Empirical Test of Put - call Parity on the Standard and Poor’s 500 Index Options (SPX) over the Short Ban 2008

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Abstract: Put call parity is a theoretical no-arbitrage condition linking a call option price to a put option price written on the same stock or index. This study finds that Put call parity violations are quite symmetric over the whole sample. However during the ban period 2008 in the U.S., puts are significantly and economically overpriced relative to calls. Some possible explanations are the short selling restriction, momentum trading behaviour and the changes in supply and demand of puts over the short ban. One interesting finding is that the relationship between time to expiry, put call parity deviations and returns on the index is highly non-linear.

Keywords: Put-call parity, SPX, short ban 2008.

1. Introduction

Section one gives a background to Put call parity (henceforth, PCP) and reviews relevant literature. Section two is the data part and the methodology adopted in the research. Section three discusses the empirical evidence. Section four investigates the link between PCP violations, trading momentum behaviour and explains others possible reasons. The final part makes some concluding remarks.

PCP condition was given in [1] that shows the relationship between the price of a European call and a European put of the same underlying stock with the same strike price and maturity date [2]. PCP for non-paying dividend options can be described as followed:

\[ c + K \exp (-r \tau) = p + S_t \]  (1)

Where:
- \( c \) and \( p \) are the current prices of a call and put option, respectively
- \( K \): the strike price
- \( S_t \): the current price of the underlying
- \( r \): the risk free rate
- \( \tau \): time to expiry

If the relationship does not hold, there are two strategies used to eliminate arbitrage opportunities. Consider the following two portfolios.

- **Portfolio A**: one European call option plus an amount of cash equal to \( K \exp (-r \tau) \)
- **Portfolio B**: one European put option plus one share

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Table 1. Arbitrage strategy based on PCP and its cash flow

<table>
<thead>
<tr>
<th>Long strategy (i.e. portfolio A is overpriced relative to portfolio B)</th>
<th>Short strategy (i.e. portfolio A is under-priced relative to portfolio B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short securities in A and buy securities in B simultaneously</td>
<td>Buy securities in A and short securities in B simultaneously</td>
</tr>
<tr>
<td>- Write a call</td>
<td>- Buy a call</td>
</tr>
<tr>
<td>- Buy a stock</td>
<td>- Short a stock</td>
</tr>
<tr>
<td>- Buy a put</td>
<td>- Write a put</td>
</tr>
<tr>
<td>- Borrow $K\exp\left(-r\tau\right)$ at risk free rate for $\tau$ time</td>
<td>- Invest $K\exp\left(-r\tau\right)$ at risk free rate for $\tau$ time</td>
</tr>
</tbody>
</table>

It leads to an immediate positive cash flow of $c + K\exp\left(-r\tau\right) - p - S_t > 0$ and a zero cash flow at expiry.

Dividends cause a decrease in stock prices on the ex-dividend date by the mount of the dividend payment [2]. The payment of a dividend yield at a rate $q$ causes the growth rate of the stock price decline by an amount of $q$ in comparison with the non-paying dividend case. In other words, for non-paying dividend stock, the stock price would grow from $S_t$ today to $S_T\exp(-q\tau)$ at time $T$ [2].

To obtain PCP for dividend-paying options, we replace $S_t$ by $S_t\exp(-q\tau)$ in equation (1):

$$c + K\exp\left(-r\tau\right) = p + S_t\exp(-q\tau)$$ (2)

2. Data and methodology

2.1. Data description

All options data is provided by OptionMetrics from 2nd September 2008 to 31st October 2008 with total of 16428 option pairs.

- Transaction costs of index arbitrage, the result from [3]’s research about SPX from 1986 to 1989 is applied. Transaction cost including commissions bid-ask spreads is around on average 0.38% of S&P 500 cash index.

- Risk – free rate: For options with time to expiry less than 12 months, daily annualised bid yield of US Treasury Bills with the matching durations is used. For options with longer time to expiry, zero coupon yields take the role of the risk-free rate. The data set is extracted from EcoWin database.

