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TABLE SPLITTING OF CLASSIFYING SPACES OF ELEMENTARY ABELIAN p-GROUPS

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t. By using representation theory and explicit sets of orthogonal idepotents which the 1 in the group ring $F_p[GL_n(Z/p)]$, we give certain splitting of the clasifying the elementary abelian *p-group* $(Z/p)^n$.

1. INTRODUCTION.

he recent years, one of the most significant problems in homotopy theory has e problem of finding a stable splitting of the classifying space of a finite abelian This can be reduced to the special case of an elementasy abelian *p-group* ([5]). $/p)^n_+$ be the classifying space of the elementary abelian *p-group* $(Z/p)^n$, together disjoint base point. Harris and Kuhn have given a splitting of $B(Z/p)^n_+$ into posable stable wedge summands, this is equivalent to a splitting of the 1 into e orthogonal idempotents in $F_p[M_n(Z/p)]$ ([5]). This splitting is finest but in most of the idempotents have not yet been known explicitly.

object of this paper is to describe some stable splitting of $B(Z/p)^n_+$ into stable ummands, completed at p. In a previous method to construct stable summands $p)^n_+$, the representation theory of the automorphism group of $(Z/p)^n$ is used [6]. A of the identity in $F_p[GL_n(Z/p)]$ into orthogonal idempotents is shown to induce splitting of $B(Z/p)^n_+$. Let $F^*_{p^n}$ denote the cyclic multiplicative group of units in ls F_{p^n}, it can be considered as a subgroup of $GL_n(Z/p)$. We construct explicitly itive orthogonal idempotents which sum to the 1 in $F_p[F^*_{p^n}]$ for each divisor m of

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2. MAIN RESULT.

F_{p^n}, choose an element ω so that ω generates the cyclic group of units in F_{p^n} and $, ..., \phi^{n-1}(\omega)\}$ forms a basis for F_{p^n} over F_p ([2]), where $\phi(a) = a^p$ is the Frobenius. $) = a_0 + a_1x + ... + a_{n-1}x^{n-1} + x^n$ be the minimal polynomial for ω. Let

$$\theta = \begin{pmatrix} 0 & 0 & \dots & 0 & -a_0 \\ 1 & 0 & \dots & 0 & -a_1 \\ 0 & 1 & \dots & 0 & -a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -a_{n-1} \end{pmatrix}$$

$n \times n$ matrix over F_p representing multiplication by ω in the basis $\{1, \omega, ..., \omega^{n-1}\}$. is a generator of $F^*_{p^n}$, we see that θ has order $p^n - 1$ in $GL_n(Z/p)$. Therefore we sider

$$F^*_{p^n} = <\theta> \subseteq GL_n(Z/p)$$

for each divisor m of n, we have $p^m - 1$ to be a divisor of $p^n - 1$ and

$$F_{p^m}^* =< \theta^{\frac{p^n-1}{p^m-1}} > \subseteq F_{p^n}^*$$

Since $F_{p^n}^*$ is abelian and p does not divide the order of $F_{p^n}^*$, there ar p^n - tinct one dimensional representations of $F_{p^n}^*$ defined over F_{p^n}. Label then by $j \in Z/(p^n - 1)$, with $R_j(\theta) = \omega^j$. Explicit idempotents in $F_{p^n}[F_{p^n}^*]$ associatedto th $e_j = -\sum_{k=0}^{p^n-2} \omega^{-kj}\theta^k$ ([1], 33.8).

We let $Z/m =< \phi_m >$ act on $Z/(p^m - 1)$ by $\phi_m(i) = ip$. Let $J_i(m)$ be the orbt con i, and let $I(m)$ be a set consisting of one element from each orbit. The cardinal of $z_i(m)$, where $z_i(m)$ is the smallest positive exponents k with $ip^k \equiv i(\mod p^m - 1)$. $m = n$, $J_i(m)$, $I(m)$, and $z_i(m)$ are exactly J_i, I, and z_i in [4].

