

## VACANCY EQUILIBRIUM CONCENTRATION OF ANHARMONIC CRYSTALS UNDER PRESSURE.

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**Abstract:** The Gibbs energy change on forming a simple vacancy is determined. The equilibrium concentration of the face-centered cubic crystals.  $N_v$  are considered. Expression of  $N_v$  at various pressure is obtained. Specific heat, compressibility coefficients  $X_T$  and  $X_S$ , linear thermal expansion coefficient are calculated and compared with experimental data for Ar crystal with point defects. Our numerical results are also compared with Zubov's one.

### I. INTRODUCTION.

The thermodynamic properties of anharmonic crystals were considered by the moment method in [1]. The point defects in crystals play the important role for these properties and were investigated by many authors [2,3,4,5,6,7,8].

In the works [6,7,8] the anharmonic crystals with lattice defects were considered by the moment method. In this paper we consider crystals with point defects at various pressures and obtain the analytic expression for vacancy equilibrium concentration depending on pressure. From this we can have the expression for  $C_v$ ,  $C_p$ ,  $X_v$ ,  $X_S$ ,  $\alpha$ . The numerical calculation results for inert gas crystal Ar at pressures  $0,5 \cdot 10^5$  Kpa and  $1 \cdot 10^5$  Kpa are in concordance with experimental data.

### ANALYTIC EXPRESSION OF THE VACANCY EQUILIBRIUM CONCENTRATION FOR CRYSTALS WITH POINT DEFECTS.

Consider the crystal containing  $N$  atoms, in which there are  $n$  vacancies. If only take into account the interaction of particles being on the first and the second coordination spheres, we have the Helmholtz free energy  $\psi$  [7]:

$$\psi = [N - (n_1 + n_2)n]\psi_0 + nn_1\psi_1 + nn_2\psi_2 + n(B - 1)\psi_0 \quad (1)$$

where,  $n_1$  and  $n_2$  are particles being on the two first coordination spheres;  $\psi_0$  is the free energy of one particle in the ideal crystal [1];  $\psi_1$  and  $\psi_2$  are the free energy of one particle being on the first coordination sphere and the second one containing one vacancy;  $(B-1)\psi_0$  is the free energy change of a particle when leaving from the node of a lattice to form a single vacancy.

The vacancy concentration is determined from [2,5]

$$N_v = \frac{n}{N} = \exp\left\{-\frac{g_v^f}{\theta}\right\} \quad (2)$$

$g_v^f$  is the Gibbs energy change on forming a simple vacancy and equal

$$g_v^f = G(P, T) - G_0(P, T) \quad (3)$$

$$G_0(P, T) = N\psi_0 + PV_0$$

is the Gibbs free energy of the ideal crystal containing  $N$  atoms in volume  $V_0$ ;  $G$  is the Gibbs free energy of one of the real crystal having a vacancy and the volume  $V$ . Using (1), (2) and (3) we can easily have.

$$g'_\nu = -(n_1 + n_2)\psi_0 + n_1\psi_1 + n_2\psi_2 + (B - 1)\psi_0 + P(V - V_0)$$

From the condition  $N_\nu < 1$  we can find:

$$1 < B < \frac{-(n_1 + n_2)\psi_0 + n_1\psi_1 + n_2\psi_2 - \psi_0}{-\psi_0}$$

The parameter  $B$  can be found by comparing experimental values of  $N_\nu$  near the melting temperature with theoretical values calculated from (2). For simplicity, we use the following approximation expression for  $B$  by an arithmetic averaging:

$$B \cong 1 + \frac{(n_1 + n_2)\psi_0 - n_1\psi_1 - n_2\psi_2}{\psi_0}$$

Using (1) and thermodynamic relation

$$PV = -\frac{a}{3} \left( \frac{\partial \psi}{\partial a} \right)_T$$

Where  $a$  is the nearest neighbour distance between two atoms, we have:

$$PV = PV_0 - \frac{a}{3} \frac{\partial}{\partial a} \left( n \frac{\partial \psi}{\partial n} \right) \quad PV = PV_0 - \frac{a}{3} \frac{\partial n}{\partial a} \frac{\partial \psi}{\partial n} - \frac{an}{3} \frac{\partial^2 \psi}{\partial a \partial n}$$