  - Dividend yields: Dividend payments on S&P 500 were paid on the last days of each quarter. During the sample period, one dividend payment was paid on 30 June 2008, as a result, for all options expired before 30 September 2008, the underlying asset did not pay dividend. For other options, the expected annualized dividend yields are estimated as 2.01% (based on the dividend historical data).

2.2. The approach adopted for identifying PCP deviation

We begin with the PCP formalised in Stoll [1], however allowing for presence of dividend, bid-offer spreads and transaction costs. Throughout the research, the following notations are adopted:

  - $c$: price of a European call option on the S&P500 index option with a strike price of $K$;
  - $p$: price of an identical put option;
  - $S_t$: current price of one S&P500 share;
  - $dy$: dividend yield on S&P500 share;
  - $T$: transaction costs for index arbitrage;
  - $r$: risk free rate
  - $\tau$: tau – time to expiry

Consider two following portfolios:

Portfolio A: one European call option plus an amount of cash equal to $K\exp(-r\tau)$. 

Portfolio B: one European put option plus an amount of cash equal to $S_t\exp(-q\tau)$. 

The difference in values between Portfolio A and Portfolio B with the same maturity results from the option and dividend payments.
**Portfolio B**: one European put option plus an amount of \( \exp(-q \tau) \) shares with dividends on the shares being reinvested in additional shares.

PCP implies the net profit from any riskless hedge should be non-positive from long strategy:

\[
c + K \exp(-r \tau) - p - S \exp(-d \tau) - T \leq 0 \quad (3)
\]

Similarly, PCP implies from short strategy:

\[
p + S \exp(-d \tau) - c - K \exp(-r \tau) - T \leq 0 \quad (4)
\]

Option prices at the midpoint of the spread are used in this research, i.e. the average of the bid and ask prices. Similarly, \( St \) – the current value of the index is estimated at the midpoint prices.

**2.3. Short sales ban and the period sample**

There are nearly 1000 financial stocks in the shorting ban list in September 2008 in which 64 stocks belong to the S&P 500 portfolio accounting for around 15% of the index’s total market capitalisation [4-7]. Adopting the timeline of events of [8], the period sample is divided into three sub-periods:

<table>
<thead>
<tr>
<th>Dummy variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>dum_preban</td>
<td>( = 1 ) for the period from 2(^{rd}) to 18(^{th}) September 2008 ( = 0 ) otherwise</td>
</tr>
<tr>
<td>dum_ban</td>
<td>( = 1 ) for the period from 19(^{th}) September to 8(^{th}) October 2008 ( = 0 ) otherwise</td>
</tr>
<tr>
<td>dum_postban</td>
<td>( = 1 ) for the period from 9(^{th}) to 31(^{st}) October 2008 ( = 0 ) otherwise</td>
</tr>
</tbody>
</table>

**2.4. Calculating the profitability of PCP violations**

On STATA, I generate two portfolios A and B as discussed in 3.1. Four variables represented for PCP violations in the research may confuse readers, therefore I supply here a list of dependent variables used in the research to make it clear. Two newly generated variables are \( A_{\text{less}}B \) and PCPdeviation are used in section 3. The two remaining including deviation and dev will used in section 4.

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{\text{less}}B )</td>
<td>( = c + K \exp(-r \tau) - p - S \exp(-d \tau) )</td>
<td>PCP deviation ignoring transaction cost</td>
</tr>
<tr>
<td>PCPdeviation</td>
<td>( = A_{\text{less}}B + 0.0038 s ) if ( A_{\text{less}}B &lt; 0 ) or ( = A_{\text{less}}B - 0.0038 s ) if ( A_{\text{less}}B &gt; 0 )</td>
<td>PCP deviation including transaction cost</td>
</tr>
<tr>
<td>deviation</td>
<td>( = A_{\text{less}}B / s )</td>
<td>PCP violation as a proportion of the underlying price but eliminating all observations which belong to the interval ([-1.38%, +1.38%])</td>
</tr>
<tr>
<td>dev</td>
<td>( = \text{PCPdeviation} \times 100 / s )</td>
<td>PCP deviation including transaction cost as a proportion of the underlying price</td>
</tr>
</tbody>
</table>

Figure 1 show the histogram is quite symmetric in which nearly 50% of deviations is on either side. The mean of the PCPdeviation is $0.852 showing that the calls are slightly overpriced with the average profit generated by applying the long strategy is $0.852. It seems to be that PCP holds, on average, however, there are some economically significant violations.
As we can see from Figure 2, the mean of profit from PCP deviations during the ban period is negative ($-3.114757$) - it implies that, on average, portfolio B is overpriced relative to portfolio A. Moreover, the number of instances with positive profit from adopting the short strategy is 2844 accounting for 55.76% of total number of PCP violations during the ban period.

