Then $W =$ t_{ik} belongs to $L_o(N, M_f)$, if there are markings M_o, M_1, ..., M_k with

$$M_o \xrightarrow{t_{i1}} M_1, M_1 \xrightarrow{t_{i2}} M_2, ..., M_{k-1} \xrightarrow{t_{ik}} M_k$$

ı the contrary $W \notin L_o(N, M_f)$.

Let $L = L(N, h) \in L_{SM(PSM)}(N, h)$ (or $L = L(N, h, M_f) \in L_{SM(PSM)}$), and β is a finite :ontaining the label such that $h(W) = \beta$ with $h : T \rightarrow nX$ (λ-free labelled function) $= t_{i1}t_{i2} \cdots t_{in}$.

milarly to the methodes applied to the above part a), we see where the word β ;s to $L(N, h)$ (or $L = L(N, h, M_f)$) by a finite number of tests. So, the membership ım for the $L_{SM(PSM)}(N, h)(L_{SM(PSM)}(N, M_f))$ is decidable.

3) *The finite problem.*

Theorem 5. The finite problem for the $L_{PSM}(N)$, $L_{PSM}(N,h)$ is decidable.

Proof. Let $L = L_o(N) \in L_{PSM}(N)$ be a language generated by the post-self-moc net $N = (P, T, \text{pre}, \text{post}, M_o)$. It is added to the place and linked every transition to p,

This place has not coming-out-way and it is used to count the firing of transiti the net N:

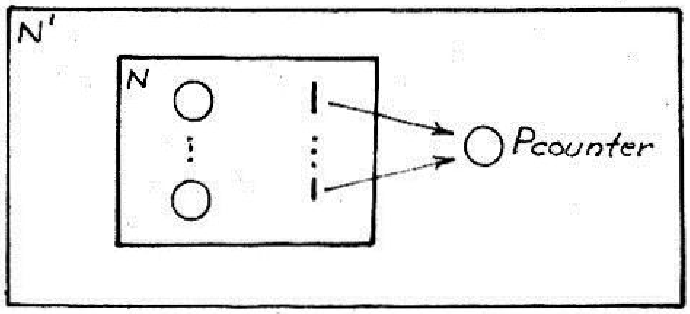

Fig.5: The net

It is easy to see that $p_{counter}$ is bounded if and only if in the net N' there is no i sequence $t_{i1}, t_{i2}, ..., t_{ik}, ...$ from M_o:

$$M_o \xrightarrow{t_{i1}} M_1, M_1 \xrightarrow{t_{i2}} M_2, ..., M_{k-1} \xrightarrow{t_{ik}} M_k$$

and then according to R.Valk [2]: since the bounded problem for the post-self-modi net is decidable, so the finite problem for the PSM-net is decidable too.

Similarly, the finite problem for $L_{PSM}(N,h)$ is decidable: since $h <> \lambda$, so the $p_{counter}$ is bounded if and only if in the labelled net $A := (N', h')$ with $h' = h$, there any infinite firing sequence of labels $a_{i1}, a_{i2}, ..., a_{ik}, ...$ starting from M_o:

$$M_o \xrightarrow{a_{i1}} M_1, M_1 \xrightarrow{a_{i2}} M_2, ..., M_{k-1} \xrightarrow{a_{ik}} M_k$$

where $a_{ij} \in X$ and $h : T \to X$.

So, as the above mentionned, the finite problem for the $L_{PSM}(N,h)$ is decidabl

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ĐỊNH LÝ VỀ DẠNG CHUẨN TẮC CỦA LƯỚI PETRI SUY RỘNG, MỘT SỐ TÍNH CHẤT ĐÓNG VÀ VẤN ĐỀ GIẢI ĐƯỢC

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Bài báo đề cập tới "Định lý về dạng chuẩn tắc của lưới Petri-suy rộng, một số tính chất ; và vấn đề giải được". Chúng tôi đã xây dựng dạng chuẩn tắc của lưới Petri-suy rộng, ıg minh một số tính chất đóng như kết hợp, kết nối, giao đồng cấu, đồng cấu ngược iao với tập chính quy. Đồng thời chứng minh một vài bài toán quyết định đối với Petri-suy rộng này như: Bài toán rỗng đối với $LSM(PSM)(N)$, $L_{SM(PSM)}(N, M_f)$; bài thuộc đối với $L_{SM(PSM)}(N)$, $L_{SM(PSM)}(N, M_f)$, $L_{SM(PSM)}(N, h)$, $L_{SM(PSM)}(N, h, M_f)$, ài toán hữu hạn đối với $L_{PSM}(N)$, $L_{PSM}(N, h)$