From (1), (2) and (4) we can find

$$\begin{aligned} \frac{a}{3} \frac{\partial n}{\partial a} \frac{\partial \psi}{\partial n} &= -\frac{1}{\theta} [-(n_1 + n_2)\psi_0 + n_1\psi_1 + n_2\psi_2 + (B - 1)\psi_0] \\ &\times \left[ (n_1 + n_2) \frac{PV_0}{N} - n_1 \frac{PV_1}{N} + n_2 \frac{PV_2}{N} + (B - 1) \frac{PV_0}{N} \right] \equiv \hat{A} \\ \frac{an}{3} \frac{\partial^2 \psi}{\partial a \partial n} &= \left[ -an \frac{\partial(PV)}{\partial n} \right] = \frac{1}{X_T} \frac{\partial Y}{\partial a} \frac{\partial a}{\partial n} - a^2 P \sqrt{2N} \frac{\partial a}{\partial n} \end{aligned}$$

here  $X_T$  is the isothermal compressibility coefficient of the crystal. In [6,7]  $\frac{\partial a}{\partial n}$  is

$$\frac{\partial a}{\partial n} = \frac{1}{N} [-(n_1 + n_2)y_0 + n_1y_1 + n_2y_2]$$

Where,  $y_0$  is the displacement of a particle from the equilibrium position in the crystal;  $y_1$  (or  $y_2$ ) the one in the crystal having a defect being on the first coordinate sphere (or the second). Therefore, the expression of  $g'_\nu$  is:

$$g'_\nu = \Delta\psi - \hat{A} - [-(n_1 + n_2)y_0 + n_1y_1 + n_2y_2] \left( \frac{3}{2X_T} - P \right) \sqrt{2} a^2$$

$$\Delta\psi = \psi - N\psi_0$$

and we have the expression of  $N_\nu$  depending pressure:

$$N_\nu = \exp\left\{-\frac{\Delta\psi}{\theta}\right\} \exp\left\{\frac{1}{\theta} [-(n_1 + n_2)y_0 + n_1y_1 + n_2y_2] \left( \frac{3}{2X_T} - P \right) \sqrt{2} a^2\right\} \exp\left\{\frac{\hat{A}}{\theta}\right\}$$

crystals we always have  $\frac{A}{\theta} \ll 1$ , so we can write (12) as following:

$$N_v = N_v(P=0) \exp\left\{\frac{1}{\theta}[-(n_1 + n_2)y_0 + n_1y_1 + n_2y_2]\left(\frac{3}{2X_T} - P\right)\sqrt{2a^2}\right\} \quad (13)$$

$$N_v = N_v(P=0) \exp\left\{-\frac{-(n_1 + n_2)\psi_0 + n_1\psi_1 + n_2\psi_2 + (B-1)\psi_0}{\theta}\right\}$$

Using the thermodynamic relations and the expression of the free energy  $\psi$  (1), the solution of the vacancy equilibrium concentration we can find specific heat  $C_v$ ,  $C_p$ , thermal compressibility and adiabatic compressibility coefficients  $X_T$  and  $X_S$ , the expansion coefficient  $\alpha$  of crystals at the various pressures. In the following paragraphs, we shall apply these results to our investigation of the inert gas crystal Ar at the pressures 0,5.10<sup>5</sup> Kpa and 1.10<sup>5</sup> Kpa.

### III. THERMODYNAMIC PROPERTIES OF THE CRYSTAL Ar WITH DEFECT AT THE PRESSURE 0,5. 10<sup>5</sup> Kpa AND 1.10<sup>5</sup> Kpa.