3. Empirical result

Statistical tests of PCP

The analysis is similar in spirit to that of Stoll [1], Mittnik and Rieken [9], who based on the regression equation:

\[ C_t - P_t = a_0 + a_1(I_t - Ke^r) + u_t \]  \hspace{1cm} (5)

This is a rearrangement of the PCP (i.e. Equation 1). PCP implies that coefficients $a_0$ and $a_1$ should be 0 and 1, respectively. The key difference of this research is that dividend and the dum_ban variable are added to examine the effect of the shorting ban on PCP. The regression equation as follows:

\[ C_t - P_t = a_0 + a_1(I_t - Ke^r) + a_2dum\_ban + u_t \]  \hspace{1cm} (6)

I estimate the regression Equation 8 by using OLS called Model 1. Option “robust” in STATA is used to avoid heteroscedasticity.

R² is 99.03% indicates that the regression fits well. The slope coefficient is quite close to 1- the theoretical expectation as Figure 3. The positive intercept is strongly significant that suggests that call options are systematically overpriced relative to puts, ceteris paribus. This result is contrast to Mittnik’s study [9] or Vipul’s result [10] in which put options are systematically overpriced more often and more significant. However, by adding dum_ban variable - there are some changes in economic interpretation:
is negative showing that during the ban, put options are likely overvalued, ceteris paribus.

- The absolute value of $\theta_2$ is greater than the absolute value of $\theta_0$, thus the combination effect is mixed. During the ban, puts are overpriced, otherwise, calls are overpriced, ceteris paribus.

- This result is consistent with Ofek’s conclusion that short sale restrictions causing limited arbitrage pushes PCP violations to be asymmetric towards overpricing puts [8].

- PCP implies that coefficients $a_0$ and $a_1$ should be 0 and 1, respectively. As the F-test done on STATA, p-value =0.0002 < 0.05 implies that $a_1$ is strongly significant different from 1 so PCP is statistically violated.

4. Explaining pcp violations

Index is essentially an imaginary portfolio of securities representing a particular market or a portion of it so investing and shorting an index are quite different from these investment strategy of ordinary individual stock. One question is how these differences of index trading affects index-PCP. Moreover, I suggest a link between PCP deviations and behavioural finance.

4.1. Investing in an index

There are three possible ways to mirror the index performance.

- Indexing is establishing a portfolio of securities that best mirrors an index. This method is costly and demanding when it involves a huge number of trading transactions.

- Buying index fund is a cheaper way to replicate the performance of an index. The first index fund tracking the S&P 500 was born in 1967 by the Vanguard Group [11]. Various new ones are Columbia Large Cap Index Fund (ticker – NINDX ), Vanguard 500 Index Fund (VFINX), DWS Equity 500 Index Fund (BTIEX), USAAS&P 500 Index Fund(USSPX) [12].

- Exchange–traded fund (henceforth ETF)- This is a security tracking one particular index like an index fund, however, it can be traded on exchange- like a typical stock with some important characteristics.

  + ETFs are priced intraday since they are actively traded throughout the day. As a result, owning ETFs, traders can take advantages of not only diversification of index funds but also the flexibility of a stock.

  + The price of an ETF reflects its net asset value (NAV), which takes into account all the underlying securities in the fund, although ETFs attempt to mirror the index, returns on ETF are not exactly same as the index performance, for instance, 1% or more deviation between the actual index’s year-end return and the associated ETFs is common [13].

