

INFLUENCE OF THE RELATION BETWEEN HOMOGENEOUS AND INHOMOGENEOUS BROADENING ON THE OPTICAL BISTABILITY EFFECT

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§1. INTRODUCTION

The homogeneous and inhomogeneous broadening take an important role in action of different lasers. Its characters of the laser output varies when these broadening change. This point is affirmed by many experimental and theoretical publications [1-4]. For lasers containing saturable absorber (LSA), the above effect has been seen also in many papers [5]. In one of our articles [10] we have studied the influence of the inhomogeneous broadening on the characteristics of the optical bistability effect (O. B.) but this research is still preliminary. In this paper we would like to examine simultaneously the influence of the relation between homogeneous and inhomogeneous broadening on the O. B. effect in LSA. We shall take the case where the emission mode is the resonant mode, because in this mode the OB effect is biggest as point in [11].

The problem is solved for a monomode LSA of ring resonator. Basic equations are presented in §2, the results in §3 and discussions in §4.

§2. BASIC EQUATIONS

Action of a ring LSA with inhomogeneous broadening for resonant mode can be described by the following equation in Rate Equation Approximation:

$$\frac{dn_o}{dt} = -\chi_o n_o + \sum \beta g(\omega_\mu - \omega_o)(n_o + 1)[N_{\mu_a} - N_{\mu_b}] \quad (1)$$

$$\frac{dN_{\mu_a}}{dt} = R_{\mu_a} - N_{\mu_a}[\beta g(\omega_\mu - \omega_o)n_o + \gamma_a] \quad (2)$$

$$\frac{dN_{\mu_b}}{dt} = R_{\mu_b} - N_{\mu_b}[\beta g(\omega_\mu - \omega_o)n_o + \gamma_b] \quad (3)$$

where

n_o - the photon density resonant mode.

N_{μ_a}, N_{μ_b} - population inversion in active and absorptive medium.

β - Einstein coefficient.

γ_a, γ_b - relaxation coefficient of the upper level in two levels schema, $\gamma_b = \xi \gamma_a$ with ξ depends on the saturation coefficient. χ_o - the resonator loss of resonant mode of frequency

$$g(\omega_\mu - \omega_o) = \frac{\Gamma^2}{\Gamma^2 + 4(\omega_\mu - \omega_o)^2}, \quad \Gamma - \text{homogeneous broadening}$$

$$R_{\mu_a} = R_o \frac{\omega^2}{\omega^2 + 4(\omega_\mu - \omega_o)^2} \quad \text{the pump energy for active medium,}$$

ϵ - inhomogeneous broadening.

R_o - the pump energy for absorptive medium and supposed to be constant.

The sum over μ exhibits the presence of inhomogeneous broadening at LSA action

We determine the photon intensity $Q_0 = \beta n_0 / \gamma$ ($\gamma \equiv \gamma_a$) by solving the system of equations (1) - (3) in stationary case

$$\frac{dn_0}{dt} = \frac{dN_{\mu_a}}{dt} = \frac{dN_{\mu_b}}{dt} = 0$$

Changing the sum over μ by integral with the form

$$\sum_{\mu} f(\omega_{\mu}) \rightarrow \frac{1}{\pi\omega} \int_{-\infty}^{+\infty} f(\omega_{\mu}) d\mu$$

after a long calculation, we obtained the following equation for Q_0

$$\begin{aligned} Q_0^3 + \left[(1 + 2\xi) + \frac{1}{1 + 2\alpha} - \frac{\alpha\sigma_0}{1 + 2\alpha} + \alpha\sigma_b \right] Q_0^2 \\ + \left[2\xi + \frac{2\xi}{1 + 2\alpha} - \frac{2\alpha\xi}{1 + 2\alpha} \sigma_0 + \frac{2\alpha(1 + \alpha)}{1 + 2\alpha} \sigma_b - \frac{\beta}{\gamma} \frac{\sigma_0}{1 + 2\alpha} + \frac{\beta}{\gamma} \alpha\sigma_b \right] Q_0 \\ - 2 \frac{\beta}{\gamma} \left[\frac{\alpha\xi}{1 + 2\alpha} \sigma_0 - \frac{\alpha(1 + \alpha)}{1 + 2\alpha} \sigma_b \right] = 0 \end{aligned}$$

Here $\alpha = \Gamma/\epsilon$ parameter characterizing the relation between homogeneous and inhomogeneous broadening.

$$\sigma_0 = \frac{\beta R_0}{\gamma \chi_0}, \quad \sigma_b = \frac{\beta R_{\mu b}}{\gamma \chi_0}$$

Equation (5) is an equation of third degree. Taking all parameters, unless σ_0 , being constant, we can obtain the expression of Q_0 as a function of σ_0 and determine the appearance of OB effect.

§3. THE INFLUENCE OF THE RELATION BETWEEN HOMOGENEOUS AND INHOMOGENEOUS BROADENING

In order to examine this influence, we have to solve equation (5) by the variation method.