Definition 2.1. *For $i \in I(m)$, let*

$$g_i(m) = \sum_{u=0}^{z_i(m)-1} \sum_{j \equiv ip^u (\mod p^m - 1)} e_j$$

Theorem 2.2. *(i) $g_i(m) \in F_p[F_{p^m}^*]$,, and (ii) the $g_i(m)$ are primitive orthogonal i tents which sum to the 1 in $F_p[F_{p^m}^*]$, i.e, $\{F_p[F_{p^m}^*]g_i(m) \mid i \in I(m)\}$ is a full set of irre representations of $F_{p^m}^*$ over F_p.* *Proof*: (i) We have

$$g_i(m) = -\sum_{u=0}^{z_i(m)-1} \sum_{k=0}^{p^n-2} \sum_{l=0}^{q-1} \omega^{-k(ip^u + l(p^m-1))}\theta^k,$$

where $q = \frac{p^n-1}{pm-1}$ and $q \equiv 1(\mod p)$, and

$$\sum_{l=0}^{q-1} \omega^{-k(ip^u + l(p^m-1))} = \begin{cases} 0 & \text{if } q \not| k. \\ q\omega^{-kip^u} & \text{if } q \mid k. \end{cases}$$

Hence $g_i(m) = -\sum_{j=0}^{p^m-2} x_{ijm}\theta^{q^j}$, where $x_{ijm} = \sum_{u=0}^{z_i(m)-1} \omega^{-qjip^u}$. We have $x_{ijn} \in F_j$ $\phi(x_{ijm}) = x_{ijm}$) and $\theta^q \in F_{p^m}^*$, therefore $g_i(m) \in F_p[F_{p^m}^*]$.

(ii). Let $E_i(m) = \{e_j \mid e_j$ occurs in $g_i(m)\}$. Then $\cup_{i \in I(m)} E_i(m) = \{e_j \mid 0 \le j \le p^n -$ $E_i(m) \cap E_{i'}(m) = \emptyset$ for any $i, i' \in I(m), i \neq i'$. Hence the $g_i(m)$ are orthogonal dem which sum to the 1. From ([4], 3.5), the idempotents $g_i(m)$ are primitive in $F_p[F_p^*$

Remarks 2.3: (i) $g_i(n)$ and $g_i(1)$ are exactly f_i in [4] and g_i in [3].

(ii) Since $F_p[F_{p^m}^*]$ is semi simple and commutative, it must be equal to a dire of fields. $F_p[F_{p^m}^*] \cong \oplus F_p[F_{p^m}^*]g_i(m)$ realizes this decomposition.

Denote the stable summand of $B(Z/p)_+^n$ corresponding to the irreducible $F_p[GL$ - module $S'_{(\lambda)}$ by $X'_{(\lambda)}$ ([5], [6], § 6). Let

$$Z_{n,m}(i) = g_i(m)B(Z/p)_+^n.$$

Colrollary 2.4.

$$B(Z/p)_+^n \simeq \vee_{i \in I(m)} Z_{n,m}(i),$$
$$Z_{n,m}(i) \simeq \vee(\lambda)\Big(\sum_{\substack{v \in I \\ v(\mod p^m - 1) \in J_i(m)}} z_v a'_{\lambda v}\Big)X'_{(\lambda)},$$

$a'_{\lambda v}$ is the number of times the representation $F_p[F^{\bullet}_{p^n}]f_i$ **occurs in a composition** for $Res^{GL_n(Z/p)}_{F^{\bullet}_{p^n}}(S'_{(\lambda)})$ and the indexing set is given in ([4], **1.5**).

roof: This follows from 2.1, 2.2 and ([4], 3.4, 4.6).

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PHÂN RÃ ỔN ĐỊNH CỦA KHÔNG GIAN PHÂN LOẠI CỦA P - NHÓM ABEN SƠ CẤP

Nguyễn Gia Định
Đại học Huế

Một trong những vấn đề có ý nghĩa nhất trong lý thuyết đồng luân là bài toán tìm phân rã ổn định của không gian phân loại của một nhóm hữu hạn, Điều này có thể về trường hợp đặc biệt của một p - nhóm Aben sơ cấp.

Trong bài báo này, bằng cách sử dụng lý thuyết biểu diễn của nhóm hữu hạn và xây một hệ tường minh các phần tử lũy đẳng trực giao có tổng bằng 1 trong nhóm $L_n(Z/p)$], chúng tôi đưa ra một phân ra ổn định của không gian phân loại của p - n aben sơ cấp $(Z/p)^n$.