SPY consistently remains the leading U.S – listed ETF, moreover, SPY together with QQQQ -Nasdaq-100 Index Tracking Stock- are the most traded and liquid stocks in the US market (www.stocks-options-trading.com). Besides SPY, there are at least 10 alternatives for traders investing in S&P500.

Table 4. 10 alternatives to SPY

<table>
<thead>
<tr>
<th>Name</th>
<th>Ticker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 RevenueShares Large Cap ETF</td>
<td>RWL</td>
</tr>
<tr>
<td>2 WisdomTree Earnings 500 Fund</td>
<td>EPS</td>
</tr>
<tr>
<td>3 First Trust Large Cap Core AlphaDEX</td>
<td>FEX</td>
</tr>
<tr>
<td>4 PowerShares Dynamic Large Cap Portfolio</td>
<td>PJF</td>
</tr>
<tr>
<td>5 ALPS Equal Sector Weight ETF</td>
<td>EQL</td>
</tr>
<tr>
<td>6 Rydex S&amp;P Equal Weight ETF</td>
<td>RSP</td>
</tr>
<tr>
<td>7 UBS E-TRACS S&amp;P 500 Gold Hedged ETN</td>
<td>SPGH</td>
</tr>
<tr>
<td>8 ProShares Credit Suisse 130/30</td>
<td>CSM</td>
</tr>
<tr>
<td>9 WisdomTree LargeCap Dividend Fund</td>
<td>DLN</td>
</tr>
<tr>
<td>10 iShares S&amp;P 500 Index Fund</td>
<td>IVV</td>
</tr>
</tbody>
</table>
4.2. Shorting an index

There are at least four approaches to short sell an index. First of all, shorting directly all securities of the index is similar to indexing that is very costly. Secondly, traders also short ETFs, for instance, one investor can short ETFs indexing S&P 500 as he/she expects the index down.

In addition, there are investment options that investors can go long but get the same results as direct shorting. They are inverse index mutual funds and inverse ETFs. These inverse fund attempt to track an index; “only their case they track the negative of a multiple of the negative of an index”[13]. For example, if the S&P 500 falls 1% today, the Ryder Inverse S&P 500 (RYURX) will rise 1%, beside that inverse-fund issuers offer a range choices such as 1.5x, and 2x leveraged ETFs, funds. URPIX – 2x inverse the S&P 500 of Profunds, for instance, will increase 2% if the index declines 1% [14].

Table 5. Inverse ETFs and inverse funds of S&P 500 index

<table>
<thead>
<tr>
<th>Name</th>
<th>Ticker</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proshares Short S&amp;P500</td>
<td>SH</td>
<td>1x Inverse ETFs</td>
</tr>
<tr>
<td>Proshares UltraShort S&amp;P500</td>
<td>SDS</td>
<td>2x Double Inverse ETFs</td>
</tr>
<tr>
<td>Ryder Inverse S&amp;P 500</td>
<td>RYURX</td>
<td>1x Inverse Mutual Funds</td>
</tr>
<tr>
<td>Rydex Inverse S&amp;P 500 2x</td>
<td>RYTPX</td>
<td>2x InverseMutual Funds</td>
</tr>
<tr>
<td>ProFunds Bear Inv</td>
<td>BRPIX</td>
<td>1x Inverse Mutual Funds</td>
</tr>
<tr>
<td>ProFunds UltraBear Inv</td>
<td>URPIX</td>
<td>2x InverseMutual Funds</td>
</tr>
<tr>
<td>Direxion funds, S&amp;P 500 Bear 1X F</td>
<td>PSPSX</td>
<td>1x Inverse Mutual Funds</td>
</tr>
<tr>
<td>Direxion Monthly S&amp;P 500 Bear 2X Inv</td>
<td>DXSSX</td>
<td>2.5x InverseMutual Funds</td>
</tr>
<tr>
<td>Ryder Inverse 2x S&amp;P 500</td>
<td>RSW</td>
<td>2x Double Inverse ETFs</td>
</tr>
</tbody>
</table>