3.1. Expressions of Q_0

Following variation method, we can receive three solutions as

$$\begin{aligned} Q_{01} &= \frac{\frac{\beta}{\gamma} [\alpha\xi\sigma_0 - \alpha(1 + \alpha)\sigma_b]}{2\xi(1 + \alpha) + \alpha + \alpha^2 - (\alpha\xi + \frac{\alpha}{2} + \frac{\beta}{\gamma})\sigma_0 + \frac{\beta\alpha}{2\gamma}(1 + 2\alpha)\sigma_b} \\ Q_{02,3} &= \bar{Q}_{02,3} - \frac{d}{3a\bar{Q}_{02,3} + 2b\bar{Q}_{02,3} + c} \end{aligned}$$

with

$$\bar{Q}_{02,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where

$$a = 1$$

$$b = \left[1 + 2\xi + \frac{1}{1 + 2\alpha} - \alpha \left(\frac{\sigma_0}{1 + 2\alpha} - \sigma_b \right) \right]$$

$$c = \left\{ 2\xi + \frac{2\xi}{1 + 2\alpha} + \frac{2\alpha}{1 + 2\alpha} [(1 + \alpha)\sigma_b - \xi\sigma_0] - \frac{\beta}{\gamma} \left[\frac{\sigma_0}{1 + 2\alpha} - \alpha\sigma_b \right] \right\}$$

$$d = -\frac{\beta}{\gamma} \left(\frac{2\alpha}{1 + 2\alpha} \right) [\xi\sigma_0 - (1 + \alpha)\sigma_b]$$

ly Q_{01}, Q_{02}, Q_{03} are functions of parameters $\alpha, \xi, \sigma_b, \sigma_o$.

2. The OB interval I_{OB}

OB interval is the one where the two solutions Q_{02} and Q_{03} are positive. The considered of d is very small (order of $\frac{\beta}{\gamma} \sim 10^{-11}$ for gas laser) the sign of $Q_{02,3}$ are identical $\bar{Q}_{02,3}$. Following criteria of the positive solutions of an second degree equation, we obtain condition for σ_o in order to have OB effect:

$$\sigma_{01} < \sigma_o < \sigma_{02} \quad (9)$$

$$\sigma_{02} = (1 + \alpha) \left(\frac{2}{\alpha} + \frac{\sigma_b}{\xi} \right) + \frac{\beta(1 + 2\alpha)\sigma_b}{2\xi} \quad (10)$$

$$\begin{aligned} \sigma_{01} = & \frac{(1 + 2\alpha)}{\alpha} \left[1 - 2\xi + \frac{1}{1 + 2\alpha} + \alpha\sigma_b - 2\frac{\beta}{\gamma} \right] + 2(1 + 2\alpha) \\ & \times \left[-\frac{2\xi\sigma_b}{\alpha} + \frac{2(1 + \alpha)}{\alpha(1 + 2\alpha)}\sigma_b + 2\frac{\beta}{\gamma} \left(\frac{\xi}{\alpha^2} - \frac{1 + \alpha}{\alpha} \right)^{1/2} \right] \end{aligned} \quad (11)$$

interval is the difference between σ_{02} and σ_{01} .

$$I_{OB} = |\sigma_{02} - \sigma_{01}| \quad (12)$$

values of σ_o placed between σ_{01} and σ_{02} will give two positive values of Q_o i.e. the OB appears. Clearly that, OB interval transforms when $\alpha = \Gamma/\epsilon$ varies.

3. Influence of the relation of α

We determine numerically values of Q_o and draw the curves of function $Q_o(\sigma_o)$ by using $\sigma_b = 20, \xi = 0,25$ and taking 3 value of α .

$$\alpha = 0,0235 \quad 0,003 \quad 0,05$$

The numerical values of I_{OB} and Q_{01}, Q_{02}, Q_{03} are presented in Tables 1, 2, 3, 4 and in 1 are also shown the curves of hysteresic cycle.

Table 1. I_{OB} interval.

	σ_{01}	σ_{02}
0,0235	159,45	168,98
0,003	137,96	151,02
0,05	105,24	126,0

Hình 1. (bên cạnh)