The interaction potential between two atoms of an inert gas is often used in the form of Lennard-Jones one. In the case of the crystal argon we have the experimental  $r = 3,405.10^{-10}m$ ,  $\epsilon/k_B = 119,8^\circ$  [8]. As in [1,7] and from the formulae (6), (13) we find the values of the vacancy equilibrium concentration, the thermodynamic quantities  $N_v$ ,  $X_T$ ,  $X_S$ ,  $\alpha$  at the pressures 0,5.10<sup>5</sup> Kpa and 1.10<sup>5</sup> Kpa. The results are written in tables 1 and 2. We also write the results of the calculation by the moment method for an ideal crystal [1], by the one particle distribution function method of Zubov [9] and the experimental data [10].

In the vicinity of the melting temperature, the vacancy equilibrium concentration is small ( $\sim 10^{-3}$ ) and the discrepancy of the thermodynamic quantities in comparison with experiments is of some percent.

From the numerical result we see that the vacancy equilibrium concentration at the same temperature is decreased when the pressure is risen. The good concordance between the theoretical calculations and the experimental data (the error  $\sim$  some percent) shows that we can use the moment method in investigating crystals with defects at the same pressure. This publication is completed with financial support from National Research Program in Natural Sciences.

Table 1: Thermodynamic properties of Ar with defect at pressure 0,5.10<sup>5</sup> Kpa.

T, K	30	40	50	60	70	80	90
$N_v$	$6,33.10^6$	$1,31.10^4$	$2,57.10^4$	$7,84.10^4$	$1,5.10^3$	$1,84.10^3$	$1,62.10^3$
$r(10^{-10} m)$	3,734	3,748	3,7634	3,7804	3,79996	3,8216	3,8475
$\epsilon(M[1])$	3,734	3,748	3,7634	3,7804	3,79996	3,8217	3,8476
$\epsilon$ (Zubov [9])				3,7859		3,8222	
$\epsilon$ (exp[10])	3,7426	3,7529	3,7650	3,7793	3,7954	3,8137	3,8349
$\alpha(10^{-10} Pa^{-1})$	3,33	3,61	3,95	4,39	4,94	5,71	6,70
$\alpha(M[1])$	3,33	3,61	3,95	4,38	4,92	5,69	6,65
$\alpha$ (Zubov [9])				4,88		6,21	
$\alpha$ (exp[10])	3,47	3,85	4,15	4,52	5,00	5,59	6,49

T <sup>0</sup> , K	30	40	50	60	70	80	
N <sub>v</sub>	6,33.10 <sup>-6</sup>	1,31.10 <sup>-4</sup>	2,57.10 <sup>-4</sup>	7,84.10 <sup>-4</sup>	1,5.10 <sup>-3</sup>	1,84.10 <sup>-3</sup>	1,6
X <sub>s</sub> (10 <sup>-10</sup> Pa <sup>-1</sup> )	3,12	3,27	3,45	3,66	3,93	4,27	4
IMM	3,12	3,27	3,45	3,68	3,97	4,35	4
Zubov [9]				4,00		4,57	
Exp [10]	3,40	3,51	3,64	3,79	3,97	4,18	4
α(10 <sup>-4</sup> k <sup>-1</sup> )	2,72	3,20	3,58	3,97	4,41	5,01	5
IMM [1]	2,72	3,20	3,58	3,96	4,40	5,00	5
Zubov [9]				4,24		5,37	
Exp [10]	2,39	2,98	3,45	3,91	4,57	4,98	5
C <sub>v</sub> (JK <sup>-1</sup> mol <sup>-1</sup> )	19,07	20,99	21,87	22,30	22,47	22,55	22
IMM [1]	19,07	20,99	21,87	22,29	22,46	22,54	22
Zubov [9]				21,22		21,59	
Exp [10]	16,84	19,65	21,12	21,91	22,29	22,50	22
C <sub>p</sub> (JK <sup>-1</sup> mol <sup>-1</sup> )	20,36	23,21	25,06	26,76	28,26	30,08	30
IMM [1]	20,36	23,21	25,06	26,52	27,86	29,46	30
Zubov [9]				25,83		29,47	
Exp [10]	17,81	21,54	24,05	26,10	27,57	30,04	30