Source: www.stockrake.com and www.associatedcontent.com

4.3. Inverse funds and effects on PCP of SPX

How inverse ETFs and inverse mutual fund work

Inverse ETFs are ideal for high-frequency traders who involve hundreds of orders everyday due to daily “reset” mechanism of these products. It means that “investors mush cash out to get the proper return”[13]. Inverse ETFs do not short individual company stocks directly, inverse ETFs utilize futures, swaps, options and other derivatives to achieve desired effects [15]. ProShares Short S&P 500 (SH) rely significantly on swaps to get short exposure – 91% of its total exposure is driven by swaps position and futures account for 9% to create inverse ETFs [15]. On the other hand, the Ryder ETFs are basically traded on options. In the case of using swaps, the inverse funds agree to pay a fixed amount and receive an amount depending on the performance of a stock index. When there is a decline in the index, the counterparty payments increase. Famous swap banks including Goldman Sachs, Morgan Stanley or Merrill Lynch are the typical counterparty. The counterparty directly short sell stocks in the index to hedge out its risk [15].

Effect of short selling ban on short sale activity on the S&P 500

Shorting directly the S&P 500 portfolio-seems to be a mission impossible because 65 stocks of the index were included in the ban list. While investors are unable to short nearly 1000 financial stocks, S&P 500 traders still have some other ways to short the index including: shorting ETFs, buying inverse unit funds as discussed above. Therefore from the short sell restrictions perspective, PCP of SPX should be less violated than PCP of stock option.

The short ban 2008 also impedes swap banks to short completely the S&P 500
portfolio. The counterparties cannot hedge away the exposure, as a result, they are less willing to write swap agreements. For instance, at least one inverse fund must stop trading because it could not find counterparties in the financial crisis 2008 [15]. However, trading volume of inverse ETFs still increased dramatically after the short ban was announced. Trading volume of Proshares Short S&P500 inverse ETFs (SH) – one of the most favourite S&P500 inverse ETFs - increased substantially over the sample period (as Figure 4). The average daily trading volume of SH in September and October 2008 is around 1,168,295 – four times higher than the figure of one year previous. It is hard to say exactly how difficult to short the index during the ban period, however, certainly, investors still able to short the index over the short ban period.

The empirical test in Section 3.3 suggests that over the whole sample, calls are overpriced relative to puts, however, puts are overvalued during the ban. To be more precise, the right hand side of Equation 2 is more likely to be greater than the left hand side.

\[ c + K \exp(-r \tau) = p + S_t \exp(-q \tau) \] (2)

The first reason for this is short sale difficulties when the short ban was applied. The analysis above suggests that the short selling ban affects the index not as severe as on ordinary stocks, and investors still can short. There should be other reasons for overpricing of the puts, possibly, behavioural finance.

I already generated \( A_{less B} \) variable proxy for the pure PCP deviations. I assume that most investors use ETFs, index funds, inverse funds to arbitrage the S&P 500 rather than shorting or indexing directly. These assets attempt to track the index, however, it is common for 1% difference between them and the S&P 500 that possibly causing PCP deviations. Moreover, transaction costs charge average 0.38% of S&P 500 cash index on arbitrageurs so deviations in the interval [-1.38%, +1.38%] of the underlying price are acceptable i.e. consistent with PCP.

I generate a new variable called: \( \text{deviation} = A_{less B} / s \). This variable represents PCP deviations as a proportion of the index value. Hence, I eliminate all deviations in the interval [-1.38%, +1.38%].

There are 1689 out of 2576 instances of PCP violations (approximately 65.57%) in which puts are overpriced during the ban. Figure 5 and 6 show that after eliminating observations assuming to be consistent with PCP, the pattern of deviation does not change.

### 4.4. Behavioural finance and PCP

#### 4.4.1. Introduction about behavioural finance

Behavioural finance has become increasingly important in explaining price fluctuations in stock market in which investors are driven by not only financial motivations but also psychology.

Recently, there are some studies focusing on positive feedback trading in the options market [16, 17]. Amin et al [16] investigated the relation between option prices of OEX written on S&P 100 index and past stock market movement. They used implied volatility as a proxy for overpricing. Amin et al (2004) reported that calls are significantly overpriced relative puts after large stock increases and reverse, puts are overvalued after a significant decrease in stock prices [16]. One point should be noted here is that when the underlying prices decline, obviously put prices will increase reflecting profit from the downward trend, however, the overpricing mentions above indicating an increase in put prices excess what it should be. One of reason for the overpricing is trend chasing or feedback trading as suggested by Shiller (2003) [18].