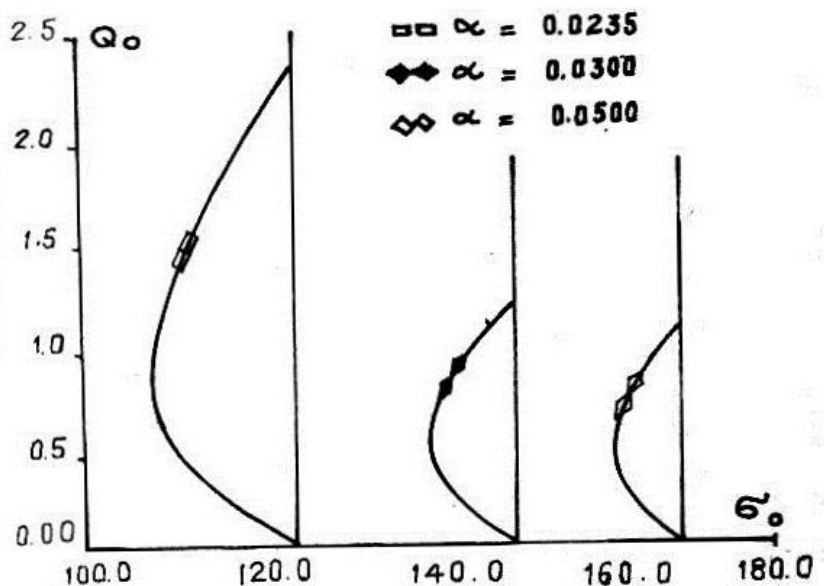


Table 2. $\alpha = 0,0125$ Table 3. $\alpha = 0,03$

σ_0	Q_{j_0}	Q_{j_1}	Q_{j_2}	σ_0	Q_{j_0}	Q_{j_1}	Q_{j_2}
159,4	2,48	0,32	0,32	137,96	1,47	0,43	0,43
160	2,64	0,43	0,23	138	1,48	0,46	0,39
160,5	2,91	0,47	0,19	138,5	1,54	0,55	0,31
161	3,006	0,51	0,17	139	1,62	0,61	0,27
162	3,46	0,57	0,13	139,5	1,69	0,65	0,24
162,5	3,74	0,60	0,12	140	1,78	0,69	0,22
163	4,07	0,62	0,10	141	1,98	0,75	0,18
164	4,93	0,67	0,08	142	2,22	0,81	0,15
164,5	5,50	0,69	0,07	143	2,52	0,87	0,13
165	6,22	0,71	0,06	144	2,90	0,92	0,10
166	8,37	0,75	0,04	145	3,42	0,97	0,08
166,5	10,09	0,77	0,03	146	4,13	1,01	0,07
167	12,69	0,79	0,02	147	5,20	1,06	0,05
167,5	17,02	0,81	0,02	148	6,965	1,106	0,039
168,5	52,45	0,85	0,006	149	10,43	1,14	0,02
168,98	401,19	0,86	$8,21 \cdot 10^{-5}$	150,5	38,57	1,20	0,006
168,98	7560	0,86	$4,29 \cdot 10^{-5}$	151	329,39	1,22	0,0007
168,98	18500	0,86	$1,60 \cdot 10^{-5}$	151,01	387,56	1,11	0,0006

Table 4. $\alpha = 0,05$

σ_0	Q_{j_0}	Q_{j_1}	Q_{j_2}	σ_0	Q_{j_0}	Q_{j_1}	Q_{j_2}
105,23	0,60	0,68	0,68	114	1,20	1,60	0,17
106,00	0,63	0,90	0,49	115	1,32	1,66	0,14
106,50	0,66	0,97	0,45	116	1,47	1,73	0,13
107	0,68	1,03	0,41	118	1,89	1,85	0,09
108	0,73	1,14	0,35	119	2,199	1,917	0,08
109	0,78	1,23	0,31	120	2,59	1,97	0,08
110	0,84	1,31	0,27	121	3,15	2,03	0,05
112	0,9	1,46	0,21	123	5,39	2,15	0,003
113	10,9	1,53	0,19	126	1399	2,31	0,0001

§4. DISCUSSIONS

From the tables and curves presented above, we see that:

Homogeneous and inhomogeneous broadening influence clearly to OB interval as the photon intensity Q_0 . These parameters role as the relaxation coefficient ξ and γ to the material constructing active and absorptive medium. These are interior parameters of OB effect.

The change of the relation $\alpha = \Gamma/\epsilon$ shows that at large values of α , we can obtain a big interval I_{OB} as well as intense OB effect. This means that for receiving a good hysteresis one can either augment the homogeneous broadening or diminish the inhomogeneous broadening. In other words the OB effect is big for the dye laser of LSA but small for gas of LSA. Author is grateful to N. Benghalem for numerical values from electronical computer.

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ẢNH HƯỞNG CỦA TỶ SỐ GIỮA MỞ RỘNG ĐỒNG NHẤT VÀ KHÔNG ĐỒNG NHẤT LÊN HIỆU ỨNG LƯỞNG ỔN ĐỊNH QUANG HỌC.

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Bài báo tìm hiểu ảnh hưởng của sự thay đổi tỷ số giữa mở rộng đồng nhất và không đồng nhất lên vùng lưỡng ổn định của đường cong trễ. Xuất phát từ hệ phương trình mô tả sự hoạt động của laser vòng có chứa vật liệu hấp thụ bão hòa đã giải bằng máy tính để tìm kiếm khoảng lưỡng ổn định. Sự thay đổi các giá trị tham số và tỷ số giữa mở rộng đồng nhất và không đồng nhất cho thấy có thể thu được hiệu lưỡng ổn định quang học tốt ở các giá trị tỷ số nói trên lớn.