Table 2: Thermodynamic properties of A<sub>r</sub> with defect at pressure 1.10<sup>5</sup> Kpa

T <sup>0</sup> (K)	30	40	50	60	70	80	90	
N <sub>v</sub>	6,15.10 <sup>-6</sup>	1,22.10 <sup>-4</sup>	2,27.10 <sup>-4</sup>	6,68.10 <sup>-4</sup>	1,74.10 <sup>-3</sup>	3,17.10 <sup>-3</sup>	1,56.10 <sup>-3</sup>	2
a(10 <sup>-10</sup> )	3,7143	3,7270	3,7409	3,7557	3,7721	3,7907	3,812	
IMM[1]	3,7143	3,7270	3,7409	3,7557	3,7722	3,7909	3,813	
Zubov [9]				3,7557		3,7879		
Exp [10]	3,7211	3,7301	3,7406	3,7526	3,7661	3,7810	3,7978	
X <sub>r</sub> (10 <sup>-10</sup> Pa <sup>-1</sup> )	2,94	3,17	3,51	3,75	4,14	4,64	5,24	
IMM[1]	2,94	3,17	3,51	3,74	4,11	4,59	5,22	
Zubov [9]				4,03		4,85		
Exp [10]	3,15	3,34	3,57	3,83	4,13	4,52	5,00	
X <sub>s</sub> (10 <sup>-10</sup> Pa <sup>-1</sup> )	2,78	2,89	3,12	3,21	3,40	3,64	3,91	
IMM	2,78	2,89	3,12	3,21	3,41	3,66	3,97	
Zubov [9]				3,42		3,77		
Exp [10]	3,01	3,09	3,18	3,30	3,40	3,56	3,73	
α(10 <sup>-4</sup> k <sup>-1</sup> )	2,35	2,78	3,02	3,38	3,71	4,11	4,56	
IMM [1]2,35	2,78	3,02	3,38	3,69	4,07	4,54	5,20	
Zubov [9]				3,35		4,24		
Exp[10]	2,05	2,57	2,97	3,32	3,67	4,05	4,49	

	30	40	50	60	70	80	90	100
	$6,15 \cdot 10^{-6}$	$1,22 \cdot 10^{-4}$	$2,27 \cdot 10^{-4}$	$6,68 \cdot 10^{-4}$	$1,74 \cdot 10^{-3}$	$3,17 \cdot 10^{-3}$	$1,56 \cdot 10^{-3}$	$2,04 \cdot 10^{-3}$
mol <sup>-1</sup> )	18,56	19,73	21,20	22,00	2,19	22,24	22,19	22,06
[1]	18,56	19,73	21,20	22,00	22,17	22,21	22,17	22,04
ov				21,14		21,68		
[10]	16,34	19,32	20,82	21,66	2,12	22,38	22,50	22,54
mol <sup>-1</sup> )	19,65	21,66	23,88	25,71	26,98	28,32	29,77	31,70
[1]	19,65	21,66	23,88	25,60	26,73	27,86	29,16	30,07
[9]				21,95		27,84		
[10]	17,10	20,87	23,34	25,18	26,82	28,41	30,13	32,14

IMM/L ideal moment method

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ĐI KHOA HỌC, KHTN, ĐHQGHN, t.XII, n<sup>o</sup> 4, 1996

## NGHIÊN CỨU QUÁ TRÌNH BIẾN ĐỔI NĂNG LƯỢNG TỰ DO KHI TẠO THÀNH MỘT VACANCY.

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ngiên cứu nồng độ vacancy cân bằng  $N_v$  đối với các tinh thể lập phương tâm diện t ra được biểu thức đối với  $N_v$  ở các áp suất khác nhau. Tính được nhiệt dung, nén  $X_T$  và  $X_S$  hệ số giãn nở tuyến tính, và thực hiện so sánh với các số liệu thực m đối với  $A_r$  có khuyết tật điểm. Các kết quả tính số của chúng tôi cũng được so với các kết quả của Zubov.