#### 4.4.2. Timeline events

Figure 7 shows that the index declined dramatically from 1274.98 point to 968.75 point – a decrease of 24% over the two months in which this index plunged more substantially and sharply during the ban – a decline of approximately 24.58% from 19 Sep 2008 to 8 October 2008. The significant downward trend
in the index value can explain the overpricing of puts over the ban period due to feedback effects or momentum-trading behaviour.

4.4.3. Empirical test of momentum trading behaviour

I generate a new variable named return- it is daily return on the S&P 500 index calculated by the following formula:

\[ \text{return}_t = \frac{S_t - S_{t-1}}{S_{t-1}} \times 100 \]

in which \( S_t \) is the closing value of the index on day \( t \) and \( S_{t-1} \) is the closing value of day \( t-1 \). Figure 8 shows a relationship between returns on the index and PCP violation in which puts tend to be overpriced (i.e. the value of \( \text{PCPdeviation} \) variable is negative) when returns on the index are negative and reverse, calls tend to be overvalued (i.e. the value of \( \text{PCPdeviation} \) variable is positive) when returns on the index are positive. This result is consistent with Amin et al’s study [16] and will be reinforced by OLS regression. I generate a new variable named “dev” which measure PCP deviations as a proportion of the underlying price as follows:

\[ \text{dev} = \frac{\text{PCPdeviation}}{S} \times 100 \]

The relationship between PCP deviations and time to expiry looks like a curve rather than linear relation, hence, to combine the maturity effect of PCP, I add \( \tau \) and \( \tau^2 = \tau^2 \) to the model 2. We have model 3 as follows:

\[ \text{dev}_i = a_0 + a_1 \times \text{return}_i + a_3 \times \tau + a_4 \times \tau^2 + u_i \]

Adjusted R-squared = 0.7334 – it increases from 0.7063 (R-squared of model 2) to 0.7334 so time to expiry is also an important variable.

STATA result

hetest

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
Ho: Constant variance
Variables: fitted values of dev
chi2(1) = 16.62
Prob > chi2 = 0.0000
.
reg dev return tau tau2, robust
Linear regression

| | Coef. | Std. Err. | t | P>|t| | [95% Conf. Interval] |
|---|---|---|---|---|---|
| dev1 | .3871455 | .0016759 | 231.00 | 0.000 | .3838605 .3904305 |
| tau | .5380065 | .0596039 | 9.03 | 0.000 | .4211765 .6548365 |
| tau2 | -.5808008 | .0297494 | -19.52 | 0.000 | -.6391129 -.5224887 |
| _cons | .3080912 | .0124981 | 24.65 | 0.000 | .2835936 .3325889 |

Figure 7 and 8 are very similar so the relationship between PCP violation and return on S&P 500 does not change when we consider PCP violation as a proportion of the underlying price. I run a regression in which \( \text{dev} \) proxy for PCP deviation is the dependent variable and \( \text{return} \) is the explanatory variable. The regression equation for model 2 as follows:

\[ \text{dev}_i = a_0 + a_1 \times \text{return}_i + u_i \]
**Economic interpretation of coefficients:**

\( \hat{\beta}_1 = 0.3871455 \) is also significantly different from 0 indicating the positive relationship between return on the underlying asset and the value of PCP deviation. The result confirms momentum trading behaviour in the sample. Due to the intercept is quite small, when return is positive, PCP deviation is predicted to be positive (i.e. call is overpriced) and reverse. Moreover \( \hat{\beta}_1 \) is the elasticity of return on PCP deviation, when return increases 1% point, the value of PCP deviation will increase 0.387% point (0.387% point deviation towards the direction that call is overpriced), ceteris paribus. Furthermore, the greater fluctuations in the underlying asset prices are, the more severe PCP is violated, for example if the return is a big negative number, arbitrageurs can generate huge riskless by employing the short strategy.

- The maturity effect: Both the coefficients associated with \( \tau \) and \( \tau^2 \) are individually and jointly significant, as a result, the relationship between time to expiry and PCP deviation is presented as a curve rather than a straight line (confirmed by F-test with \( p \)-value=0.000). By using the command “nlcom”, we can find the turning point of the curve:

\[
\text{test } \tau \text{ tau2}
\]

\[
\begin{align*}
( 1) & \quad \tau = 0 \\
( 2) & \quad \tau^2 = 0 \\
F(  2, 16424) =  637.29 \\
\text{Prob > } F = 0.0000
\end{align*}
\]

\[
. \text{nlcom}_{\tau_{\text{turning point}}} = \frac{-b[\tau]/(2*b[\tau^2])}{b[\tau]/(2*b[\tau^2])}
\]

| dev | Coef. | Std. Err. | t | P>|t| | [95% Conf. Interval] |
|-----|-------|-----------|---|-----|-----------------|
| tau_turning_point | 0.4631592 | 0.0297825 | 15.55 | 0.000 | 0.4047824 to 0.5215361 |

The result shows that when time to expiry \( \tau = 0.46316 \) – around 169 days, the value of PCP deviation is highest, after that the longer time to expiry, the more overvalued put. By using the result from model 3, I draw a line that PCP holds exactly (i.e. dev=0). Let dev=0, value of \( \tau \) ranges from 0 to 4 years, I use the Goal seek function on Excel to find the corresponding value of return.

According to Figure 9, we can generate a simple trading rule based on prediction from model 3. PCP holds exactly for all points along the red line. All points above the red line indicates that call is overpriced while the underneath area implies that put is overpriced, therefore traders can easily use appreciate strategy to arbitrage PCP violations.

### 4.5. Supply and demand of puts during the ban

The question whether trading on options can substitute for short selling underlying asset thus is considered by many researchers after the ban was announced [19, 20]. Blau and Wade (2009) documented that when short sellers face high costs of borrowing stocks, the demand of put option is likely to rise [19].

However, who will be willing to write puts during the short ban? The nature of writing put is a party with advantages of low shorting costs for example “an institution with ability to borrow stock in house” [19] As we known about “delta hedging”, when a call buyer hold call options, he or she must short sell a delta units of the underlying asset per each unit of calls to hedge the position. Similarly, put
writers also short the underlying stock to hedge their risk. As a result, the short ban limits the put supply to some extent. The combined effects of short ban on put options market is an increase in put demand and a decline in put supply. Grundy et al examined which effect is stronger by tracking put option volume [19]. However, based on a basic demand-supply theory, we can see these effects above pushing put prices up. This idea partly explains for the overpricing of puts over the ban period in line with PCP violations during the ban.

5. Conclusion

Although attempting to replicate the real financial market by considering dividend, time to expiry, trading momentum, some factors have not been taken into account that may constraint traders to arbitrage PCP violations. Firstly, borrowing rates do not equal lending rates. Moreover, constraints on the use of short-sale proceeds, the presence of taxation, dividends on the index are not known, must be estimated – all of these make arbitrage opportunities no longer riskless. From my point of view, the real PCP violations are less severe and less frequent as empirical results. Furthermore, due to working on daily data so the research cannot investigate the effect of delay in order execution on PCP. The trading rule could be more realistic when investors can generate arbitrage profit, for example, every minute if intraday data is examined.

References

APPENDIX: Figure Captions

Figure 1. Histogram of PCPdeviation over the whole sample.

Figure 2. Histogram of PCPdeviation during the ban period.
Figure 3. The fitted line.

Note: $x = e^{by} - Ke^{rt}$

Figure 4. Daily trading volume of SH over the short ban period.

Source: http://finance.yahoo.com/q/hp?s=SPY+Historical+Prices
Figure 5. Histogram of deviation over the whole sample.

Figure 6. Histogram of deviation over the ban period.
Figure 7. S&P 500 over last five years.

Figure 8. Lowess smoother of PCP violation against return on S&P 500 (1).
Figure 9. Lowess smoother of PCP violation against return on S&P 500(2).

Figure 10. Relationship between return on SP500, time to expiry and PCP.